Institute of Actuaries of India

CT4 – Models

Indicative Solution

November 2008

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

1.

(a)

(i) Age of the birthday falling in the policy year of death .
Rate Year = Policy Year

(ii) Age at nbd at the policy anniversary , falling in the calendar year of death
Rate Year = Calendar Year

(iii) Age last birthday at the 1st of July falling in the calendar year of death.
Rate Year = Calendar Year

(b) The maximum likelihood estimate of force of mortality

$$\mu = \frac{d}{E_x^c} = \frac{40}{32000} = 0.00125$$

(Total 5 Marks)

2.

(i) $\overset{o}{e}_x = \int_0^\infty t.f_x(t)dt = \int_0^\infty t._t P_x.\mu_{x+t} dt$

$$= \int_0^\infty t..\left(-\frac{\partial}{\partial t}{}_t P_x\right)dt$$

$$= -\left[t._t P_x\right]_0^\infty + \int_0^\infty {}_t P_x dt$$

$$= \int_0^\infty {}_t P_x dt$$

(ii) ${}_t P_x = \exp\{-\int_0^t \mu_{x+s} ds\}$

$$= \exp\{-\int_0^t \mu ds\} = \exp\{-[\mu s]_0^t\} = \exp\{-[\mu t]\}$$

As per (i)

$$\overset{o}{e}_x = \int_0^\infty \exp[-\mu t]dt = [\frac{\exp(-\mu t)}{-\mu}]_0^\infty$$

$= 1/\mu = 1/0.02 =$ 50 years

(Total 6 Marks)

3.

(i)

Type I censoring:

If the censoring times $\{c_j\}$ are known in advance (a degenerate case of random censoring), then the mechanism is called "Type I censoring".

Type II censoring:

If observation continues until a pre-determined number of deaths has occurred, then "Type II censoring" is said to be present. This can simplify the analysis, because then the number of events of interest is non-random.

(ii)

tj	nj	cj	dj	dj/nj	$\Lambda_t = \sum \frac{d_j}{n_j}$	$\exp(-\Lambda_t)$
0	*20*	*0*	*0*	*0.000*	*0.000*	*1.000*
4.5	20	3	2	0.100	*0.100*	*0.905*
10.5	15	6	2	0.133	*0.233*	*0.792*
31.5	7	2	1	0.143	*0.376*	*0.686*
43.5	4	0	1	0.250	*0.626*	*0.535*
46.5	3	2	1	0.333	*0.960*	*0.383*

(iii)

Two similarities:

- High initial level of failures, similar to high infant rate mortality
- Increasing failure for older good, similar to riser mortality in older age

(Total 9 Marks)

4.

(a)

(i) When $i \neq j$.

Integrated form of the Kolmogorov backward equations

$$P_{ij}(s,t) = \sum_{l \neq i} \int_0^{t-s} e^{-\int_s^{s+w} \lambda_i(u)\,du} \mu_{il}(s+w) P_{lj}[s+w,t]\,dw$$

or

$$P_{ij}(s,t) = \sum_{l \neq i} \int_0^{t-s} P_{-ii}(s, s+w)\mu_{il}(s+w)P_{lj}[s+w, t]\,dw$$

Integrated form of the Kolmogorov forward equations

$$P_{ij}(s,t) = \sum_{k \neq j} \int_0^{t-s} P_{ik}(s, t-w)\mu_{kj}(t-w)e^{-\int_{t-w}^{t} \lambda_j(u)\,du}\,dw$$

or

$$P_{ij}(s,t) = \sum_{k \neq j} \int_0^{t-s} P_{ik}(s, t-w)\mu_{kj}(t-w)P_{-jj}(t-w, t)\,dw$$

(ii) When i = *j*

Integrated form of the Kolmogorov backward equations

$$P_{ii}(s,t) =$$

$$\sum_{l \neq i} \int_0^{t-s} P_{-ii}(s, s+w)\mu_{il}(s+w)P_{lj}[s+w, t]\,dw + P_{-ii}(s,t)$$

Integrated form of the Kolmogorov forward equations

$$P_{ii}(s,t) =$$

$$\sum_{k \neq j} \int_0^{t-s} P_{ik}(s, t-w)\mu_{kj}(t-w)P_{-ii}(t-w, t)\,dw + P_{-ii}(s,t)$$

(b) Derivation of the integrated form of the forward equations

(i) When i $\neq$ *j*. and i = *j*

Start with the forward equations:

$$\frac{\delta}{\delta t}P_{ij}(s,t) = \sum_k P_{ik}(s,t)\mu_{kj}(t)$$

We are aiming for an expression that gives *pij*(*s*,*t*) in terms of the other transition probabilities, so we first rewrite the forward equation in the form:

$$\frac{\delta}{\delta t} P_{ij}(s,t) - P_{ij}(s,t)\mu_{jj}(t) = \sum_{k \neq j} P_{ik}(s,t)\mu_{kj}(t)$$

Also note that $\mu_{jj}(t) = -\sum_{k \neq j} \mu_{jk}(t) = -\lambda_j(t)$ so that:

$$\frac{\partial}{\partial t} p_{ij}(s,t) + p_{ij}(s,t)\lambda_j(t) = \sum_{k \neq j} p_{ik}(s,t)\mu_{kj}(t)$$

The integral of the coefficient is the definite integral $\int_s^{\cdot} \lambda_j(u)\,du$

and the integrating factor is $e^{\int_s^t \lambda_j(u)du}$. We therefore multiply by this integrating factor to give:

$$e^{\int_s^t \lambda_j(u)du} \frac{\partial}{\partial t} p_{ij}(s,t) + e^{\int_s^t \lambda_j(u)du} p_{ij}(s,t)\lambda_j(t) = e^{\int_s^t \lambda_j(u)du} \sum_{k \neq j} p_{ik}(s,t)\mu_{kj}(t)$$

$$\frac{\partial}{\partial t}\left[p_{ij}(s,t) e^{\int_s^t \lambda_j(u)du} \right] = e^{\int_s^t \lambda_j(u)du} \sum_{k \neq j} p_{ik}(s,t)\mu_{kj}(t)$$

Changing the variable from t *to v* and integrating both sides of the equation wrt to v , between the limits of s and t , the equation becomes,

$$p_{ij}(s,t) e^{\int_s^t \lambda_j(u)du} = p_{ij}(s,s) + \int_s^t e^{\int_s^v \lambda_j(u)du} \sum_{k \neq j} p_{ik}(s,v)\mu_{kj}(v)\,dv$$

The main integral on the right-hand side can be changed with the substitution $v = t - w$ Then:

$$dv = -dw$$

$$v = s \Rightarrow w = t - s$$

and:

$$v = t \Rightarrow w = 0$$

So we have:

$$p_{ij}\left(s,t\right) e^{\int_s^t \lambda_j(u) du} = p_{ij}\left(s,s\right) + \int_0^{t-s} e^{\int_s^{t-w} \lambda_j(u) du} \sum_{k \neq j} p_{ik}\left(s,t-w\right) \mu_{kj}\left(t-w\right) dw$$

Finally, we note that $p_{ij}\left(s,s\right) = \delta_{ij}$, and multiply both sides by $e^{-\int_s^t \lambda_j(u) du}$ to get:

$$p_{ij}\left(s,t\right) = \delta_{ij} e^{-\int_s^t \lambda_j(u) du} + \int_0^{t-s} e^{-\int_s^t \lambda_j(u) du} e^{\int_s^{t-w} \lambda_j(u) du} \sum_{k \neq j} p_{ik}\left(s,t-w\right) \mu_{kj}\left(t-w\right)$$

This simplifies to:

$$p_{ij}\left(s,t\right) = \delta_{ij} e^{-\int_s^t \lambda_j(u) du} + \sum_{k \neq j} \int_0^{t-s} p_{ik}\left(s,t-w\right) \mu_{kj}\left(t-w\right) e^{-\int_{t-w}^t \lambda_j(u) du} dw$$

$$= \delta_{ij}\, p_{\overline{ii}}\left(s,t\right) + \sum_{k \neq j} \int_0^{t-s} p_{ik}\left(s,t-w\right) \mu_{kj}\left(t-w\right) p_{\overline{ii}}\left(t-w,t\right) dw$$

So we have:

$$p_{ij}\left(s,t\right) = \sum_{k \neq j} \int_0^{t-s} p_{ik}\left(s,t-w\right) \mu_{kj}\left(t-w\right) p_{\overline{ii}}\left(t-w,t\right) dw \quad \text{for } i \neq j$$

and:

$$p_{ii}\left(s,t\right) = \sum_{k \neq j} \int_0^{t-s} p_{ik}\left(s,t-w\right) \mu_{ki}\left(t-w\right) p_{\overline{ii}}\left(t-w,t\right) dw + p_{\overline{ii}}\left(s,t\right)$$

(Total 10 Marks)

5 (i) $h(x,t) = h_0(t).\exp(\beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k)$

where $h(x, t)$ is the hazard at duration t, $h_0(t)$ is some unspecified baseline hazard, $x_1 \ldots. x_k$ are covariates and $\beta_1 \ldots.. \beta_k$ are their associated parameters.

(ii) Lives who are:

- recently taken the policy
- accepted based on a non medical
- female

(iii) The value of the parameter associated with the duration is 0.41. The standard error associated with the parameter is 0.065.

The approximate 95% confidence interval is therefore 0.41 ± 1.96(0.065) = (0.2826, 0.5374), which does not include 0. Therefore, the analysis shows that duration affects the risk of death.

(iv) The hazard for male policyholder who was accepted without any medical examination is $h_0(t)$exp(0.035*1+0.41*0 + -0. 75*0).

The hazard for male policyholder of the same age who was accepted with medical examination and took out a policy 1.5 years ago is

$h_0(t)$ exp(0.035*1+0.41*1.5 + -0. 75*1).

The ratio is thus

$$\frac{h_0(t)\exp(0.035*1)}{h_0(t)\exp(0.035*1+0.41*1.5 - 0.75*1)} = \exp(0.145) = 1.156$$

So the model implies that the probability of death of male policyholder who was accepted without any medical examination is 15.6% greater of male policyholder who was accepted with medical examination.

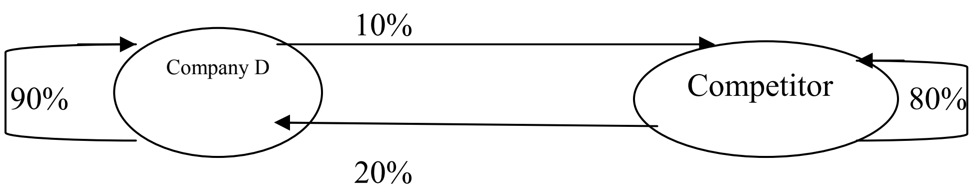
There could be another answer assuming old policyholder was accepted on non medical basis.

(Total 11 Marks)

6.

a) Each year a customer can either be buying D's cereal or the competitors.

The transition diagram is given by



The transition matrix is given by

State1 = Customer buying D's Cereals
State2 = Customer buying competitor's cereals

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$$

ii)

- D's market share in Year 2

We know that D' has 25% market share , hence we have a row matrix representing the initial state of system given by

$$\begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\begin{bmatrix} 0.25 & 0.75 \end{bmatrix}$$

As per the markov theory, in period t, the state of system is given by row matrix S_t, where

$$S_t = S_{t-1}(p) = S_{t-2}(p)(p) = SP^{t-1}$$

We already know the state of the system in year one (S_1), so that the state of system in year two is given by

$$S_2 = S_1 \mathrm{P}$$

$$= \begin{bmatrix} 0.25 & 0.75 \end{bmatrix} \begin{pmatrix} 0.90 & 0.10 \\ 0.20 & 0.80 \end{pmatrix}$$

$$= [\, 0.25*0.90+0.75*0.20 \;,\; 0.25*0.10+0.75*0.8 \,]$$

$$= [\, 0.375 \;,\; 0.625 \,]$$

Results make intuitive sense.

e.g. of the 25% currently buying D's cereal 90% continue to do so, whilst of the 75% buying the competitor's cereal 20% change to buy D's cereal – giving a (fractional) total of (0.25)(0.90) + (0.75)(0.20) = 0.375 buying D's cereal.

Hence in year 2, 37.5% of the people are in state 1 , ie buying D's cereal.

- D's market share in Year 3

  In year 3 the state of the system is given by

$$= [\, 0.375 \;,\; 0.625. \,] \begin{pmatrix} 0.20 & 0.10 \\ 0.20 & 0.80 \end{pmatrix}$$

$$= [\, 0.4625, 0.5375 \,]$$

Hence in year 3, 46.25% of the people are buying D's cereal.

(iii) In long run

The idea of the long-run is based on the assumption that, eventually, the system reaches "equilibrium" (often referred to as the "steady-state") in the sense that st = st-1. This is not to say that transitions between states do not take place, they do, but they "balance out" so that the number in each state remains the same.

$$[x_1,x_2] = [x_1,x_2] \begin{bmatrix} 0.90 & 0.10 \\ 0.20 & 0.80 \end{bmatrix}$$

$x_1 = 0.90^* x_1 + 0.20^* x_2$

$x_2 = 0.10^* x_1 + 0.80^* x_2$

$x_1 + x_2 = 1$

Solving these equations we get

$x_1 = 2/3$

$x_2 = 1/3$

Hence in the long run Ds market share will be 66.66%.

(Total 11 Marks)

7.

i) The transition matrix is given by

We use 5 states Queue, Ground Floor first visit (G1), Ground Floor second visit (G2) , First Floor (F) and out (O).

$$\begin{array}{ccccc} Q & G1 & G2 & F & O \end{array}$$

$$\begin{pmatrix} -1/20 & 1/20 & 0 & 0 & 0 \\ 0 & -1/30 & 0 & 4/150 & 1/150 \\ 0 & 0 & -1/10 & 0 & 1/10 \\ 0 & 0 & 1/600 & -1/60 & 3/200 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

ii)

We have $e_Q = 20 + e_{G1}$

$= 20 + [30 + 0.80 \times e_F + 0.20 \times 0]$

$= 20 + 30 + 0.80\,[60 + 0.90 \times 0 + 0.10 \times e_{G2}]$

$= 98 + 0.08\,[10 + 0]$

$= 98.8$

iii)

Yes. Any particular individual will start in the Queue and end up outside, so the stationary distribution must be (0, 0, 0, 0, 1) iv)

With one in one out policy, we can ignore queue and think of exit from G2 feeding straight back into G1. We are now dealing with the 3 state markov jump process with states G1, G2 and F and transition matrix is given by

$$\begin{array}{ccc} G1 & G2 & F \end{array}$$
$$\begin{pmatrix} -4/150 & 0 & 4/150 \\ 1/10 & -1/10 & 0 \\ 0 & 1/60 & -1/60 \end{pmatrix}$$

If we wait long enough the process should settle down into the stationary distribution. We can find this by solving the matrix equation

$$\left(\pi_{G1}, \pi_{G2}, \pi_F\right)\begin{pmatrix} -4/150 & 0 & 4/150 \\ 1/10 & -1/10 & 0 \\ 0 & 1/60 & -1/60 \end{pmatrix} = 0$$

This is a markov jump process so $\pi p = \pi$ becomes $\pi A = 0$

Solving these equations and using $\sum \pi_i = 1$

-4/150 π_{G1} + π_{G2}/10 = 0

π_{G1} = 15/4 π_{G2} - 1

-1/10 π_{G2} + π_F/60 = 0

π_F = 6 π_{G2} - 2

15/4 π_{G2} + π_{G2} + 6 π_{G2} = 1

π_{G2} = 4/43

π_{G1} = 15/43

$\pi_F = 24/43$

[15/43, 4/43, 24/43]

The proportion of visitors on the ground floor is therefore 15/43 = 44.2 %

(Total 14 Marks)

8.

i) Give and describe any 3 examples of stochastic process .

<u>White noise</u> :-

White noise is a stochastic process that consists of a set of independent and identically distributed random variables $Z_1, Z_2, \ldots, Z_n$. The random variables can be either discrete or continuous and the time set can be either discrete or continuous.

<u>General Random walk</u>

Start with a sequence of independent and identically distributed random variables, $Z_1, Z_2, \ldots, Z_n$,…, and define the process:

$$X_n = \sum_{j=1}^{n} Z_j + X_0$$

With some initial condition $X_0 = x_0$ or X0 has some specified distribution (not necessarily the same as the common distribution of the Z_i s). Then <Xn>, n=0,1,2,… is known as a random walk. The Z_i s are called the 'steps' or increments of the random walk.

<u>Poisson process</u>

Poisson process with rate λ is a continuous-time integer-valued process C,
t ≥ 0 with the following properties:
(i) N0 = 0
(ii) Nt has independent increments
(iii) Nt has Poisson distributed stationary increments:

P[Nt – Ns = n] = $\frac{[\lambda[t-s]^n e^{-\lambda(t-s)}}{}$ / n!, s< t , n =0,1,2,3

b) How would you classify any stochastic process and explain the classification.

Strictly stationary

A stochastic process is said to be *strictly stationary* (or strongly stationary, simply, stationary) if the joint distributions of $X_{t+1}, X_{t+2},\ldots,X_{t+n}$ and $X_{k+1}, X_{k+2},\ldots,X_{k+n}$ are the same for all t, k and all integers n.

If a stochastic process is not stationary then it is said to be *non-stationary*.

Remarks

- Stationarity means that the statistical properties of the process is unaffected by a shift in time.
- In particular, X_t and X_k must have the same distribution. Hence $E[X_t]$ and $Var[X_t]$ must be constant over time.

Weakly stationary

A stochastic process $< X_t >$ is said to be *weakly stationary* if

a) $E[X_t]=E[X_k]$ for all *t* and *k* (i.e., the expected level of the process is constant), and,
b) $Cov[X_t, X_{t+m}]$ is a function only of *m* (the lag), for all *t* and *m*. [In particular, $Cov[X_t, X_{t+m}]$ does not depend on *t*.]

Remarks:

- Strong stationarity implies weak stationarity, but not vice versa.

Independent increments

A stochastic process $< X_t >$ is said to have *independent increments* if the m-increment, $X_{t+m} - X_t$, is independent of the past of the process $\{X_s, 0 \le s \le t\}$ for all *t* and *m*. That is,

$$P[X_{t+m} - X_t \mid X_s, s \le t] = P[X_{t+m} - X_t] \text{ for all } t \text{ and } m.$$

So if a process has independent increments, the next movement will be independent of the current state and the path taken by the process to get to its current state.

Markov Property

Let $<X_t>$, $t \in \Re$ (the real numbers) be a continuous time stochastic process. Then $<X_t>$, is said to have the *Markov property* if, for all *t*, and all sets A

$$P[X_t \in A | X_{s1}, X_{s2},\ldots, X_s]=P[X_t \in A | X_s=x] \qquad \text{where } s_1<s_2<\ldots<s<t.$$

C) How would you classify (i) according to (ii) and explain your answer

Process\Classification	*Strcitly Stationary*	*Weakly Stationary*	*Independent Increments*	*Markov Property*
White Noise	√	√	χ	√
Random Walk	χ	χ	√	√
Poisson process	χ	χ	√	√

Key: √ =yes χ = no

Stationary

- Clearly, directly from the definitions, white noise is stationary.
- A random walk is never stationary.. For the mean to be constant as is implied by stationarity, demands that the increment of the random walk has zero mean and, equally, for each entry in the random walk to have the same variance requires that the variance of the increment is also zero. The only sequence satisfying these constraints is the constant sequence x_0, x_0, x_0,…. So, aside from this degenerate case, a random walk is not stationary.
- Poisson process :- This is a Markov jump process with state space $S = \{0,1,2,...\}$. It is not stationary: as in the case for the random walk, both the mean and variance increase linearly with time.

Independent Increments

- white noise does not have independent increments. Let Z_1, Z_2, Z_3, be white noise. Consider the covariance between the level and the subsequent increment of the process, i.e,

$$Cov[Z_{n+1} - Z_n, Z_n] = E[(Z_{n+1} - Z_n - 0)(Z_n - \mu_Z)]$$
$$= E[Z_{n+1}Z_n - Z_n^2 - \mu_Z(Z_{n+1} - Z_n)]$$
$$= E[Z_{n+1}]E[Z_n] - E[Z_n^2] - \mu_Z.(\mathrm{E}[Z_{n+1}] - \mathrm{E}[Z_n]) \quad \text{as } Z_i\text{s are independent}$$
$$= \mu_Z^2 - (\sigma_Z^2 + \mu_Z^2) - 0$$
$$= -\sigma_Z^2$$

As this is not in general zero, it allows us to conclude that

$$P[Z_{t+1} - Z_t \mid Z_t] \neq P[Z_{t+1} - Z_t]$$

- A random walk has, by definition, has independent increments.
- Poisson process by definition as above has independent increments.

Markov property

- White noise clearly has the Markov property. In fact, not only is the future of the white noise process independent of the past, it is also independent of its current value. Finally, to conclude the table,
- It can be demonstrated that a random walk has the Markov property. This follows from the result in d)

- Poisson process is a Markov jump process with state space $S = \{0,1,2,...\}$.

D) Prove that stochastic process with independent increments has markov property

Consider a stochastic process <X_t> with independent increments. If it is a discrete time process then

$$P[X_{n+1} \mid X_n, X_{n-1},..X_0] = P[X_{n+1} - X_n + X_n \mid X_n, X_{n-1},..X_0]$$
$$= P[X_{n+1} - X_n + x \mid X_n = x, X_{n-1},..X_0]$$

but increments are independent, so

$$P[X_{n+1} \mid X_n, X_{n-1},..X_0] = P[X_{n+1} - X_n + x \mid X_n = x]$$
$$= P[X_{n+1} \mid X_n]$$

Or, the same argument made when time is continuous:

$P[X_t \in A \mid X_{s_1} = x_1, X_{s_2} = x_2, ..., X_{s_n} = x_n, X_s = x]$, where $s_1<s_2<\ldots<s<t$.

$= P[X_t - X_s + x \in A \mid X_{s_1} = x_1, X_{s_2} = x_2, ..., X_{s_n} = x_n, X_s = x]$

$= P[X_t - X_s + x \in A \mid X_s = x]$ as it has independent increments

$= P[X_t \in A \mid X_s = x]$

(Total 15 Marks)

9

(i) H_0: Standard mortality table represents the true underlying mortality rates of the experience.

Age: x	*Initial exposed to risk:* Ex	*Standard Mortality rates* qx	*Expected Deaths* E	*Actual deaths* A	*A-E*	*Chi-squared*
50	2305	0.0064	14.75	15	0.25	0.00417
51	2475	0.0069	17.08	16	-1.08	0.06798
52	2705	0.0075	20.29	22	1.71	0.14455
53	2900	0.0081	23.49	23	-0.49	0.01022
54	3170	0.0087	27.58	27	-0.58	0.01216
55	6730	0.0094	63.26	66	2.74	0.11850
56	6875	0.0101	69.44	67	-2.44	0.08556
57	8190	0.0109	89.27	88	-1.27	0.01810
58	8200	0.0117	95.94	102	6.06	0.38278
59	7680	0.0119	91.39	80	-11.39	1.42001
60	7160	0.0121	86.64	85	-1.64	0.0309
Total	58390		599.12	591.00	-8.12	2.295

(a) If H_0 is true, the Chi-squared statistic will follow the χ^2 sampling distribution.

Since we are an experience with a standard table, the degrees of freedom = 11

Observed value of χ^2 = 2.295

Tabular value of χ^2 for upper 95% point = 19.68

The observed value of test statistic is less than 5% significance point. Hence, there is no evidence to reject the null hypothesis.

(b) If H_0 is true test statistic $Z(c) \sim N(0, 1)$

Here, observed value of $Z(c) = \frac{-8.12}{\sqrt{599.12}} = -0.332$

This is a two-tailed test. We compare the value of statistic with the upper and lower 2.5% points of N(0,1). As -1.96 < -0.332 < 1.96, there is insufficient evidence to reject the null hypothesis.

(c) If H_0 is true:

Age	*zx*	*zx^2*
50	0.0646	0.00417
51	-0.2607	0.06798
52	0.3802	0.14455
53	-0.1011	0.01022
54	-0.1103	0.01216
55	0.3442	0.11850
56	-0.2925	0.08556
57	-0.1345	0.01810
58	0.6187	0.38278
59	-1.1916	1.42001
60	-0.1758	0.03089

Number of groups of positive signs = 4

Number of positive signs = 4

Number of negative signs = 7

From the table , we can find that the value for k for which

$$\sum_{t=1}^{k} \frac{\binom{n1-1}{t-1}}{\binom{m}{n1}} \binom{n2+1}{t} < 0.05$$

Where n1=4; n2=7 & m=4+7=11

The tabular value of k is 1 which is less than number of groups of signs and hence there is no evidence to reject the null hypothesis.

(ii) **Summary**

Absolute sizes of deviations are very small (chi-squared test): standard table is closer to actual experience.

The deviations are consistently negative over the age range (cumulative deviations test): this implies that the true rates are actually lower than standard table.
As the actual number of runs is more than the expected number, it indicates that the actual rates follow the standard rates closely.

(iii) (a)

Three methods used for carrying out graduation are:

- Graduation by parametric formula
- Graduation by reference to a standard table
- Graphical method

(b) Considering that the actual rates follow closely with a standard table, graduation by reference to standard table could be used. Further, graduation method by reference to a standard table can be used to fit relatively small data sets, which is very much true in this case.

A graphical method would probably not be accurate enough. A parametric formula might be difficult to fit if the data are unreliable.

(Total 19 Marks)

[Total 100 Marks]

********END********