INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

12th November 2008

Subject ST6 — Finance and Investment B

Time allowed: Three hours (14.15* pm – 17.30 Hours)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception

2. * You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.

3. You must not start writing your answers in the answer sheet until instructed to do so by the supervisor

4. The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.

5. Attempt all questions, beginning your answer to each question on a separate sheet.

6. Mark allocations are shown in brackets.

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor Separately.

Q1) Prices of zero coupon bonds reveal the following pattern of 1 year forward rates (with annual compounding):

Year	Forward Rate
1	5%
2	7%
3	8%

In addition to the zero-coupon bond, investors may also purchase a 3-year bond making annual payment of Rs. 60 with par value Rs 1000.

a) What is the price of the coupon bond? (1)

b) Show that the yield to maturity of the coupon bond is 6.6%? (1)

c) If you forecast that the yield curve in (a) will be flat at 7% (with annual compounding) in one year's time, what is your forecast for the expected rate of return on the bond for the 1-year holding period? (2)

d) Under what conditions would your answers in part b and c be equal? (1)

[5]

Q2) Long term Treasury bonds currently are selling at yields to maturity of nearly 8%. You expect interest rates to fall. The rest of the market thinks that they will remain unchanged over the coming year. In each question, choose the bond that will provide the higher holding period return over the next year if you are correct. Briefly explain your answer.

a) i.) A Baa-rated bond with annual coupon rate 8% and time to maturity 20 years.
   ii.) An Aaa-rated bond with annual coupon rate of 8% and time to maturity 20 years. (1)

b) i.) An A-rated bond with annual coupon rate 4% and maturity 20 years, callable at 105.
   ii.) An A-rated bond with annual coupon rate 8% and maturity 20 years, callable at 105. (1)

c) i.) A 6% coupon non-callable T-bond with maturity 20 years and YTM = 8%.
   ii.) A 9% coupon non-callable T-bond with maturity 20 years and YTM = 8%. (1)

[3]

Q3) a) Gamma Brothers believe that market volatility will be 20% annual for the next three years. Three-year at -the –money call and put options on the market index sells at an implied volatility of 22%. Using Gamma's estimate of volatility, 3-year at-the-money options have $\Phi(d_1) = 0.6$ where $\Phi(x)$ represents the probability that standard normal variable is less than x and d_1 is as defined in Black-Scholes equation in the tables. What option portfolio can Gamma Brothers establish to speculate on its volatility belief without taking a bullish or bearish position on the market? (3)

b)   You are a corporate treasurer who will purchase Rs. 50 million of bonds for the sinking fund in three months. You believe rates will soon fall, and you would like to repurchase the company's sinking fund bonds (which currently are selling below par) in advance of requirements. Unfortunately, the regulation allows you to purchase at most 2 week earlier than the scheduled purchase. What action can you take in the futures market to hedge any adverse movements in bond yields and prices until you can actually buy the bonds? Will you be long or short? Why?   (2)

c)   Suppose that the relationship between the rate of return on Infosys stock, the market index and a IT industry index can be described by the following regression equation: $r_{Infosys} = 0.5r_M + 0.75r_{Industry}$. If futures contract on the IT industry is traded, how would you hedge the exposure to the systematic and industry factor affecting the performance of Infosys stock. How many rupees' worth of the market and industry index contract would you buy or sell for each rupee held in Infosys?   (1)
[6]

Q4)   The common stock of Reliance Industries Limited has been trading in a narrow range around Rs. 2500 per share for months, and you believe that it is going to stay in that range for the next 2 months. The price of a 2-month European at the money put option is Rs. 150.

a)   If the risk-free interest rate is 6% per annum with continuous compounding, what must be the price of a 2-month European at the money call option on Reliance stock? (The stock pays no dividends.)   (1)

b)   What would be a simple option strategy using a put and a call to exploit your conviction about the stock price's future movement? What is the most money you can make on this position? How far can the stock price move in either direction before you lose money?   (2)

c)   How can you create a position involving a put, a call and riskless lending that would have the same payoff as the stock at expiration? What is the net cost of establishing the position now?   (2)
[5]

Q5)   Consider this arbitrage strategy to derive the parity relationship for spreads: (1) enter a long futures position with maturity date T_1 and future price $F(T_1)$; (2) enter a short position with maturity T_2 and futures price $F(T_2)$; (3) at T_1, when the first contract expires, buy the asset from the spot market and borrow $F(T_1)$ rupees at rate r_f; (4) pay back the loan with interest at T_2.

a)   What are the total cash flows to this strategy at times 0, T_1, and T_2?   (3)

b)   Why must profits at time T_2 be zero, if no arbitrage opportunities are present?   (1)

c)   What must the relationship between $F(T_1)$ and $F(T_2)$ be for the profit at T_2 to be equal to zero?   (1)
[5]

Q6)   Alpha Limited enter a 6-year swap with Beta Limited to pay MIBOR in return for a fixed 10% rate on a notional principal of Rs. 100 crores. Suppose three years from now on, the market rate on 3-year swaps is MIBOR for 9% at this time and the three year zero coupon rate is 10% at this time, at the same time Beta Limited goes bankrupt and default on its swap obligation.

   a)   Why is Alpha Limited harmed by the default?   (2)

   b)   What is the market value of the loss incurred by Alpha Limited as a result of the default?   (2)

   c)   Suppose instead that Alpha Limited had gone bankrupt. How do you think the swap would be treated in the reorganization of the firm?   (2)
                                                                                                        [6]

Q7)   a)   What is your understanding about risk-neutral world and how does it relate to the world of stochastic calculus?   (2)

   b)   "The expected future value of an interest rate in a risk neutral world is greater than it is in the real world. " What does this statement imply about the market price of risk for i) an interest rate and ii) a bond price. Do you think the statement is likely to be true? Give reasons.   (2)

   c)   The variable S is an investment asset providing income at rate q measured in currency A. It follows the process
         $dS = \mu S dt + \sigma S dz$
         in the real world. Defining new variables as necessary, give the process followed by S and the corresponding market price of risk, in a world that is the traditional risk-neutral world for currency A (Using Ito Lemma)   (4)
                                                                                                        [8]

Q8)   At time zero the price of a non-dividend paying stock is S_0. Suppose that the time interval between 0 and T is divided into two subintervals of length t_1 and t_2 . During the first subinterval, the risk free interest rate and volatility are r_1 and σ_1 respectively. During the second interval they are r_2 and σ_2 respectively. Assume that the world is risk-neutral.

   a)   Determine the stock price distribution at time T under Black-Scholes world.   (2)

   b)   Suppose that $\bar{r}$ is the average interest rate between time zero and T and that $\bar{V}$ is the average variance between times zero and T. What is the stock price distribution as a function of T in terms of $\bar{r}$, $\bar{V}$, T and S_0?   (2)

   c)   What are the results corresponding to a and b above when there are three subintervals with different interest rates and volatilities?   (2)

   d)   Argue that if the risk free rate, r, and the volatility σ, are known functions of time, the   (2)

stock price distribution at time T in a risk neutral world is

$Ln(S_T) \sim$ Normal with mean of $\ln(S_0) + \left(\bar{r} - \dfrac{\bar{V}}{2} \right) T$ and variance of $\bar{V} T$.

    e)   What is the implication of the above for the Black-Scholes formula for call option?   (2)

[10]

Q9)   a)   Under what circumstances is it possible to make a European option on a stock index both gamma neutral and vega (kappa) neutral by adding a position in one other European option on the same stock index? Derive the conditions by setting up the appropriate equations.   (5)

    b)   What volatility smile is likely to be observed for 6-month options when the volatility is uncertain and positively correlated to the stock price?   (2)

[7]

Q10) You are offered the position of CEO in Delta Co. A part of the remuneration is dependent on the stock price at the end of the year. The agreement states that you would receive Rs 1 mn as bonus if the stock price at the end of the period rises to 110. Currently the stock is trading at 100. Assume that over the next year the stock prices will increase by 10% or decrease by 10%. The risk-free rate is 5% per annum with anual compounding. Unfortunately, no such options are traded on Delta Co.

    a)   What is the name of this option? Suppose the desired option were traded. How much would he receive today if he writes this option?   (3)

    b)   What portfolio position in stock and T-bill will ensure you a payoff equal to the payoff that would be provided by the contract? What can he do to lock in the amount calculated above so that his final pay-off becomes independent of the share price at the end of the year.   (2)

    c)   Why is the binomial model generally not a good model to estimate the value of the above type of option contracts?   (2)

[7]

Q11) A life insurance company has just written 700 Crores of 5-year single premium bonds over the six week period ending on 31st March 2008 (the "closing date"). The policyholders invested an average of Rs 250,000 and were guaranteed a return equal to the greater of:
ı) 95% of their original investment and
ıı) the amount of their original investment multiplied by the ratio of the Nifty index five years from the closing date to the Nifty index on the closing date.
Early surrender values are linked to the value of the underlying derivative contracts but the amount payable on death is equal to the amount of the original single premium investment. The liability to policyholders is backed by five different derivative contracts each with a different investment bank. You have just taken over the post of appointed actuary of the company and have sufficient expertise to assess the value of the derivative contracts

independently of the investment banks. You have been asked by the board of the company to assess the suitability of existing risk management practices for this block of single premium business.

Discuss the issues that you would consider in responding to the board's request.   **[10]**

Q12) a) List the key features that a good model of the entire interest rate yield curve should have.   (2)

b) Write down the stochastic differential equation defining the Vasicek model under the risk neutral probability measure.   (1)

c) Starting from your equation in b derive an equation expressing r(T) in terms of r(t)   (4)

d) Use your equation in c to deduce the conditional distribution of r(T) given r(t) under the risk neutral probability measure   (5)
          [12]

Q13) Suppose that the risk-free zero curve is flat at 8% per annum with continuous compounding and that default can occur halfway through each year in a new 5-year credit default swap. Suppose that the recovery rate is 30% and the default probabilities each year conditional on no earlier default is 3%. Estimate the credit default swap spread. Assume payments are made annually.   **[10]**

Q14) Suppose that X_t and Y_t are standard Brownian motions and they are independent. Let F_t denote the natural filtration with respect to both of these Brownian motions. Show that the process Z_t defined below is a martingale.

$$Z_t = X_t^2 Y_t - \int_0^t Y_u du$$
          [6]

****