INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

5th November 2008

Subject CT4 - Models

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.* **However, answers to objective type questions could be written on the same sheet.**

4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q 1)   In a mortality investigation, carried out for a large group of assured lives, the following tabulations of deaths were suggested-

    i).   Age at the birthday falling in the policy year of death

    ii). Age nearer birthday at the policy anniversary falling in the calendar year of death

    iii). Age last birthday at the 1st July falling in the calendar year of death.

    a)   State the rate year implied by each of these methods of tabulations of death.   (3)

    b)   Consider an investigation period covering period of four years, where the force of mortality is assumed to be constant, there were 40 deaths and population remained constant at 8000. Find out the maximum likelihood estimate of force of mortality.   (2)
        [5]

Q 2)   i)   Derive an expression, in terms of tPx only, for the complete expectation of life at age *x*.   (3)

    ii)   In a certain population the force of mortality is constant at 0.02 for all ages. For a life aged exactly 25 calculate the complete expectation of life.   (3)
        [6]

Q 3)   A manufacturer of electronic good is attempting to estimate the useful working lifetime of his products. To do so, he has been tracking 20 products sold on 1 January 2003. The purchaser of each item was contacted at three-monthly intervals up to 1 January 2008 to check whether the product was still functioning correctly. The results are summarised below (where B means observation ceased because the product stopped functioning and O means observation ceased for some other reason).

Observation Ceased period ending	Number of observation	Reason
1 July 2003	2	B
1 October 2003	3	O
1 January 2004	2	B
1 July 2004	2	O
1 October 2004	1	O
1 July 2005	3	O
1 October 2005	1	B
1 April 2006	1	O
1 July 2006	1	O
1 October 2006	1	B
1 January 2007	1	B
1 January 2008	2	O

Note: Assume that products breakdown and censoring happens mid-way between three monthly checks.

      i)   Define Type I and Type II censoring.   (2)

      ii)   Calculate the Nelson-Aalen estimate of survival function for this product.   (5)

      iii)   Suggest two ways in which the breakdown rate for electronic good is similar to typical patterns for human mortality.   (2)

[9]

Q 4)   Given a stochastic process $P_{ij}(s,t)$, which is the probability of going from state "i" to state "j" between time "s" to time "t",

    a) Write down the integrated form of Kolomogorov backward and forward equations when

        (i) $i \neq j$ (3)
        (ii) $i = j$ (1)

    b) Derive the integrated form of the forward equations when

        (i) $i \neq j$
        (ii) $i = j$ (6)

[10]

Q 5)   An investigation into the risk factors associated with the mortality rate of the policyholders of a life insurance company was undertaken. The groups identification and respective vector of covariates are given as under:

       ▪ Z1 = 1, if lives were underwritten medically; otherwise it is 0 (i.e. for non medical lives)

       ▪ Z2 = period from date of issuance of policy to the date of investigation in years

       ▪ Z3 = 0 if female; 1 if male

"β" is a vector of parameters; the table given below provides the parameter values and standard error for each covariates:

Covariates	Parameter	Standard error
Medical/Non medical	-0.75	0.007
Duration	0.410	0.065
Gender	0.035	0.015

    (i)   Defining all the terms you use, write down a general expression for the Cox proportional hazards model in terms of a set of covariates, their associated parameters and a baseline hazard function.   (2)

    (ii)   In the context of the investigation described above, state the class of policyholders to which the baseline hazard refers.   (2)

(iii)   Is the duration covariate significant in determining mortality? Justify your answer.   (4)

(iv)   Compare the probability of death for a new male policyholder who was accepted without any medical examination to that of a same type of policyholder of the same age who took out a policy 1.5 years ago.   (3)

[11]

Q 6)   A company D manufactures breakfast cereals, currently has 25% of the market share. In order to increase their market share, the company D employed a market research organization to analyze the needs of the customers. The market research organization worked on the previous years data and came out with the results that: -

▪   90% of D's customers remained loyal in that year, but 10 % switched to competition.

▪   In addition 80% of the competition's customers remained loyal to the competition but 20 % of the competitor's customers switched their loyalty to D

Assuming that these trends continue, to determine D's market share,

i)   Depict the above problem as a transition diagram and a transition matrix.   (2)

ii)   Using Markov chains or Markov process, determine D's market share in:
        a. 2 years   (3)
        b. 3 years   (2)
        c. In long run.   (4)

[11]

Q 7)   A new mall has been opened in town and the mall is spread across two floors- ground floor and first floor. The mall has only one entry point- from the ground floor and two exits points – one in ground floor and one in first floor. In order to attract the customers, the management team decided to offer large discount on all items for a period of two months.

On the opening day, a large crowd had gathered and the visitors had to wait for 20 minutes in queue outside the mall before they could enter the mall.

The visitors spent an average of 30 minutes in ground floor and 80% of them proceeded to the first floor and the balance 20 % left the mall from the ground floor exit without visiting the first floor.

The 80% of the visitors who went to the first floor spent an average of 60 minutes in the first floor and 90 % of them left the mall from the first floor exit. The balance 10 % visited the ground floor again and spent 10 minutes in the ground floor of the mall and left the mall from the ground floor exit.

i)   Model the above problem as a markov jump process giving the states and transition matrix.   (3)

ii)   What is the total expected length of time the visitor spends in the mall?   (3)

iii)   Does the process modeled by you in i) above, has a stationary distribution? Explain the reasoning and give the distribution if it exists.   (2)

iv)   The mall becomes very popular and on a particular day, the crowd was double the usual and hence the management, in order to control the crowd, operated a one in and one out policy. They also blocked the first floor exit, so that all the visitors could exit the mall from the ground floor. Assuming that there will always be one person waiting in queue, what proportion of the visitors will be on the ground floor on an average on that particular day.   (6)

[14]

Q 8)   i)   Give and describe any 3 examples of stochastic process   (4)

ii)   How would you classify any stochastic process and explain the classification   (4)

iii)   How would you classify (i) according to (ii) and explain your answer.   (4)

iv)   Prove that stochastic process with independent increments has markov property.   (3)

[15]

Q 9)   The mortality experience of certain group of annuity policyholders has been compared with the published standard mortality table. The following is an extract from the data.

Age:	Initial exposed to risk:	Standard Mortality rates	Expected Deaths	Actual deaths
x	Ex	qx	E	A
50	2305	0.0064	14.75	15
51	2475	0.0069	17.08	16
52	2705	0.0075	20.29	22
53	2900	0.0081	23.49	23
54	3170	0.0087	27.58	27
55	6730	0.0094	63.26	66
56	6875	0.0101	69.44	67
57	8190	0.0109	89.27	88
58	8200	0.0117	95.94	102
59	7680	0.0119	91.39	80
60	7160	0.0121	86.64	85
Total	58390		599.12	591.00

i)   Carry out a comparison between the actual and expected mortality experience, using the following statistical tests.

       (a)   Chi-squared test.
       (b)   Cumulative deviations test.
       (c)   Grouping of signs test.

You should state the appropriate Null Hypothesis and, for each test, the conclusion reached with regard to this hypothesis.   (12)

ii)   Summarise what you can infer about the mortality experience of these
        policyholders from your analysis, giving your reasons.   (3)

iii)
     a) List three methods used for carrying out a graduation.

     b) You have now asked to carry out a graduation for the data given in part (i).
         Indicate, with reasons, which one of the three methods of graduation described
         above would be most appropriate.   (4)
                                                                                  [19]

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