INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

4th November 2008

Subject CT3 – Probability & Mathematical Statistics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2) ***Mark allocations are shown in brackets.***

3) ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet</u>.***

4) ***In addition to this paper you will be provided with graph paper, if required.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) The sample given below represents auto claim data from motor damage claims (in Rs.'000) of commercial vehicles.

258	639	739	839	579	679	588	688	654	673
838	412	638	619	369	759	859	679	779	628
728	659	471	571	671	534	693	738	106	206
465	608	708	808	908	998	639	739	489	674
556	656	329	567	509	523	428	528	628	495

a) Construct a stem and leaf display with stem labels 1,2,3,…,9 and leaves ordered. (3)

b) Make a frequency table by summarizing the data into class intervals of width 100,000 the first interval ending at 100,000 and draw the histogram. (4)

c) Comment on the *skewness* of the data. (1)

d) If the observation 998,000 is omitted, find the median and Q_1 using the stem-leaf display only. (2)

[10]

Q. 2) If X has the *pdf*

$$f(x) = \lambda e^{-\lambda x}; x > 0, \lambda > 0 \ ,$$

Obtain the distribution of Y=1-exp(-λX)

[3]

Q. 3) The joint *pdf* of (X,Y) is given by

$$f(x, y) = x\exp[-x(1+y)]; x > 0, y > 0$$

a) Find the marginal *pdf s* of X and Y (2)

b) Show that E[Y] does not exist (2)

c) Calculate $E\,[Y/X\,]$ and comment in light of b) (3)

[7]

Q. 4) Let $X_1, X_2, \ldots, X_n$ be *iid* exponential random variables with mean $1/\lambda$

a) Using *mgf* , find the distribution of $Y=X_1+X_2+\ldots+X_n$. (3)

b) Using *cgf* of Y, find the mean and variance of Y. (3)

[6]

Q. 5) The blood type distribution in India is: Type A: 41%; Type B: 9%; Type AB: 4%; and Type O: 46%. It is estimated during a war, 4% of soldiers with type O blood were classified as having type A ; 88% of those with type A were correctly classified ; 4% with type B blood were classified as Type A ; and 10% with type AB were classified as Type A. A soldier was wounded and brought to surgery and was classified as having type A blood. What is the probability that he had been classified as type A.

[6]

Q. 6) Let X_1, and X_2 be *iid* random variables with $P(X_i = \pm 1) = 1/2,\ i = 1,2$. Let $X_3 = X_1 X_2$

Show that X_1, X_2, X_3 are pair- wise independent but not completely.

[3]

Q. 7) A study is performed to investigate the connection between maternal smoking during pregnancy and birth defects in children. Of the mothers studied 40% smoke and 60% do not. When the babies were born, 20 babies were found to have some sort of birth defect. What is the probability that 12 or more of the affected children had mothers who smoked? Assume that there is no relationship between maternal smoking and birth defects.

[3]

Q. 8) Let $X_1, X_{2,\ldots,} X_{50}$ be independent random variables following *N(0,1)* distribution. Define

$$Y = \sum_{k=1}^{50} X_k^2 \quad .$$

a) Calculate the mean of Y (2)

b) Calculate, using central limit theorem, $P\,[\,Y < 60\,]$. (3)

[5]

Q. 9) Let X follow Poisson distribution with parameter λ, and let λ follow the gamma distribution

$$f(\lambda) = \frac{r^p}{\left|\overline{p}\right.} \quad e^{-r\lambda} \lambda^{p-1} \ ; \ \lambda > 0 \ ; \ r > 0 \ ; \ p > 0 \quad (p \text{ integer})$$

Find the distribution of X, its mean and variance.

[5]

Q. 10) Let W denote the time of the first occurrence of the event, with reference to a Poisson process. Show that W has an exponential distribution with mean $1/\lambda$.

[3]

Q. 11) A textile fibre manufacturer is investigating a new drapery yarn, which the company claims that the thread elongation this yarn follows normal distribution with mean 12 kg and sd 0.5 kg. The company wishes to test the hypothesis H_0: $\mu = 12$ against H_1: $\mu < 12$ using a random sample of 4 specimens.

a) What is the probability of type I error if the critical region used is $\bar{x} < 11.5$ kg. (2)

b) Find the power for the case in a) when the true mean elongation is 11.25 kg. (2)

[4]

Q. 12) In the examination of 400 sales-people at a national sales convention, 64 were found to exhibit the classic symptoms of ego distortion.

a) Estimate the standard error of sample the proportion. (1)

b) Construct a *99%* confidence interval for the true proportion. (2)

[3]

Q. 13) A study is conducted to estimate the difference in the mean occupational exposure to radioactivity in utility workers in the years 1973 and 1979. These data, based on independent samples drawn from normal populations, are summarized below.

1973	1979
$n_1 = 16$	$n_2 = 16$
$\bar{x}_1 = 0.94$	$\bar{x}_2 = 0.62$
$s_1^2 = 0.040$	$s_2^2 = 0.028$

a) Explain by comparing variances at α=0.2 level, whether the pooling of sample variances is appropriate. (3)

b) Find 95% confidence interval for the difference between the average occupational exposure to radioactivity in utility workers in the years 1973 and 1979. (3)

c) In which year the mean exposure to radioactivity is higher? Justify (1)

[7]

Q. 14) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with *pdf*

$$f_{\alpha,\beta}(x) = \left(\frac{\alpha}{\beta}\right)\left(\frac{x}{\beta}\right)^{\alpha-1} \quad ; 0 < x < \beta \ , \ \alpha > 0$$

$$= 0 \qquad \textit{otherwise}$$

a) Find the *MLE* s of α ,β (6)

b) The length (in mm s) of cuckoo's eggs can be modeled with this distribution. For the data given below, compute MLE s of α ,β

22.0, 23.9, 20.9, 23.8, 25.0, 24.0, 21.7, 23.8, 22.8, 23.1, 23.1, 23.5, 23.0, 23.0 (4)

[10]

Q. 15) It is suspected that there is an association between the day of the week on which an item is produced and quality of the item. The following table gives the categories of the quality as well as the day of production of 500 items produced.

Quality	Days				
	Mon	Tue	Wed	Thur	Fri
Excellent	44	74	79	72	31
Good	14	25	27	24	10
Fair	15	20	20	23	9
Poor	3	5	5	0	0

a) The usual guidelines for testing the hypothesis of no association is that 20% of the expected cell frequencies should not be less than 5 and none should be less than 1. Check whether this criterion is satisfied. (2)

b) Test for independence in a 2 × 2 table combining excellent with good, fair with poor, Mon with Tue and other days together. (2)

[4]

Q. 16) The following table gives the yield (pounds per plot) of three varieties of wheat obtained with different kinds of fertilizers

	Variety of Wheat		
	A	B	C
Fertilizer	1.69	1.56	1.30
F1	0.64	1.22	0.75
F2	0.90	1.32	1.26
F3	1.41	1.39	0.69
F4	1.01	1.33	0.62

Test the hypothesis of equality in the average yields of the tree varieties of wheat, on assuming that four kinds of fertilizers are equally effective, at *5%* level.

[4]

Q. 17) As humidity influences evaporation, the solvent balance of water-reducible paints during spray-out is affected by humidity. A study is conducted to examine the relation between humidity (X) and the extent of solvent evaporation (Y). The following data summary is obtained:

$$n = 25, \quad \Sigma x = 1314.90 \quad \Sigma y = 235.70,$$
$$\Sigma x^2 = 76{,}308.53, \quad \Sigma y^2 = 2286.07 \quad \Sigma xy = 11{,}824.44$$

a) Find the estimated correlation coefficient between X and Y and test the hypothesis $H_0 : \rho = 0$, against $H_1 : \rho \neq 0$; ρ being the population correlation coefficient between X and Y. (4)

b) Stating the assumptions, fit a regression line of the model $Y_i = \beta_0 + \beta_1 x_i + e_i$ for the above data. (3)

c) Obtain the unbiased estimator of σ^2. (2)

d) Test the hypothesis $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$ (3)

e) Obtain 90% confidence interval for β_0 and 99% confidence interval for β_1. (4)

f) Obtain the coefficient of determination R^2. (1)

[17]

****