# Institute of Actuaries of India 

## Subject ST8 - General Insurance: Pricing

## May 2015 Examination

## INDICATIVE SOLUTIONS

## Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

## Solution 1:

It's a type of reinsurance arrangement.
The fronting insurer underwrites a risk.
All or nearly all of the risk is ceded.
The fronting insurer receives a fee or commission to cover its expenses and profit.
The size of the fee takes into account which party is carrying out administration and claims handling.
In event of "reinsurer" default the liability falls upon the fronting insurer.
[2 Marks]

## Solution 2 :

Driverless cars are expected to result in fewer accidents, thus logic stands to say this will decrease insurance claims for accidents.

It could also result in fewer life threatening situations for the people meeting the accident. This could potentially result in Third party Liability premiums as well.

Own Damage Premiums may become cheaper if the potentially higher costs to repair or replace damaged vehicles is more than offset by the lower accident frequency rate.

Impact on third party liability needs to be studied. For example, a third party liability claims arising out of an accident due to the technology not reading the situation appropriately. Will the manufacturer be liable or the owner of the car? This needs to be understood. This could also impact the product liability market, with car being the product and the manufacturer being the insured.

Coverage for physical damage due to a crash and for losses not caused by crashes but by wind, floods and other natural elements and by theft (comprehensive coverage) is less likely to change.

Many of the traditional risk factors, such as the past claims history of the insured, expected number of KMs driven, where the car is garaged etc. will still apply, but the make, model and style of car may assume a greater importance.

Impact will be known with lesser uncertainty with a lag, as more volume of experience is accumulated.

Information about the driving usage such as whether the car was driving on congested roads or free-ways etc could already be gathered by this new technology with more accuracy and could be possibly be used for pay-as-you-go premium rates.

## Solution 3 :

(i) Model X assumes that the claim rate is the same for each observation because the linear Predictor $\eta_{i}$ is always equal to $\alpha$.

Model Y assumes that Drivers 1 to 25have the same claim rate and that Drivers 26 to 40 have the same claim rate.

Model Z assumes that each driver claims at a different rate.
(ii) The models are nested (because one is a subset of the other) and the scaled deviances are given (i.e. the scale parameter is known) so we can use a $\chi^{2}$ test.

We will use a 5\% significance level in the tests below.

The difference in the scaled deviance between Model X and Model Y is 6.83 . Model X has 1 parameter and Model Y has 2 . So we compare the difference in the scaled deviance with $\chi_{1}^{2}$.

The upper $5 \%$ point of $\chi_{1}^{2}$ is 3.841 . Since 6.83 is greater than 3.841 , we conclude that Model Y is a significant improvement over Model X.

The difference in the scaled deviance between Model Y and Model Z is 20.13.Model Y has 2 parameters and Model $Z$ has 40 . So we compare the difference in the scaled deviance with $\chi_{38}^{2}$.

The upper $5 \%$ point of $\chi_{38}^{2}$ is 53.38 . Since 20.13 is lesser than 53.38 , we conclude that Model $Z$ is not a significant improvement over Model Y.

## Solution 4 :

Directors and Officers ( $\mathrm{D} \& \mathrm{O}$ ) insurance is purchased by companies to protect against the directors and officers of the company being sued for acts they have performed in their capacity as directors and officers of the company.

Deliberate fraud by directors and officers will be excluded.

The perils include the following:

- allowing a company to continue operating in circumstances when it should have been declared insolvent
- any act resulting in the insured being declared unfit for his or her role
- Allowing false financial statements to be published.

Cover is likely to be on a claims-made basis.
Typical exposure measures are turnover and net assets and liabilities of the company.
Reporting of claims tends to be fairly quick.
However settlement may take time, due to the high incidence of litigation.
Therefore this tends to be a relatively long-tailed class of business.
Typical risk / rating factors are:

- nature of business
- past experience
- state of the economy
[6 Marks]


## Solution 5 :

## (i)

Factors that influence the settlement amount such as Age, annual wages, jurisdiction, etc. Wage levels indicate the extent of loss of income to the family of the victim. Higher wage levels lead to higher claim sizes.

Judicial inflation is based on how courts arrive at the award after taking into consideration various factors such as age of the victim, the wage levels and other factors. More lenient court awards lead to greater claim sizes, other factors remaining the same.
[2]

## (ii)

These claims take considerable time to settle since the time of accident. When claim amounts are influenced by judicial decisions, the final settlement amounts may be more influenced by the judicial award levels at the time of settlement and may not depend so much on year of accident or year of claim intimation.

## (iii)

The underlying mix of claims in terms of geography, age groups, wage levels etc could be different by year. For example, there could be relatively more accidents involving in regions with higher average wage levels and this would lead to a higher average claim size even if there is no wage inflation or judicial inflation.
(iv)

Selection of explanatory factors that influence the award amount such as Age, wage levels etc. Use the corresponding number of claims as weight. Choice of distribution and link function (for ex, Gamma with link function).

Use Settlement Year as the additional explanatory variable and use the parameter estimates for various settlement years as the trend factors for judicial inflation by settlement year.

Here increase in wages over the years will not lead impact the relativities/parameter estimates for year of settlement since wage amounts (especially if brought to wage levels prevailing at the time of settlement) are already being used an explanatory variable. Therefore the driver in claim inflation due to increase in wage levels will be excluded out. [3]
[7 Marks]

## Solution 6 :

(i)

- Surveyor's report
- Type of trade or business
- Type of use of building
- Dangerous materials/processes
- Value of cash stored on premises
- Known mine workings or similar underground hazard
- Part of building unoccupied
- Age of building
- Time since last renovation
- Construction type
- Location of building/postcode
- Floor area
- Section-level limit: rebuild SI
- Section-level limit: value of contents SI
- Overall policy limit
- Excess/deductible
- Fire protection equipment e.g. sprinklers
- Security features
- Value of property
- Number of properties in the policy size - may get a size credit
- Exclusions
- EML/PML
- Period of cover
- Coverage e.g. BI included, flood, subsidence, terrorism
- Number of floors in building
- Distance from hazard, eg coast or river
- Height above sea level
- Loss history/claims experience
- Proximity to Fire Station
(ii)

Exposure/policy details

- Rating factors
- History of changes to rating factors
- Particularly, rating factors at time of claim
- Policy dates/period on risk

Claims details

- Claim reference
- Link to policy
- Risk identifier (if the policy covers multiple properties)
- Claim status - open/closed
- Claim dates:
- Incurred
- Reported
- Settled
- of payments
- Definition of claim amount, ie ground up or after deductible
- Payment type.g. indemnity, loss adjuster fee
- Amount of payment
- Estimated amount outstanding
- Date of estimate of outstanding amount
- Basis of estimate
- Recovery amount
- Policy section/type of claim eg stock
- Type of peril e.g. flood, fire

General: Currency of values

## Solution 7 :

(i)

The objective is to minimize E [ $\exp \{-$ Surplus $\}]$ where Surplus $=\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}$ and represents the total profit to the insurer, net of reinsurance, out of this contract. When Surplus is $+\mathrm{ve}, \exp \{-$ Surplus $\}<1$. On the other hand, it takes a value greater than one when surplus is -ve .

If the probability distribution of the Surplus random variable is symmetrical about zero, E [ $\exp \{$-Surplus $\}$ will be greater than one.
(Since, $\exp (-x)+\exp (x)=\left(1-x+x^{2} / 2!-x^{3} / 3!+\ldots\right)+\left(1+x+x^{2} / 2!+x^{3} / 3!+\ldots\right)=2\left(1+x^{2} / 2!+\right.$ $\left.x^{4} / 4!+\ldots\right)>2$ ).

This is greater than $\mathrm{E}[\exp \{$-Surplus $\}$ ] when Surplus is zero with probability 1, in which case
E [exp \{-Surplus\} ] is 1 . Preference of $100 \%$ certain no-loss and no-profit situation over a situation in which surplus random variable that is symmetrically distributed about zero indicates risk-aversion.

## OR

For example, Surplus is 1 unit with probability 0.5 and -1 unit with probability 0.5 . Then $\mathrm{E}[\exp \{-$ Surplus $\}]=0.5 * \exp (-1)+0.5 * \exp (1)=1.54>1$. $\mathrm{E}[\exp \{-$ Surplus $\}]$, when Surplus is zero with probability 1 , is 1 . Therefore, the second scenario is preferred. This indicates risk aversion on the part of the insurer.
(ii)

$$
\begin{aligned}
& \mathrm{P}=120 \% * \mathrm{E}[\mathrm{~N}] * \mathrm{E}[\mathrm{X}]=1.2 \lambda / \theta \\
& \mathrm{P}_{\mathrm{R}}=150 \% * \mathrm{E}[\mathrm{~N}] * \mathrm{E}\left[\mathrm{X}_{\mathrm{R}}\right]=1.5 \lambda e^{-\theta M} / \theta \\
& \text { (Since } \left.\int_{M}^{\infty}(x-M) \theta e^{-\theta x} d x=e^{-\theta M} / \theta\right)(1 \text { Mark }) \\
& \mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right]=e^{\lambda / \theta\left(1.5 e^{-\theta M}-1.2\right)} * \mathrm{E}\left[e^{S_{I}}\right] \\
& \mathrm{E}\left[e^{S_{I}}\right]=\mathrm{MGF}(\mathrm{t}) \text { of } \mathrm{S}_{\mathrm{I}} \text { at } \mathrm{t}=1
\end{aligned}
$$

For $\mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right.$ ] to be minimized, its first derivate with respect to M should be zero and second derivate with respect to M should be positive.
$\operatorname{MGF}(\mathrm{t})$ of $\mathrm{S}_{\mathrm{I}}=\mathrm{M}_{\mathrm{N}}\left[\log \mathrm{M}_{\mathrm{X}}(1)\right] \quad$ (Since S is a compound distribution)
(where X denotes the claim amount distribution to the insurer, after reinsurance)

$$
=e^{\lambda\left(M_{X}(1)-1\right)}
$$

$\mathrm{M}_{\mathrm{X}}(1)=\int_{0}^{M} e^{x} \theta e^{-\theta x} d x+e^{M} \int_{M}^{\infty} \theta e^{-\theta x} d x==\{\theta /(1-\theta)\}^{*}\left\{e^{(1-\theta) M}-1\right\}+e^{(1-\theta) M}$
$\lambda\left(\mathrm{M}_{\mathrm{X}}(1)-1\right)=\lambda\left[e^{(1-\theta) M}-1\right] /[1-\theta]$
$\mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right]=\exp \left\{\lambda / \theta\left(1.5 e^{-\theta \mathrm{M}}-1.2\right)+\lambda\left[\mathrm{e}^{(1-\theta) \mathrm{M}}-1\right] /[1-\theta]\right\}$
$=\exp \left\{\lambda / \theta\left(1.5 \mathrm{e}^{-\theta \mathrm{M}}-1.2\right)+\lambda\left[\mathrm{e}^{(1-\theta) \mathrm{M}}-1\right] /[1-\theta]\right\}$

Therefore, minimizing $\mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right]$ is the equivalent of minimizing

$$
\left\{\lambda / \theta\left(1.5 \mathrm{e}^{-\theta \mathrm{M}}-1.2\right)+\lambda\left[\mathrm{e}^{(1-\theta) \mathrm{M}}-1\right] /[1-\theta]\right\}
$$

Therefore, setting equal to the first derivate of $\mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right]$ with respect to M :

$$
\begin{aligned}
& -1.5 \lambda \mathrm{e}^{-\theta \mathrm{M}}+\lambda \mathrm{e}^{(1-\theta) \mathrm{M}}=0 \quad \Rightarrow \lambda \mathrm{e}^{-\theta \mathrm{M}}\left\{-1.5+\mathrm{e}^{\mathrm{M}}\right\}=0 \\
& \mathrm{M}=\ln 1.5
\end{aligned}
$$

We now have to show that second derivate of $\mathrm{E}\left[\exp \left\{-\left(\mathrm{P}-\mathrm{P}_{\mathrm{R}}-\mathrm{S}_{\mathrm{I}}\right)\right\}\right]$ with respect to M at $\mathrm{M}=\ln 1.5$ is positive:

That is, $1.5 \lambda \theta \mathrm{e}^{-\theta \mathrm{M}}+\lambda(1-\theta) \mathrm{e}^{(1-\theta) \mathrm{M}}>0$ when $\mathrm{M}=\ln 1.5$
Substituting $\mathrm{M}=\ln 1.5$, we get: $1.5 \lambda \theta \mathrm{e}^{-\theta \ln 1.5}+\lambda \mathrm{e}^{(1-\theta) \ln 1.5}>0$
Therefore, $\mathrm{M}=\ln 1.5$ is the optimal choice.
[8 Marks]

## Solution 8 :

(i)
$\mu\left(\Theta_{\mathrm{i}}\right)=\operatorname{Var}\left(\Theta_{i}\right)=\Theta_{i}$

We could use sample mean as the estimate for $\mathrm{E}\left[\operatorname{Var}\left(\Theta_{i}\right)\right]=0.2$

Use sample variance as the estimate for $\operatorname{Var}\left(X_{i}\right)=0.25$

Since $\operatorname{Var}\left(X_{i}\right)=\operatorname{Var}[\mu(\Theta)]+\mathrm{E}[\operatorname{Var}(\Theta)]$ : Use estimate for $\operatorname{Var}[\mu(\Theta)]$ as the estimate for $\operatorname{Var}\left(X_{i}\right)$ minus the estimate for $\mathrm{E}[\operatorname{Var}(\Theta)]=$ Sample Variance - Sample Mean $=0.25-0.2=$ 0.05

Using Bühlmann-Straub Formula, the credibility factor $Z=1 /(1+0.20 / 0.05)=1 / 5=0.2[4]$
(ii)
a) Loss Cost of a larger group that includes the class:

Advantages:

- Independent from the base statistic.
- Easily available and easy to compute too.

Disadvantages:

- Contains an intrinsic bias and prediction error that is unknown
- May not have much in common with the group; or customers in this class may complain if their experience is better than that of the group
b) Loss Cost of a related large risk class:

Advantages:

- Easier to explain since there is some relationship between base class and related class

Disadvantages:

- Contains an intrinsic bias and prediction error that is unknown
- To reduce bias, adjustments are needed
c) Existing Rates adjusted for frequency and severity trends:

Advantages:

- Minimize rate fluctuations
- Easily available and easy to compute too.

Disadvantages:

- High prediction error for classes with high process variance
- Doesn't minimize the prediction error of the credibility weighted estimate
d) Competitor's Rates:

Advantages:

- Have prediction errors that are independent of the loss costs of the concerned class

Disadvantages:

- Regulators may complain that the competitor's rates are unrelated to the subject company's own loss costs
- Not readily available


## Solution 9 :

(i)

Company X is directly responsible for $40 \%$ of $37,500,000=15,000,000$ under the coinsurance agreement.

Company R1 takes $5 \%$ of $15,000,000=750,000$ under the quota share agreement.

There are two possible approaches (stating the assumption is very critical):

Assumption 1 Company X takes the full three lines of cover, ceding $75 \%$ to the surplus reinsurers. Therefore, its net claim is $25 \% \times(15,000,000-750,000)=3,562,500$.

Assumption 2 Company $X$ keeps the maximum 2,500,000retention, so that its proportionate retention is $2,500,000 \div(95 \% \times 40 \% \times 25,000,000)=1 \div 3.8$. Therefore its net claim is $(15,000,000-750,000) \div 3.8=3,750,000$.

## (ii)

As company Y is a coinsurer alongside X the insolvency does not affect the amount to be paid by X.

As company R 2 is a reinsurer of X the insolvency will affect the recoveries X can make. In the most extreme case, X will lose the amount it is owed by R 2 ; in practice, there will almost certainly be a partial recovery from the liquidators of R2. The actual answer will lie between the answer in (i) and a revised amount, assuming that there is no recovery at all from R2. The answer in that case depends on the assumption made in (i) above.

## Assumption 1

In this case A will have to pay an extra $75 \% \times 50 \% \times(15,000,000-750,000)=5,343,750$, so the total will be $8,906,250$.

## Assumption 2

In this case, A will have to pay an extra $50 \% \times(15,000,000-750,000-3,750,000)=5,250,000$, for a total of 9,000,000.

## Solution 10:

(i)

Having a deductible leads to reduced number of claims, especially small claims which also require resources for the purpose of claims processing.

The customer has more options and might also prefer to get small repairs done at the garage without involving the insurer. For larger claims, the customer will utilize insurance while bearing the deductible amount from his/her own pocket. The customer might be willing to opt for this because of reduction in premium.
(ii)

The decrease in expected claims is $\mathrm{E}[\mathrm{X} \mid \mathrm{X}<\mathrm{D}] * \operatorname{Pr}\{\mathrm{X}<\mathrm{D}\}+\mathrm{D} * \operatorname{Pr}\{\mathrm{X}>=\mathrm{D}\}$, which is always less than D , where random variable X denotes the claim amount without the deductible and D denotes the amount of deductible. This is the amount that should be passed on to the customer in terms of reduction in premium.

Generally speaking, vehicles with higher values have greater claims severity. If the premium reduction is a percentage of original premiums, the policies with higher premiums tend to benefit more than the policies with lesser premiums. Therefore, second alternative is more appropriate. In fact, if discount is a fixed percentage of premiums, the discount amount could even exceed the deductible amount itself for vehicles with high IDV.

## (iii)

NCB allows the use of actual experience to compensate for the fact that risk classes are still heterogeneous despite the use of many a priori rating variables.

Rating systems penalizing insureds responsible for one or more accidents by premium surcharges (or maluses), and rewarding claim-free policyholders by awarding them discounts (or bonuses) are now in force in many developed countries.

Besides encouraging policyholders to drive carefully (i.e. counteracting moral hazard), they aim to better assess individual risks.

## (iv)

The amount of premium is adjusted each year on the basis of the individual claims experience using techniques from credibility theory. However, credibility formulas are difficult to implement in practice, because of their mathematical complexity.

For this reason, bonus-malus scales have been proposed by insurance companies. Such scales have to be seen as commercial versions of credibility formulas. The typical customer can figure out what the premium will be for any given claims history.
(v) There are two additional costs to the insurer which should be considered:

1. In the event of exactly one claim, customer will still be charged discounted premiums in the future. So the expected reduction in premiums over the future years should be considered.
To estimate the reduction in premiums, the probabilities of NCB levels in the future years with and without NCB protection have to be estimated, subject to the condition of exactly one claim
2. Without NCB Protection, customer would have restrained in making small claims due to bonus hunger if no claims are already lodged. Now, the customer might not restrain in making that first claim. This would be lead to higher expected claims.
The amount below which the customer might choose not to lodge a claim, without NCB protection, owing to bonus hunger is difficult to estimate.

## Solution 11:

(i)
(a) Near-Aliasing: When modeling in practice, a common problem occurs when two or more factors contain levels that are almost, but not quite, perfectly correlated. In a When levels of two factors are "nearly aliased" in this way, convergence problems can occur.

Grouping of various levels of a factor: It is now mainly adopted as a method to thin out redundant codes from the data that has little exposure. This method simply assigns a single parameter to represent the relativity for multiple levels of the factor.

Parameter Smoothing: Instead of grouping various levels of a factor or summarizing the data, a much better job can be done by retaining most of the granularity in the data and then using the patterns in the data itself to help define the grouping and smoothing to apply. If the data is grouped prior to loading into the modelling package, the trend in model predictions be easy to discern.
(b) Distribution for Normal with mean $\mu$ and variance $\sigma^{2}$ is as follows:

$$
\mathrm{e}^{-\frac{(\mathrm{y}-\mu)^{2}}{\sigma^{2}}-\frac{\ln \left(2 \pi \sigma^{2}\right)}{2}}
$$

Therefore, Likelihood for the $\mathrm{i}^{\text {th }}$ observation is: $\mathrm{e}^{-\frac{\left(\mathrm{y}_{\mathrm{i}}-\mu_{\mathrm{i}}\right)^{2}}{\sigma^{2}}-\frac{\ln \left(2 \pi \sigma^{2}\right)}{2}}$

Therefore, Log-Likelihood for the $\mathrm{i}^{\text {th }}$ observation is: $\quad-\frac{\left(y_{i}-\mu_{i}\right)^{2}}{\sigma^{2}}-\frac{\ln \left(2 \pi \sigma^{2}\right)}{2}$
The linear predictor for the GLM is as follows:

$$
\ln \mu_{i}=\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}
$$

Using the linear predictor in the Log-Likelihood function for $i^{\text {th }}$ observation, we get: [2]

$$
-\frac{\left(y_{i}-\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}\right)\right)^{2}}{\sigma^{2}}-\frac{\ln \left(2 \pi \sigma^{2}\right)}{2}
$$

(c) Since, the observations are independent of each other, the overall log-likelihood is the sum of log-likelihood of all observations.

So, the overall log-likelihood is as follows:

$$
\sum_{i=1}^{4}\left\{-\frac{\left(y_{i}-\exp \left(\beta_{0}+\beta_{1} x_{i 1}+\beta_{2} x_{i 2}\right)\right)^{2}}{\sigma^{2}}-\frac{\ln \left(2 \pi \sigma^{2}\right)}{2}\right\}
$$

Substituting the values from the data, the overall log-likelihood reduces to

$$
\begin{gathered}
-\frac{\left(1000-\exp \left(\beta_{0}\right)\right)^{2}}{\sigma^{2}}-\frac{\left(800-\exp \left(\beta_{0}+\beta_{2}\right)\right)^{2}}{\sigma^{2}}-\frac{\left(2000-\exp \left(\beta_{0}+\beta_{1}\right)\right)^{2}}{\sigma^{2}} \\
-\frac{\left(1400-\exp \left(\beta_{0}+\beta_{1}+\beta_{2}\right)\right)^{2}}{\sigma^{2}}-2 \ln \left(2 \pi \sigma^{2}\right)
\end{gathered}
$$

For the values of $\beta_{0}, \beta_{1}$ and $\beta_{2}$ which maximize the log-likelihood, the first (partial derivate) with respect to each of these parameters must be zero and second (partial) derivate with respect to each of these parameters should be negative.

## WITH RESPECT TO $\beta_{1}$ :

For the first partial derivative to be zero:
$\left(2000-e^{\widehat{\beta} 0+\widehat{\beta} 1}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1}+\left(1400-e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}$ should be zero

Substituting the values given, we get
$(2000-1,980.305)^{*} 1,980.305+(1400-1,427.323) * 1,980.305=39,002.70-38,998.75=$ $3.95 \sim 0$

Second partial derivate:
$\left(2000-2 e^{\widehat{\beta} 0+\widehat{\beta} 1}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1}+\left(1400-2 e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2} \quad$ should be negative

Substituting the values given, we get
$(2000-2 * 1,980.3) * 1,980.3+(1400-2 * 1,427.3) * 1,980.3$, which is negative

## WITH RESPECT TO $\beta_{2}$ :

For the first partial derivative to be zero:
$\left(800-e^{\widehat{\beta} 0+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 2}+\left(1400-e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}$ should be zero

Substituting the values given, we get
$(800-747.853) * 747.853+(1400-1,427.323) * 1,427.323=38,998.29-38,998.71=-0.46$ ~ 0

Second partial derivate:
$\left(800-2 e^{\widehat{\beta} 0+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 2}+\left(1400-2 e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2} \quad$ should be negative

Substituting the values given, we get
$(800-2 * 747.85) * 747.85+(1400-2 * 1,427.32) * 1,427.32$, which is negative

## WITH RESPECT TO $\beta_{0}$ :

For the first partial derivative to be zero:
$\left(1000-e^{\widehat{\beta} 0}\right) e^{\widehat{\beta} 0}+\left(2000-e^{\widehat{\beta} 0+\widehat{\beta} 1}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1}+$
$\left(800-e^{\widehat{\beta} 0+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 2}+\left(1400-e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}$ should be zero

Substituting the values given, we get
$(1000-1,037.59) * 1,037.59+(2000-1,980.305) * 1,980.305+(800-747.853) * 747.853+$
$(1400-1,427.323) * 1,427.323$
$=-39,003.01+39,002.70+38,998.29-38,998.75=-0.77 \sim 0$

Second partial derivate:
$\left(1000-2 e^{\widehat{\beta} 0}\right) e^{\widehat{\beta} 0}+\left(2000-2 e^{\widehat{\beta} 0+\widehat{\beta} 1}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1}+$ $\left(800-2 e^{\widehat{\beta} 0+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 2}+\left(1400-2 e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}\right) e^{\widehat{\beta} 0+\widehat{\beta} 1+\widehat{\beta} 2}$ should be zero

Substituting the values given, we get
$(1000-2 * 1,037.6)^{*} 1,037.6+(2000-2 * 1,980.3) * 1,980.3+(800-2 * 747.85) * 747.85$ $+(1400-2 * 1,427.3) * 1,427.3$, which is negative.

## Solution 12 :

(i)

A technical premium is one that reflects all of the expected costs and profits arising from the policy based on technical analysis. It consists of the following elements:
Risk premium

- the pure risk rate
- a loading for catastrophe and/or large loss claims

Office premium

- a loading for the cost of reinsurance
- a loading for expenses including commission
- a capital charge to reflect the cost of capital
- investment income
(ii) Following adjustments might be required to be made to the base data:

Claims experience tends to fluctuate over time. For certain classes, experience in a single year can be unusually heavy or light, particularly where the risk is affected by the climate. If the experience in the ideal base period does not appear to be typical, one should:

- choose another base year that is more typical
- aggregate more years' experience or
- apply an adjustment factor to the affected base year.

Large or exceptional claims - In addition to the attritional (normal) claims experience, the premium should also reflect the expected loss due to catastrophic and large claims. Often these claims are removed from the base attritional data to enable a reliable analysis. However, the premium needs to allow for the actual (expected) cost of these claims. We can estimate the required loading from the insurers' own data, provided sufficient experience is available. Otherwise we use external data for such claims, or a catastrophe model.

Trends in claims experience - if we detect trends in the base data for any of the components of the risk premium, we should give as much weight as possible to the latest years' experience or adjust the earlier years' experience.

We should investigate any trends that we have detected in the base data to see if they are likely to continue into the future or if they are a result of a one-off change - for example, in office or market practice. If we expect them to continue, we will need an assumption to allow for them in the projection of the risk premium.

The data needs to be adjusted for changes in risk and/or cover provided. Changes in the risk over time can be very awkward to deal with. Changes in the risk may arise because of changes in:

- the mix of underlying risks
- cover / policy conditions
- claims handling / underwriting strategy
- the method of distribution
- the level of reinsurance coverage.

Other adjustments:
Adjusting for inflation
Adjusting for environmental changes such as legislative factors, advances in technology, medical advances, changes in the construction of property.

Appropriateness of using open and closed claims data for pricing the policy:

Closed claim amounts do not usually change.

Open claims have an estimate of outstanding reserve. The final amount may be quite different, particularly when nothing has been paid.

Some companies may reserve (case estimates) prudently, in which case the estimates may be biased upwards.

Some companies may put an automatic reserve on all notified claims immediately without considering likelihoods.

All claim amounts look to have been rounded, so not sure how reliable they are.
Incurred amounts for open claims might not contain anything for business interruption or if they do then they may be based on unverified loss of profits.

There might be some losses that haven't been notified yet because of reporting delays or because the estimated amount isn't big enough yet.

We should analyse how open claims have moved historically and possibly adjust the open claims:

- closed claims may also re-open and move
- we should adjust for inflation
- older claims data may be less relevant than newer ones
- claims below excesses/deductibles may be missing
(iii)
a) Remove the extra excess amount from each claim.
b) We can add the gap between the excess amounts (old $\mathrm{v} / \mathrm{s}$ new excess) to each of the claims.
However, there may be claims we do not know about below old excess amount.
Ideally we would get all ground up claims from the insured.
Otherwise make an adjustment to the claims we have based on benchmark data such as an exposure curve.
c) Limit all claims to the revised limit.
d) We need to add on additional claims between the old limit and new limit.

We may not know this information for the limits claims i.e. the claims capped at old limits.
Ideally we would get all GU claims from the insured. or make an adjustment to the claims we have based on benchmark data, e.g. an exposure curve.

