Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2015 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1:

(i) Gross premium prospective reserve is the expected present value of future benefits and future expenses less the expected present value of future gross premiums.

Gross premium retrospective reserve is the expected accumulation of past gross premiums received, less past expected expenses and benefits.

[2]

(ii) Gross premium retrospective and prospective reserves will be equal if:

- the mortality, interest rate and expense basis used is the same as used to determine the original gross premium; and
- the gross premium is that determined on the original basis (mortality, interest, expenses) using the equivalence principle

[2]

(iii) Let P be the single premium. Then, by equivalence principle:

$$P = (B+R)a_x + I$$

$$\Rightarrow\ P = (B+R)(a_{x:\overline{t}|} + v^t\,{}_tp_x a_{x+t}) + I$$

$$\Rightarrow\ P - I - (B+R)a_{x:\overline{t}|} = (B+R)v^t\,{}_tp_x a_{x+t}$$

$$\Rightarrow [P - I - (B+R)a_{x:\overline{t}|}]\frac{(1+i)^t}{{}_tp_x} = (B+R)a_{x+t}$$

$$i.e.\ {}_tV^{retro} = {}_tV^{pro}$$

[4]

[8 Marks]

Solution 2:

(i) $$\ddot{a}_x = E(a_{\overline{K_x}|}) = E\left(\frac{1-v^{K_x+1}}{1-v}\right)$$

$$\Rightarrow\ \ddot{a}_x == \frac{1-E(v^{K_x+1})}{d} = \frac{1-A_x}{d}$$

$$\Rightarrow\ P_x == \frac{A_x}{\ddot{a}_x} = \frac{dA_x}{1-A_x}$$

[3]

(ii) Consider an insured *(x)* who borrows the single premium A_x for the purchase of a single premium unit whole life assurance.
The insured agrees to pay interest in advance of dA_x on the loan at the beginning of each year during survival and to repay the A_x from the unit death benefit at the end of the year of death.

In essence, the insured is paying an annual benefit premium of dA_x for a whole life insurance of amount $1- A_x$

Therefore, for a whole life insurance of amount 1, the annual premium must be $dA_x/(1-A_x)$ **[3]**

[6 Marks]

Solution 3:

We first calculate the expected present value for an n-year term insurance on (x) for which the benefit payable at the end of the year, in case death occurs in year k+1, is $\ddot{s}_{\overline{k+1|}}$. The present value random variable of this benefit at policy issuance is given by:

$$Z = \begin{cases} v^{K+1}\ddot{s}_{\overline{K+1|}} = \dfrac{1}{d^{(j)}}[v^{K+1}(1+j)^{K+1} - v^{K+1}] & ;0 \le K < n \\ 0 & ;K \ge n \end{cases}$$

where the present values are calculated at interest rate i and $d^{(j)}$ is the discount rate equivalent to interest rate j.

$$\Rightarrow E(Z) = \frac{A'^{1}_{x:\overline{n|}} - A^{1}_{x:\overline{n|}}}{d^{(j)}}$$

where $A'^{1}_{x:\overline{n|}}$ is calculated at the rate of interest $i'=(i-j)/(1+j)$

Now, let P be annual premium for the endowment assurance in question. Then

$$P\ddot{a}_{[40]:\overline{20|}} = 1{,}000{,}000\, A_{[40]:\overline{20|}}^{\ \ \ \ \ 1} + P\left(\frac{A'^{1}_{[40]:\overline{20|}} - A^{1}_{[40]:\overline{20|}}}{d^{(j)}}\right)$$

where j=1.923% and $A'^{1}_{[40]:\overline{20|}}$ is calculated at the rate of interest $i' = \dfrac{6\% - 1.923\%}{1+1.923\%} = 4\%$

$$\ddot{a}_{[40]:\overline{20|}}(@6\%) = 12.000 \qquad\qquad d^{(j)}(@6\%) = \frac{6\%}{1+6\%} = 0.01887$$

$$A_{[40]:\overline{20|}}^{\ \ \ \ \ 1}(@6\%) = \frac{1}{(1+6\%)^{20}} \times \frac{l_{60}}{l_{[40]}} = 0.3118 \times \frac{9,287.2164}{9,854.3036} = 0.29386$$

$$A^{1}_{[40]:\overline{20|}}(@6\%) = A_{[40]:\overline{20|}} - A_{[40]:\overline{20|}}^{\ \ \ \ \ 1} = 0.32076 - 0.29386 = 0.0269$$

$$A^{1}_{[40]:\overline{20|}}(@4\%) = A_{[40]:\overline{20|}} - \frac{D_{60}}{D_{[40]}} = 0.46423 - \frac{882.85}{2,052.54} = 0.0341$$

$$\therefore\ P = 25,292.55$$

[6 Marks]

Solution 4:

(i) Let P be the single premium. Then, the equation of value is:

$$P = 1{,}000\ddot{a}_{50} + 90 + 10\ddot{a}_{50} + 0.02P + P \times A_{50}$$
$$\Rightarrow P = (1{,}000 + 10) \times 17.444 + 90 + P(0.02 + 0.32907) \Rightarrow P = 27{,}204.83$$

[2]

(ii) $_{60}V = 1{,}000\ddot{a}_{60} + 10\ddot{a}_{60} + P \times A_{60}$

$$\Rightarrow {}_{60}V = (1{,}000 + 10) \times 14.134 + 27{,}204.83 \times 0.45640 = 26{,}691.62$$

[2]

(iii) $_{59}V = \dfrac{_{60}V(1 - q_{59}) + Pq_{59}}{1+i} + 1{,}000 + 10$

$$\Rightarrow {}_{59}V = \frac{26{,}691.62 \times (1 - 0.007140) + 27{,}204.83 \times 0.007140}{1.04} + 1{,}010 = 26{,}678.54$$

[2]

(iv) The death strain at risk per policy in the 10th year is given by:

$$DSAR = P - {}_{60}V = 513.21$$

$$\therefore EDS = DSAR \times 500 \times q_{59} = 1{,}832.16 \text{ and}$$
$$ADS = DSAR \times 1 = 513.21$$

Therefore, the mortality profit in the 10th year is given by:

$$EDS - ADS = 1{,}318.95$$

The total profit (from all sources) in the 10th year is given by:

$$500 \times ({}_{59}V - 1{,}000 - 9)(1 + 4\%) - P \times 1 - (500 - 1) \times {}_{60}V$$
$$= 1{,}837.59$$

Note that the interest rate profit in the 10th year is 0 since the actual interest rate earned is same as that expected on the valuation basis.

The expense profit in the 10th year is therefore the difference between the total profit and the mortality profit i.e. 1,837.59 – 1,318.95 = 518.64 **[6]**

[12 Marks]

Solution 5:

(i) The loss function is given by:

$L = 120,000 a_{\overline{K_{70}}|} - P$ where P is the single premium.

$E(L) = 0$

$\Rightarrow P = 120,000 E(a_{\overline{K_{70}}|}) = 120,000 a_{70} = 120,000(11.562 - 1) = 1,267,440$

[2]

(ii) $\Pr(L > 0) = \Pr(120,000 a_{\overline{K_{70}}|} > 1,267,440) = \Pr(a_{\overline{K_{70}}|} > 10.562)$

Note that $a_{\overline{13}|} = 9.9856$ and $a_{\overline{14}|} = 10.5631$

$\therefore \Pr(L > 0) = \Pr(K_{70} \geq 14) = {}_{14}p_{70}$

$$\Rightarrow \Pr(L > 0) = \frac{l_{84}}{l_{70}} = \frac{5,339.057}{9,238.134} = 57.79\%$$

[3]

(iii) If the single premium $P_{(k)}$ is set such that:

$L_k = 120,000 a_{\overline{k}|} - P_{(k)} = 0$ i.e. $P_{(k)} = 120,000 a_{\overline{k}|}$
then, the loss will be positive only if $K_{70} > k$ since for annuities, the loss will increase the longer the person lives.

$$\therefore \Pr(L_k > 0) = \Pr(K_{70} > k) = \Pr(K_{70} \geq k+1) = {}_{k+1}p_{70} = \frac{l_{k+71}}{l_{70}}$$

$\therefore \Pr(L_k > 0) \leq 0.05 \Rightarrow l_{k+71} \leq 0.05 l_{70}$ i.e. $l_{k+71} \leq 461.9067$

Note that $l_{97} = 611.905$ and $l_{98} = 459.696$

$\therefore \Pr(L_k > 0) \leq 0.05 \Rightarrow k + 71 = 98 \Rightarrow k = 27$

Hence the required premium is given by:

$120,000 a_{\overline{27}|} = 120,000 \times 16.3296 = 1,959,552$ **[5]**

[10 Marks]

Solution 6:

Let P be the initial premium. The premium at time j is then $P(1+i)^j$ where i = 4%. The equation of value is therefore given by:

$$100,000A_{[30]} = \sum_{j=0}^{\infty} P(1+i)^j \, {}_j p_{[30]} v^j$$

$$\Rightarrow 100,000A_{[30]} = P(1+\sum_{j=1}^{\infty} {}_j p_{[30]})$$

$$\Rightarrow P = \frac{100,000A_{[30]}}{1+e_{[30]}}$$

The per policy in force reserve at the end of 20 years is given by:

$${}_{20}V = 100,000A_{50} - \sum_{j=0}^{\infty} P(1+i)^{j+20} \, {}_j p_{50} v^j$$

$$\Rightarrow {}_{20}V = 100,000A_{50} - P(1+i)^{20}(1+\sum_{j=1}^{\infty} {}_j p_{50})$$

$$\Rightarrow {}_{20}V = 100,000[A_{50} - \frac{A_{[30]}(1+i)^{20}(1+e_{50})}{(1+e_{[30]})}]$$

$$A_{[30]} = 0.16011,\ A_{50} = 0.32907$$
$$e_{[30]} = 48.764,\ e_{50} = 29.565$$
$$\therefore {}_{20}V = 100,000[0.32907 - \frac{0.16011 \times 2.19112 \times 30.565}{49.764}] = 11,359.66$$

[8 Marks]

Solution 7:

$\ddot{a}_{\overline{50:50}:\overline{20|}} = \ddot{a}^{(m)}_{50:\overline{20|}} + \ddot{a}^{(f)}_{50:\overline{20|}} - \ddot{a}_{50:50:\overline{20|}}$

$\ddot{a}_{50:\overline{20|}} = \ddot{a}_{50}$ - $v^{20} * l_{70} / l_{20} * \ddot{a}_{70}$

For males, this is

$\ddot{a}^{(m)}_{50:\overline{20|}}$ = 18.843 - 1.04^(-20) * 9,238.134 / 9941.923 * 11.562 = 13.940

For females this is

$\ddot{a}^{(f)}_{50:\overline{20|}}$ = 19.539 - 1.04^(-20) * 9,392.621 / 9952.697 * 12.934 = 13.968

$\ddot{a}_{50:50:\overline{20|}} = \ddot{a}_{50:50}$ - 1.04^(-20) * 9, 238.134/9941.923 * 9,392.621/9952.697 * $\ddot{a}_{70:70}$

= 17.688 - 1.04^(-20) * 9, 238.134/9941.923 * 9,392.621/9952.697 * 9.7666

= 13.780

Therefore, $\ddot{a}_{\overline{50:50}:\overline{20|}}$ = 13.940 + 13.968 – 13.780 = 14.128

[6 marks]

Solution 8:

(i) The Standardised mortality ratio is the ratio of actual deaths in the population divided by the expected number of deaths in the population if the population experienced standard mortality. **[1]**

(ii) Actual number of deaths for Actuaria = 159+165+171 = 495

Age	Standard Population		Actuaria		Expected deaths (2/1*3)
65	2,500,000	22,462	12,700	159	114
66	2,200,000	23,000	11,000	165	115
67	2,000,000	23,791	10,876	171	129
Total	**6,700,000**	**69,253**	**34,576**	**495**	357

Expected number of deaths for Actuaria

= 22,462/2,500,000 × 12,700 + 23,000/2,200,000 × 11,000 + 23,791/2,000,000 × 10,876 = 357

SMR = 495/357 = 138.5% **[3]**

(iii) The mortality rates in Actuaria are significantly higher (around 39%) compared to the standard population. **[1]**

(iv) Mortality trends in Actuaria could be adverse due to the following:

- Sub-standard housing
- Poor quality nutrition
- Low levels of education
- Relatively less healthy occupations
- Genetic effects
- Difficult geography and climate **[3]**

[8 Marks]

Solution 9:

(i)

Year	Age	Profit vector	qx	Px	p(x-1)	Profit signature	Present value
1	30	-250	0.00059	0.99941	1	(250.00)	(235.85)
2	31	-400	0.000602	0.999398	0.99941	(399.76)	(355.79)
3	32	-600	0.000617	0.999383	0.998808	(599.29)	(503.17)
4	33	1500	0.000636	0.999364	0.998192	1,497.29	1,185.99

NPV @ 6% = 91.18 **[3]**

(ii) 2V = 600 / 1.03 = 582.52

1V = (400 + 582.52 * 0.99941) / 1.03 = 953.57

Year 1 revised cashflow = -250 - 953.57 * 0.99941 = -1203

NPV = -1203 / 1.06 + 1185.99 = 51.08 **[3]**

(iii) The NPV after zerosising negative cashflows to achieve a single financing phase is smaller. This is because the negative cashflows have been accelerated hence being discounted less. **[1]**

[7 Marks]

Solution 10:

(i) Decrement table

Age	Ind prob death	Ind prob surr	Dep prob surr	Dep prob mort	Prob surv at end	Prob surv at start
61	0.006433	0.06000	0.05981	0.00624	0.93395	1.00000
62	0.009696	0.06000	0.05971	0.00941	0.93089	0.93395
63	0.011344	0.06000	0.05966	0.01100	0.92934	0.86940
64	0.012716	0.06	0.05962	0.01233	0.92805	0.80797

Unit fund

Year	1	2	3	4
Premium	30,000	30,000	30,000	30,000
Fund at start	-	11,790	38,723	69,525
Allocated premium	12,000	27,000	30,000	33,000
Less: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Fund before interest	11,400	37,440	67,223	100,875
Plus: Interest	570	1,872	3,361.14	5,043.76
Fund after interest	11,970	39,312	70,584	105,919
Less: FMC	180	590	1,059	1,589
Fund at end	11,790	38,723	69,525	104,330

Non-unit fund

Unallocated prem	18,000	3,000	-	(3,000)
Add: Bid offer spread	600	1,350	1,500	1,650
Policy fee	-	-	-	-
Less: Commission	6,000	600	600	600.00
Less: Expenses	5000	2000	2000	2000
Fund before interest	7,600	1,750	(1,100)	(3,950)
Add:Interest	266.00	61.25	(38.50)	(138.25)
Less: Maturity benefit				4,841.16
Add: FMC	180	590	1,059	1,589
Less: Additional death benefit	550.43	576.32	335.34	-
Non-unit cash flow	7,495.12	1,824.62	(415.08)	(7,340.63)

Profit margin

Year	1	2	3	4	
Profit	7,495.12	1,824.62	(415.08)	(7,340.63)	
Prob surv at start	1	0.93395	0.86940353	0.80797	
Discount factor profit	0.934579439	0.873438728	0.816297877	0.762895212	
PV profit	7,004.79	1,488.43	(294.58)	(4,524.73)	**3,673.91**
Discount factor premium	1.00	0.93	0.87	0.82	
PV pemium	30,000.00	26,185.60	22,781.12	19,786.29	**98,753.01**
Profit magin	**3.72%**				

[15]

(ii) Profit margin could be higher or lower depending on the definition of the surrender benefit. If a sizeable surrender penalty is imposed such that the unit fund mostly covers the surrender benefit, profit margins could improve. **[2]**

(iii) To cover high initial expenses **[1]**

[18 Marks]

Solution 11:

Selection is the process by which lives are divided into separate groups so that the mortality (or morbidity) within each group is homogeneous. That is, the experience of all lives within a particular group can be satisfactorily modelled by the same stochastic model of mortality (or morbidity). Lives in different classes will be charged according to different premium scales, which reflect the mortality differences between the classes.

Types of mortality selection

1. Temporary initial selection: Each group is defined by a specified event (the select event) happening to all the members of the group at a particular age, eg buying a life assurance policy at age x, retiring on ill-health grounds at age x. A select mortality table (representing the stochastic model of mortality) is estimated for each group. The mortality patterns in each group are observed to differ only for the first s years after the

select event. The length of select period is s years. The differences are temporary, producing the phenomenon called temporary initial selection.

2. Class selection: Each group is specified by a category or class of a particular characteristic of the population, eg sex with categories of male and female, occupation with categories of manual and non-manual employment. The stochastic models (life tables) are different for each class. There are no common features to the models, they are different for all ages. This is termed class selection.

3. Time selection: Within a population mortality (or morbidity) varies with calendar time. This effect is usually observed at all ages. The usual pattern is for mortality rates to become lighter (improve) over time, although there can be exceptions, due, for example, to the increasing effect of AIDS in some countries.

4. Adverse selection: Adverse selection usually involves an element of self-selection, which acts to disrupt (act against) a controlled selection process which is being imposed on the lives. This adverse selection tends to reduce the effectiveness of the controlled selection.

5. Spurious selection: When homogeneous groups are formed we usually tacitly infer that the factors used to define each group are the cause of the differences in mortality observed between the groups. However, there may be other differences in composition between the groups, and it is these differences rather than the differences in the factors used to form the groups that are the true causes of the observed mortality differences.

[6 Marks]

Solution 12:

$tp_{40}^{\overline{SS}}$ = exp ($\int_0^t (\rho_{x+s} + \gamma_{x+s}$)ds

Since the transition intensities are assumed to be constant, the expression simplifies to:

$tp_{40}^{\overline{SS}} = e^{-t\,(\rho+\,\nu)}$

EPV of sickness benefit = 20000 * $\int_0^{20} e^{-\delta t}\ * tp_{40}^{\overline{SS}}$ dt

= 20000 * $\int_0^{20} e^{-(\delta+\rho+\,\nu)t}$ dt

= -20000 / $(\delta+\rho+\ \nu)$ * $e^{-(\delta+\rho+\,\nu)t}$ from 0 to 20

= 20000 / (ln 1.04 + 0.05) * [1 - e$^{-20\,(\ln 1.04+0.05)}$]

=186527.2

[5 Marks]

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