INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

14th May 2015

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Following is a part of conversation between two friends:

A: XYZ bank is expected to declare good quarter end results tomorrow. You can buy its shares.

B: Since you already know this information, many other people know it too. I think the current stock price already reflects all the information made public and also that is yet to be made public. I would rather make higher returns based on trading rules looking at the historical price charts and patterns therein instead of relying on 'tips'. After all there is also a chance that the news is wrong and the results are bad in reality.

i) If the Efficient Market Hypothesis is assumed to be true. State giving reasons whether B's actions are consistent with his statements. (3)

Stock did rally the next day after the quarter end results were publically announced.

ii) What level of market efficiency does this market exhibit? (2)

The price patterns indicated that the stock could be sold now and bought at a lower level. However B did not proceed with the trade fearing the news turns out to be true.

iii) Explain which theories of behavioural finance may be applied to B's approach? (2)
[7]

Q. 2) In country X, there are 1250 people with the following wealth distribution.

Wealth Bracket	Number of People
< 0.5 unit	100
0.51-1.5 units	500
1.51-2.5 units	600
> 2.51 units	50

The population can invest only in one deposit as of now. The Company pays 20% interest on the deposit but there is 10% probability that the company may default on the interest and repays only the principle. It is assumed that the Utility Theory holds and all the investors have a utility function defined by:

$$U(w) = w^{-2} + w$$

i) Determine the risk characteristics of the population. (5)

The government wants to raise funds by offering one year bonds to the population.

ii) State giving reasons if the Bank will be successful in mobilizing the funds if it offers 15% guaranteed rate of return. If no, why not and If yes, how many individuals will subscribe to the bond issue? (4)

The government suspects that at any point in time the older members will place higher utility on their wealth than the younger people. The advisors to the government have suggested two other utility functions to reflect such behaviour:

$$Eq1: U(w,x) = w^{-2} + w + e^{-x/100}$$

$$Eq2: U(w,x) = w^{-2} + w + \ln(x+w)$$

iii) Determine which function appropriately captures the age related behaviour for the given level of wealth. (4)

[13]

Q. 3) Consider an at the money American put option on the geometric average of price of a non dividend paying stock currently priced at Rs. 200. Risk free interest is 10% per annum, volatility is 35% per annum and time to maturity is 2 months.

i) Determine values of u and d assuming stock price follows Geometric Brownian motion with drift μ and volatility σ stating any other assumption used. (5)

ii) Evaluate the value of an American put option using a 2-time step. The geometric average is measured from today till the option matures (4)

[9]

Q. 4) Consider a European put option on a non –dividend paying stock when the stock price is Rs. 300, strike price is Rs. 320, risk free rate is 6% per annum, and the time to maturity is one year. Suppose that the average variance rate during the life of an option has a 0.20 probability of being 0.06, a 0.5 probability of being 0.09 and a 0.3 probability of being 0.12. Estimate the value of the option assuming that the Volatility is uncorrelated with the stock price. State any other assumptions that you make. **[8]**

Q. 5) An overseas investment bank trades options on dollar-euro exchange rates. The exchange rate (dollars per euro) is 0.88. The dollar-euro exchange rates have a delta of 30,000 and a gamma of (-80,000).

i) Explain how these numbers can be interpreted. (2)

ii) What position would the bank take to make the position delta neutral? (1)

iii) After a short period of time, the exchange rate moves to 0.92. Estimate the new delta. (1)

iv) What additional trade is necessary to keep the position delta neutral? (1)

v) Assuming the bank did set up a delta-neutral position originally, has it gained or lost money from the exchange rate movement? (1)

[6]

Q. 6) In some country, the interest rate are modelled as per Cox–Ingersoll–Ross (CIR) model with the following parameters
$\propto = 0.2$, $\mu = 0.08$, $\sigma = 0.1$
The bond price under CIR model is given by:
$B(t,T) = e^{a(\tau)-b(\tau)r(t)}$
Where

$$b(\tau) = \frac{2(e^{\theta\tau}-1)}{(\theta+\alpha)(e^{\theta\tau}-1)+2\theta}$$

$$a(\tau) = \frac{2\alpha\mu}{\sigma^2}\log\left(\frac{2\theta(e^{(\theta+\alpha)\tau/2}}{(\theta+\alpha)(e^{\theta\tau}-1)+2\theta}\right)$$

$$\theta = \sqrt{\alpha^2+2\sigma^2}$$
$$\tau = T - t$$

i) Write the Stochastic Differential Equation (SDE) for the interest rate under Q (0.5)

Corporate bonds are also traded in above country and an analyst intends to model the corporate bond by allowing for the credit risk by tweaking the SDE for the interest rates. He adds another term $\varphi\, r(t)dt$ to the SDE.

ii) Write the revised SDE and rework the value of the parameters for CIR model if $\varphi = 0.06$ (1.5)

iii) Find out the corporate bond price at time 5 and 10 years with the parameters determined in (ii) above if the current rate of interest is 7% (7)

iv) Define giving formulae what is spot rate, forward rate and instantaneous forward rate and their relationship with the bond price? Evaluate the spot rate at time 10 and the forward rate between time 5 and 10 for the corporate bond. (5)

[14]

Q. 7) A company declares dividend on its shares (face value Rs. 100) currently priced Rs. S_0 as per following:

3% on the face value with a probability of 95%
0% with a probability of 5%

If the Company declares dividend in the preceding year, the share price increases by 5%. If no dividend is declared the investor sells the shares immediately at a price as that at the start of the year.

i) Determine the expected cumulative cash-flows in terms of S_0 at the end of 1^{st}, 2^{nd} and k^{th} year. (4)

ii) Using the results in (i) and assuming that the share is trading at its fair price, determine the fair value of the share price if risk free rate of return is 8.00% (5)

[9]

Q. 8) **i)** Assume that the mean and standard deviation of market portfolio is (μ_m, σ_m). An investor constructs a portfolio with a% in security B and (1-a)% in M. Write the equation of returns and variance on the investor's portfolio. (2)

ii) Prove that the slope of the frontier on μ-σ space traced by the investor's portfolio at a=0 is given by:

$$\frac{\mu_b - \mu_m}{(\sigma_{bm} - \sigma_m^2)/\sigma_m}$$ (4)

iii) You are also given that the slope of the efficient frontier at (μ_m, σ_m) is $\frac{\mu_m - \mu_z}{\sigma_m}$, (where 'z' is any frontier portfolio uncorrelated with 'm'). Equate this with the slope obtained in step (ii) and simplify the equation. (2)

iv) Identify the equation obtained in (iii) and state where it can be used. (2)

[10]

Q. 9) Let the stock price S_t follow Geometric Brownian motion with drift μ and volatility σ

i) Find the mean, variance and 95% confidence interval of S_t (7)

ii) You are given that the fair price to pay at time t for a derivative paying X at time T is $V_t = e^{-r(T-t)} E_Q[X|\, F_T]$ where Q is the risk-neutral probability measure and F_t is the filtration with respect to the underlying process. Show that the fair price to pay at time t for a forward on this share, with forward price K and time to expiry T - t , is: $V_t = S_t - Ke^{-r(T-t)}$ (4)

iii) Verify the above using an alternate approach (3)

[14]

Q. 10) Suppose $f = f(t, S_t) = S_t^{-2r/\sigma^2}$ where S_t follows Geometric Brownian motion with drift μ and volatility σ

i) Show that the process is a martingale for certain value of μ while stating your assumptions. Can this be price of a traded security? (6)

ii) Suppose Ornstein Uhlenbeck process is a solution to the equation

$dX_t = -\gamma X_t dt + \sigma dB_t$, γ and σ are positive parameters

Find the expression of such a process. Is this process a martingale and why? (4)

[10]

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