INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

06th May 2015

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) Claims of an Insurance company follow Poisson process with parameter λ and the individual claim amounts follow exponential distribution with parameter β. The company reinsures 80% of its portfolio exceeding the retention level of M. The reinsurers retained part is given by following formula:

$$Z = \begin{cases} 0, x < M \\ 0.8(x - M), x \geq \text{M} \end{cases}$$

$$where, x\ is\ claim\ amount.$$

The premium loading factor used by the insurer and reinsurer are 25% and 60% respectively.

i) Describe the range of M such that the insurer does not incur any losses $[\beta = 0.01]$ (5)

ii) Deduce the moment generating function $M_Y(R)$; Y is the net claim payable by the insurer. (3)

iii) Hence prove that the adjustment co-efficient equation will follow the below equation:

$$0.8\,\beta^2 e^{MR} - \beta e^{M\beta}(\beta - 0.2R) + (\beta - 0.2R)(\beta - R)[1.25e^{\beta M} - 1.28] = 0$$ (3)

iv) Find out adjustment co-efficient, R using M = 50, $\beta = 0.01$
Use the following approximation:

$$e^x = 1 + x + \frac{x^2}{2}$$ (2)

v) If the insurance company had a proportional reinsurance with 20% retention by the insurer then find out the R (2)

vi) If the company's initial surplus is 100 then calculate maximum long-term ruin probability under part (iv) and (v) and comment. (2)

[17]

Q. 2) An Actuary loves watching theatre. Each Sunday, he visits one of four possible locations; he goes to INOX with probability 1/3, to PVR with probability 1/6, to Cinemax with probability 1/6, to Fun cinemas with probability 1/3. If he goes to INOX there is a 70% chance that he will get the tickets for the show; corresponding figures for PVR, Cinemax and Fun Cinemas are 30%, 50% and 80% respectively.

i) If on a particular Sunday, he comes home without watching theatre (because of not getting tickets), where is it most likely that he has been? (4)

ii) Find the probability that he watches theatre on at least two of the three consecutive Sundays. (2)

iii) His friend, who also goes for the theatre every Sunday, chooses the location with above probabilities. Find the probability that the two friends will meet at least once in the next two weekends. (3)
Assume: The Actuary and his friend chooses locations independently in different weeks and independent of one another. They will meet in the theatre after getting the tickets.

[9]

Q. 3) A street food stall owner specializes in making following three items: “masala dosa”, “noodles” and “pizza”. He gets an option to showcase his cooking skills in an upcoming mall for a day. However he is allowed to cook and sell only one of the above mentioned food items. His profit would depend on the average footfall in the mall which can be low, normal and high. He also knows that it is equally likely that the mall will experience low or high footfalls. The chance that the mall will experience normal footfall is twice the chance of experiencing high footfall.

He estimates his profit under each possible scenario as

	Low	**Normal**	**High**
Masala Dosa	850	950	1200
Noodles	900	800	1500
Pizza	500	875	1400

i) Determine the best of the worst possible criterion and the maximax solution to this problem. (2)

ii) Determine the Bayes Criterion Solution to this problem. (2)
[4]

Q. 4) An insurance company introduces a one year health insurance product which pays a fixed benefit upon surgical procedures as specified in the policy contract. The maximum no of claims permissible under the contract is limited to 2.

The benefit payable upon surgery is divided into two categories: minor and major where Minor surgery Benefit = Rs. 100000 and Major Surgery Benefit = Rs. 200000.

The probability associated with minor and major surgical claims are 0.7 and 0.3 respectively.

Assuming that the no of claims from each policy follows a discrete distribution with the following probability function:

Probability (number of claims equals 0) = 0.7

Probability (number of claims equals 1) = 0.2

Probability (number of claims equals 2) = 0.1

Derive the distribution function of the aggregate claim amount from an individual policy over the coming year. **[7]**

Q. 5) An Insurance company ABC has an excess of loss reinsurance contract with retention Rs. 100000 under a particular class of business.

Last year the following claims data were observed

55000, 33200, 71080, 43120, 13215, 91030, 80000 and 4 claims over 100000

It is assumed that the individual claims gross of reinsurance follow an exponential distribution with parameter μ

i) **a)** Estimate μ by applying the method of percentiles (hint: use the median claim). (3)

b) Estimate μ using the method of maximum likelihood estimate. (5)

Another Insurance Company ABC has a policyholder excess of Rs. 10000 on every policy sold under a particular class of business.

The individual claim amounts are assumed to follow a Pareto distribution with parameters (α, 150000).

ii) **a)** Prove that the conditional distribution of the amount paid by the insurer has a Pareto (α,160000) distribution with pdf

$$s(y) = \alpha * 160000^{\alpha} / (160000+y)^{\alpha+1}$$ (4)

The amount paid by the insurer after deducting the excess on the last 10 claims were observed to be 100000, 80000, 73200, 225300, 45000, 60100, 30000, 300000, 145000, 133330.

b) Use this information to find the maximum likelihood estimate of α. (5)

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Q. 6) The past year's claim data of an insurance company is given in the table below:

Claim Incidence Year	Claim Intimation Year	No. of Claims	Total Claim paid
2012	2012	50	45,000
2012	2013	35	32,375
2012	2014	25	24,375
2013	2013	95	80,275
2013	2014	60	57,000
2014	2014	80	72,800

Past inflation experience is 5% and expected future inflation is 10%.

i) Calculate the outstanding claim reserve using the average cost per claim method with weighted average development factors. (7)

ii) State the assumptions (1)

iii) Test the fitting of the model for the Total claim paid and comment on the output (4)

iv) If all the individual development factors follow the log normal distribution with below parameters, then calculate the probability that the outstanding claim reserve for Incidence year 2014 will lie between 1,50,000 and 2,00,000 (6)

Parameters for average cost per claim $\mu_1 = 0.001675$ and $\sigma_1 = 0.03825$
Parameters for number of claims $\mu_2 = 0.190225$ and $\sigma_2 = 0.45475$

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Q. 7) **i)** $X_t = A_t + B_t$; $A_t = 0.5X_{t-1} + e_t^{(A)}$
$B_t = 0.7(A_{t-1} - A_{t-2}) + e_t^{(B)}$; $Y_t = \begin{pmatrix} A_t \\ B_t \end{pmatrix}$

where $e_t^{(A)}$ and $e_t^{(B)}$ are zero mean white noise process.

Determine whether Y_t *is a stationary process.* (3)

ii) A time series model is given by the following equation:
$X_t = (\alpha + 1)X_{t-1} - (\alpha + 0.25\alpha^2)X_{t-2} + 0.25\alpha^2 X_{t-3} + e_t$
where e_t is a white noise process with variance σ^2

a) Check whether the above can be expressed as ARIMA(ρ, d, ν) process, specifying the range of α (2)

b) If the auto-covariance function (γ_K) of the above ARIMA process follows the below equation:
$\lambda_1 + k\lambda_2 = (0.5\alpha)^{-K}\gamma_K$,

then find λ_1 and λ_2 in terms of α and σ^2 (11)

c) If $x_1, x_2, \ldots x_{50}$ are the observed values of X_t then forecast the next two observation x_{51} and x_{52}, assume $\alpha = 0.04$. (2)

[18]

Q. 8) Profit Insurance Company sells 3500 policies under a particular category of business last year. The policies are assumed to be independent and at most one claim can be made on any policy. The probability of making a claim q is the same for all policies.

The total number of claims in the previous year was found to be p.

The prior distribution of q is Beta (α, β)

i) Find maximum likelihood estimate of q and posterior distribution of q given the past year data. (5)

ii) Find the Bayesian estimate of the posterior distribution under quadratic loss function. If p = 500, α =1 and β = 4, find the value of the Bayesian estimate. (2)

iii) Can the Bayesian estimate of q be written in the form of a credibility estimate? If yes, express the same in the form of a credibility estimate and compute the credibility factor for the above values mentioned in part (ii). (3)

[10]

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