INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

07th May 2015

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet.*

4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) T_x is a random variable representing the future lifetime of a person aged x. F (Tx) is the distribution function of T_x, and S_x is defined as 1- F (T_x).

i) Explain what is represented by Sx and derive a relation for the density function of Tx in terms of Sx and the force of mortality μ. (3)

ii) Explain what is meant by expectation of life (e_x) and derive a relation between e_x and Sx. (2)

iii) The expected future lifetime of a new born (e_0) is 60 years, that of a person aged 40 years (e_{40}) is 25 years and that of a person aged 55 years (e_{55}) is 12 years. Give reasons as to why e_{40} is not = e_0 – 40 i.e 20 years and why e_{55} is not = e_0 – 55 i.e 5 years. (2)

[7]

Q. 2) A CEO of an insurance company wants to understand the difference in stochastic and deterministic modeling of actuarial work. He asked his actuary to prepare a memorandum for the same. In this context a senior actuary asked his junior to prepare a note on the following:

i) List the benefits of modeling in actuarial work. (2)

ii) Describe the difference between a stochastic and a deterministic model. (2)

iii) Outline the factors you would consider in deciding whether to use a stochastic or deterministic model to study a problem. (3)

iv) Explain how a deterministic model might be used to validate model outcomes where a stochastic approach has been selected. (2)

[9]

Q. 3) At the end of each year, The Credit Rating Company assesses the credit worthiness of the debt issue by company. The credit rating A (the most credit-worthy), B and D (debt defaulted) Historic evidence supports the view that the credit rating of a debt can be modeled as a Markov chain with transition matrix

$$\mathbf{P} = \begin{array}{c} \\ A \\ B \\ D \end{array} \begin{vmatrix} A & B & D \\ 1-a-a^2 & a & a^2 \\ a & 1-2a & a \\ a^2 & a & 1-a-a^2 \end{vmatrix}$$

for some parameter .

i) Draw the transition graph of the chain. (2)

ii) Determine the range of values for '**a**' for which the matrix *P* is a valid transition matrix. (2)

iii) Explain whether the chain is irreducible and/or aperiodic. (1)

iv) For a = 0.2, calculate the proportion of employees who, in the long run, are instate D. (3)

v) Given that a= 0.2, calculate the probability that a debt rating in the third year, $X3$, is D:

a) in the case that the debt s rating in the first year, $X1$, is A.
b) in the case $X1 = B$.
c) in the case $X1$ = D. (3)

[11]

Q. 4) **i)** Derive the likelihood function for observing exactly 'd' deaths in an investigation covering 'N' identical, independent lives aged exactly 'x' for one year, assuming the number of deaths follows a Binomial model. (2)

ii) An insurance company conducted the mortality investigation of its annuitants aged 70 years using the above approach. 10,000 annuitants aged exactly 70 years were observed for 1 year and 1,820 deaths were observed. Give the 95% confidence interval for the rate of mortality q_{70}. (3)

iii) A company is considering using the above model for mortality investigation of its 'employees'. State any constraints that may arise in using the model. (2)

iv) Suggest any approximations that can be made to the model to overcome the problems cited in (iii). (2)

[9]

Q. 5) Consider a time-homogeneous Markov jump process $\{X(t) : t \geq 0\}$ with two states denoted by 0, 1, and transition rates $\sigma_{0,1} = \lambda$, $\sigma_{1,0} = \mu$.

i) State Kolmogorov's forward equation for the probability $P_{0,0}(t)$ that X is in state 0 at time t, given that it starts in state 0. (1)

ii) Show that $P_{0,0}(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} * \exp -(\lambda + \mu)\, t$. (3)

iii) Let O_t denote the total amount of time spent in state 0 up until time t, which

may be expressed as $O_t = \int_0^t Is\, ds$ where, $Is = \begin{cases} 1, & Xs = 0 \\ 0, & Xs\ otherwise \end{cases}$.

Derive using the result in part (ii), an expression for $\mathbf{E}[O_t|X(0) = 0]$, the expected occupation time in state 0 by time t for the two-state continuous-time Markov chain starting in state 0. (2)

iv) Write down the expected occupation time in state 1 by time t for the two state continuous-time Markov chain starting in state 0. (1)

v) A health insurance scheme labels members as “healthy” (state 0) or “unhealthy” (state 1) at any time. When in state 0, members pay contributions at rate ‘a’; when in state 1 they receive benefit at rate ‘b’.

Expenses amount to a constant ‘c’ per member per unit time.

a) Explain how the above model can be used to calculate ‘a’ in terms of ‘b’ and ‘c’.

b) State the assumptions which you make in applying the model.

c) Discuss whether they are likely to be satisfied in practice. (3)

[10]

Q. 6) A life company writes a significant proportion of health insurance business. The premiums for the policy are based on age last birthday at the start of the policy. The policy covers multiple claims during the term i.e the policy does not exit on a claim, it only exits due to lapsation or the policy term getting completed. The company is interested in analyzing the morbidity experience and has asked its health team to collect suitable data for the investigation. The team has submitted the following data.

- h_x number of claims at age x where x is defined as ‘Age last birthday on the policy anniversary prior to making the claim’
- $P_{x,t}$ = Number of policies in-force as at 1st April classified as age x last birthday at time t (t = time elapsed from 1st April 2013),

for the period from 1st April 2013 to 31st March 2015.

i) Explain what is meant by a ‘rate interval’ and give the rate interval as per the above data. (2)

ii) Derive an expression for the central exposed to risk E_x^c of the analysis, clearly stating the assumptions made. (3)

The following is the summary of the data for ages 44 to 46

	$P_{x,t}$		
t/x	**44**	**45**	**46**
0	30,520	29,947	29,120
1	30,901	30,479	30,103
2	30,990	30,253	30,437
3	31,529	30,839	29,602

x	Number of claims
44	1,214
45	1,321
46	1,452

iii) Estimate μ_{45} and state the exact age to which the μ applies. (4)

iv) It is given that about 40% of the new business is sold in the last quarter of the financial year. Describe if your results would be different, in light of this information. (2)

v) Explain how you would estimate q_x from the above data. (2)
[13]

Q. 7) A large life insurance company selling various products like term assurance, unit-linked, with-profits, savings, pension analyses its mortality experience on an annual basis. This experience is compared with the standard mortality table (IALM 2006-08) and the company sets its expected mortality rates for the future as a % of the standard table. Currently the expected rate is 70% of the standard table.

The mortality analysis has been done this year as well and a student has proposed that the experience should be compared with the last year's experience and if the mortality underlying the portfolio is same as that of last year then the expected mortality rates can be left unchanged.

The deaths in each of last year and this year, and the expected deaths as per the company's expected mortality rates (for quinquennial ages) is given below.

	Last Year		**This Year**	
Age	**Expected Claims**	**Actual Claims**	**Expected Claims**	**Actual Claims**
25	810	808	724	719
30	1,708	1,851	1,433	1,322
35	1,084	1,400	444	500
40	1,705	1,562	1,397	1,207
45	1,572	1,366	1,465	1,177
50	1,643	1,296	1,209	905
55	2,911	2,200	2,436	1,798

i) Test the hypothesis of the student using a chi – squared test. (8)

ii) Explain why the approach suggested by the student may not be appropriate. (2)

iii) Another student has suggested that crude death rates observed from the data should be graduated before comparing with the standard table. Explain why graduation may be needed. (2)
[12]

Q. 8) **i)** Explain the difference between informative and non-informative censoring with suitable examples. (2)

ii) A group of 10 patients (K-T) suffering from cancer are being investigated for the effects of a new medicine. A group of another 10 patients (A-J) who have been administered an earlier medicine are also being observed. The details of these patients are given below.

Old Medicine		
	Week of exit	**Reason for exit**
A	8	Death due to cancer
B	6	Death due to heart attack
C	5	Death due to cancer
D	-	-
E	6	Death due to cancer
F	9	Went abroad for further treatment
G	3	Death due to cancer
H	-	-
I	5	Death due to cancer
J	10	Was discharged for marriage of son and returned in 12th week

New Medicine		
	Week of exit	**Reason for exit**
K	11	Death due to cancer
L	10	Went to village to spend rest of lifetime there
M	3	Committed suicide
N	8	Death due to cancer
O	9	Discharged on family insistence
P	-	-
Q	7	Death due to heart attack
R	10	Death due to cancer
S	-	-
T	-	-

Calculate the Kaplan-Meier estimate of Survival function for each of the above 2 treatments and comment on your results. (6)

iii) A patient who was on the old medicine was administered the new medicine after 8 weeks on the old medicine. Find the probability of surviving for another 9 weeks on the new medicine. (1)

[9]

Q. 9) **i)** Define following stochastic process:

a) Poisson process
b) Compound Poisson process
c) Thinning of poisson process
d) Markov jump process. (4)

ii) Explain the condition needed for such a process to be time homogeneous. (2)

iii) Outline the principal difficulties in fitting a Markov jump process model with time-inhomogeneous rates. (2)

[8]

Q.10) A team of medical researchers is interested in assessing the effect of a certain condition on mortality. The condition is, in itself, non-fatal and is curable, but is believed to increase the risk of death from heart disease. The team proposes to use a model with four states: (1) "Alive, without condition", (2) "Alive, with condition", (3) "Dead from heart disease" and (4) "Dead from other causes".

i) Draw a diagram showing the possible transitions between the four states. (2)

Let the transition intensity between state i and state j at time x+t be μ_{x+t}^{ij}.
Let the probability that a person in state i at time x will be in state j at time x+t be ${}_tP_x^{\,ij}$.

ii) Show, from first principles, that

$$\frac{d}{dt}\left({}_tP_x^{\,24}\right) = {}_tP_x^{\,21}\,\mu_{x+t}^{14} + {}_tP_x^{\,22}\,\mu_{x+t}^{24}$$ (5)

An empirical investigation using data for persons aged between 60 and 70 years produces the following results:

Waiting time in state "Alive, without condition" is 2,046 person-years
Waiting time in state "Alive, with condition" is 1,139 person-years
10 deaths from heart disease to persons "Alive, without condition"
30 deaths from other causes to persons "Alive, without condition"
25 deaths from heart disease to persons "Alive, with condition"
20 deaths from other causes to persons "Alive, with condition"

iii) Show that there is a statistically significant difference (at the 95% confidence level) between the death rates from heart disease for persons with and without the condition. (5)

[12]

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