# INSTITUTE OF ACTUARIES OF INDIA

# EXAMINATIONS

## 05$^{th}$May 2015

## Subject CT3 – Probability & Mathematical Statistics

## Time allowed: Three Hours (10.30 – 13.30 Hrs.)

## Total Marks: 100

### INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

| **AT THE END OF THE EXAMINATION**<br><br>**Please return your answer book and this question paper to the supervisor separately.** |
|---|

**Q. 1)** **i)** Given that, for any two events A and B, P [A U B] = 0.6 and P [A U $B^C$] = 0.8.
Find P [A] (3)

**ii)** In a certain large population, 2% of people have 6 fingers. A random sample of 400 people is chosen from this population. Calculate approximate value for the probability that 8 or more number of people in the sample have 6 fingers. (3)

**iii)** Simulate three observations from exponential distribution with mean 5 using the following three random numbers from U(0,1)

0.0923, 0.8657 and 0.3494 (3)

**iv)** The Annual pay packages (in Lakhs of Rupees) of 9 actuarial students are given below:

2, 6, 16, 8, 5, 7, 5, 9, 3.

Present this data graphically using the Boxplot and comment on the distribution. (3)
**[12]**

**Q. 2)** A company sells a particular brand of a mobile at Rs. 10,000 per piece. It guarantees refund as follows. Full cost of the mobile will be refunded to the buyer if it fails during the first year, half the cost if it fails after one year but before two years, and no refund if it fails after two years. The life time of the mobile follows exponential distribution with mean two years,

**i)** Find the probabilities of the mobile failing in first year, in second year and after two years. (3)

**ii)** If the company sells 500 mobiles, calculate expected amount of refund by the company. (3)
**[6]**

**Q. 3)** A laptop retailer procures 40%, 30%, 20% and 10% of laptops from four manufacturers A, B, C and D respectively. It is known that 15%, 10% 5% and 3% of laptops received from the respective manufacturers are faulty. A laptop is randomly chosen from the retailer and is found to be defective.

Find the probability that it is manufactured by B? **[4]**

**Q. 4)** A general insurance company has designed a one year motor insurance policy in such a way that if a policyholder claims for the first time, he will get Rs. 5000 and for the subsequent claims he will get Rs. 2500 each. Obviously, the policyholder will not get any amount if he has not filed any claim during a policy year.

An actuary has made an assumption that for all integers n ≥ 0, $P_{n+1} = 0.6\,P_n$ where $P_n$ represents the probability that the policyholder files n claims during the period.

**i)** Find the distribution of the number of claims arising on the motor insurance policy. (5)

**ii)** Find expected value and standard deviation of the claim amount for each policy. (3)
**[8]**

**Q. 5)** Consider the discrete random variable X with probability function

$P(X = x) = \frac{2}{3^{x+1}}; \; x = 0,1,...$

**i)** Find the moment generating function of X (3)

**ii)** Obtain E (X) and Var (X). (3)

**[6]**

**Q. 6)** In a bunch of insurance policies, let N be the number of claims with the following distribution.

| N | 0 | 1 | 2 |
|---|---|---|---|
| $P(N = n)$ | $^2/_5$ | $^1/_2$ | $^1/_{10}$ |

The claim amounts $X_i$ : i = 1 and 2 are iid random variables with $P(X_i = 1000) = 0.7$ and $P(X_i = 2000) = 0.3$. Assume that N and $X_i$ are independent.

**i)** Find the distribution of the total claim amount $S_N$. (5)

**ii)** Find $E(S_N)$ using the distribution of $S_N$ (2)

**iii)** Verify (ii) using $E(S_N) = E(N)E(X_i)$ (2)

**[9]**

**Q. 7)** Suppose $X_i : i = 1,2, \ldots n$ are $n$ independent and identically distributed random variables, each with mean μ and variance $\sigma^2$. Let $\bar{X} = \sum_1^n X_i$ be the sample mean and $S^2 = \frac{1}{n-1}\sum_i^n (X_i - \bar{X})^2$ be the sample variance.

**i)** Show that the mean and variance of $\bar{X}$ is μ and $\sigma^2$/ n respectively. (2)

**ii)** Show that E $(S^2) = \sigma^2$. (2)

Assume $X_i$ 's are from a normal population. Using the distribution of $(n-1)S^2/\sigma^2$ as chi square

**iii)** Show that variance of $S^2$ is 2 $\sigma^4$/ (n-1). (1)

**iv)** Find the probability that $S^2$ will fall between plus or minus 50% of its expected value when n = 10 and $\sigma^2$ = 100. (3)

**[8]**

**Q. 8)** The probability density function of a random variable X, is given by

$$f(x) = \frac{c\,\beta^3}{(x+\beta)^4} \quad ; x > 0, \beta > 0$$

where c is a constant and β is a parameter.

**i)** Determine the value of c and calculate the mean and variance of X as a function of β by using Formulae and Tables for Actuarial Examinations or otherwise. (3)

It is required to estimate β based on a random sample $X_{1,}, X_2, \dots, X_n$

**ii)** Show that the method of moments estimator $\hat{\beta}$ is $2\overline{X}_n$ and verify the unbiasedness and consistency of this estimator. (4)

**iii)** Consider the set of estimators of the form $b\overline{X}_n$, where b is a constant. Show that the value of b that minimizes the MSE of $b\overline{X}_n$ is 2 / (1 + 3/n) (5)

**iv)** Compare the unbiasedness and consistency of the estimator in (iii) with minimum b using the corresponding properties of the estimator in (ii). (2)

**[14]**

**Q. 9)** An insurer believes that the distribution of the number of claims on a particular type of policy is binomial with parameters n = 4 and p. A random sample of 180 policies revealed the following information.

| **No. of claims** | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| **No. of policies** | 86 | 75 | 16 | 2 | 1 |

**i)** Obtain the maximum likelihood estimate of p. (4)

**ii)** Carry out a goodness of fit test for the number of claims on each policy conforms to the binomial model. (4)

**[8]**

**Q. 10)** A sample of size 1 is taken from a Poisson distribution with mean $\mu$. Let $H_0: \mu = 1$ and $H_1: \mu = 2$. A test rejects the null hypothesis if $x > 3$.

**i)** Calculate the probability of type I error of the test. (2)

**ii)** Calculate the power of the test. (2)

**[4]**

**Q.11)** The following table shows the data on rainfall and the yield of a crop for last eight years:

| **Rainfall *X*** | 55 | 73 | 60 | 152 | 106 | 42 | 29 | 84 |
|---|---|---|---|---|---|---|---|---|
| **Yield *Y*** | 122 | 135 | 127 | 188 | 145 | 108 | 46 | 141 |

$\bar{X}$= 75.125; $\bar{Y}$ = 126.500; $S_{XX}$= 10,804.875; $S_{YY}$ = 11,250.000 and $S_{XY}$= 9,818.500

**i)** Fit a Linear regression model: $Y = \alpha + \beta X + e$ for the above data. (2)

**ii)** Construct ANOVA table for the regression model fitted in part (i) and test $\beta = 0$ against $\beta \neq 0$ at 5% level. (4)

**iii)** Find 95% confidence limits for $\beta$. (2)

**iv)** Test population correlation coefficient $\rho = 0$ against $\rho \neq 0$ at 1% level. (3)

**v)** Calculate coefficient of determination and interpret. (2)

**[13]**

**Q. 12)** An insurance company has recruited four agents during the last few months. The number of policies $y_{ij}$ generated by *i* th agent on *j* th month is given below. The agents joined in successive months with the Agent 1 being the first to join.

| **Agent** | **Number of policies** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 10 | 2 | 1 | 7 | 6 | 3 | 4 |
| 2 | | 3 | 7 | 5 | 6 | 1 | 2 | 3 |
| 3 | | | 1 | 8 | 5 | 5 | 7 | 5 |
| 4 | | | | 5 | 1 | 6 | 2 | 7 |

The summary measures are: $y_{1.}$ = 38; $y_{2.}$ = 27; $y_{3.}$ = 31; $y_{4.}$ = 21; $y_{..}$ = 117
$\sum\sum y_{ij}^2$= 677

**i)** Examine whether there is any difference among the mean numbers of policies generated by the agents, stating any assumptions made. (5)

**ii)** Test for the significance of difference between the means for the pair of agents whose mean number of policies generated, is the highest and lowest using a least significant difference approach at the 5% level. (3)

**[8]**

********************