

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

12th May 2015

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. Mark allocations are shown in brackets.*
- 3. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 4. Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1)** The nominal rate of discount per annum convertible quarterly is 8%.
- i)** Calculate the equivalent force of interest. (1)
 - ii)** Calculate the equivalent effective rate of interest per annum. (1)
 - iii)** Calculate the equivalent nominal rate of discount per annum convertible monthly. (1)
- [3]**

- Q. 2)**
- i)** Briefly describe the Call and Put Options (2)
 - ii)** Compare the debenture stocks and unsecured loan stocks (2)
- [4]**

- Q. 3)** A fixed interest stock bears a coupon of 7% per annum payable half yearly on 1st April and 1st October. It is redeemable at par on any 1st April between 1st April 2015 and 1st April 2021 inclusive at the option of the borrower.

On 1st July 2002 an investor purchased INR 10,000 nominal of the stock at a price to give a net yield of 6% per annum effective after allowing for tax at 25% on the coupon payments.

On 1st April 2010 the investor sold the holding at a price which gave a net yield of 5% per annum effective to another purchaser who is also taxed at a rate of 25% on the coupon payments.

- i)** Calculate the price at which the stock was bought by the investor. (3)
 - ii)** Calculate the price at which the stock was sold by him. (3)
- [6]**

- Q. 4)** A continuous cash flow is to be paid at a rate of $\rho(t) = 10 + 10t$ for $3 < t < 4$ and $\rho(t) = 50 - t^2$ for $5 < t < 6$. The force of interest applicable during the period is:

$$\delta(t) = \begin{cases} 0.01 + 0.01t & 0 \leq t < 4 \\ 0.15 - 0.003t^2 & 4 \leq t < 6 \\ 0.06 & t \geq 6 \end{cases}$$

Find the total accumulated value of the above mentioned payment streams at time 10. **[6]**

- Q. 5)** In a particular country, income tax and capital gains tax are both collected on 1st April each year in relation to gross payments made during the previous 12 months.

A fixed interest bond is issued on 1st January 2013 with a term of 25 years and is redeemable at 110%. The security pays a coupon of 8% per annum, payable half-yearly in arrears.

An investor, who is liable to tax on income at the rate of 25% per annum and at 30% per annum on capital gains, bought INR 10,000 nominal of the stock at an issue price of INR 9,900.

- i) Assuming an inflation rate at 3% per annum over the term of the bond and assuming the investor holds the stock to redemption
- a) Show that the money yield obtained by the investor is between 6% to 7% per annum (6)
- b) Also calculate the net real yield obtained by him. (2)
- ii) Explain how and why your answer to (i) above, would alter if tax were to be collected on 1st June instead of 1st April each year. (1)
- [9]
- Q. 6)** $f_{t,r}$ is the forward rate applicable over the period t to $t + r$ and i_t is the spot rate over the period 0 to t . The gross redemption yield from a one year, two year and three year bond all with a 6% annual coupon is 6%, 6.3% and 6.6% per annum effective respectively. If all the bonds are redeemed at par and are exactly one year from the next coupon payment, then
- i) Calculate i_1 , i_2 and i_3 assuming no arbitrage. (4)
- ii) Calculate $f_{0,1}$, $f_{1,1}$ and $f_{2,1}$ assuming no arbitrage. (4)
- iii) Explain why the forward rates increase more rapidly with term than the spot rates. (1)
- [9]
- Q. 7)** The annual yields from a particular fund are independent and identically distributed. Assuming, the distribution of $1 + i$ to be log-normal with parameters $\mu = 0.07$ and $\sigma^2 = 0.006$, where i denotes the annual yield on the fund, calculate
- i) The mean accumulation in ten years' time of an investment in the fund of INR 20,000 at the end of each of the next ten years, together with INR 150,000 invested immediately. (5)
- ii) The single amount that should be invested in the fund immediately to give an accumulation of at least INR 600,000 in ten years' time with a probability 0.99. (6)
- [11]
- Q. 8)** A young couple, Mr. & Mrs. Jones took a personal loan for an amount of INR 20,000 to refurbish their home. The loan is being repaid with payments of INR 427.90 made monthly in arrears for 5 years.
- i) Calculate the Annual Percentage Rate (APR) of the couple's loan. (5)
- However, the couple have been struggling to repay the loan for last few months, since Mr. Jones lost his job. After exactly one year, the loan company offers to help the couple by restructuring their loan with new monthly payments of INR 274.49 made in arrears.
- ii) Assuming the APR remains unchanged from the original loan; calculate the term of the new loan. (3)

- iii) Mr. & Mrs. Jones' initial reaction towards the restructured loan is positive, both in terms of the monthly repayment amount and the term of the loan. However, they are unsure if this may mean that they are paying too much as compared to the original loan. Calculate how much more interest in total the couple will pay on their restructured loan than on the original loan.

(2)
[10]

- Q. 9)** A pension fund has a liability to pay INR 100,000 at the end of one year, INR 105,000 at the end of two years, and so on, the amount increasing by INR 5,000 each year to INR 195,000 at the end of 20 years, this being the last payment. The fund values these payments using an effective interest rate of 7% per annum. This is also the interest rate at which the current prices of all bonds are calculated.

The fund invests an amount equal to the present value of these liabilities in the following two assets:

- A** Zero coupon bond redeemable in 25 years, and
- B** A fixed interest bond redeemable at par in 12 years' time which pays a coupon of 8% per annum annually in arrears

- i) Calculate the present value and the duration of the liabilities. (6)
- ii) Calculate the amount of cash that should be invested in each asset, if the duration of the assets is equal to that of the liabilities. (9)

[15]

- Q. 10) i)** Define the Terms with reference to an investment Project

a) Discounted payback period

b) Payback period (2)

- ii) Compare both the discounted payback period and the payback period with the net present value for determining whether to proceed with an investment project and explain why the latter is a superior measure. (3)

- iii) An investor is looking to invest in an infrastructure project. He has two choices available between two different cities. For each of the projects, the expected cashflows have been advised to the investor as being as follows.

For Project A, an initial expense of INR 170,000 has been estimated. This is followed by a further outgo of INR 20,000 at the end of first year and INR 10,000 at the end of second year. However, the project promises impressive incoming cash flows of INR 20,000 each at the end of year 1 and 2, and income of INR 200,000 at the end of year 3.

On the other hand, Project B demands an initial investment of INR 30,000 more than that of Project A, but no further ongoing expenses. In return, this project is expected to provide an income of INR 14,000 pa at the end of each of the first 6 years. This is followed by the return of the entire initial investment at the end of the 6th year.

- a) Calculate the internal rate of return (correct to 1 decimal place) for each project. (5)
- b) Calculate the net present value of each project using a risk discount rate of 6% *pa*. (2)
- c) The investor is looking to explore the option of funding the projects through borrowing money from a bank. However, he is unsure about the impact this would have on the profitability of the projects. Calculate the maximum interest rate that could be charged by the bank, above which each project becomes unprofitable. Mention any other factors that the investor should consider when deciding between the projects. (2)

[14]

Q. 11) Mrs. Brown, an actuary, had received a payment of INR 300,000 by selling her apartment during the month of June 2005. The money was deposited in her bank account for an initial few months, until she decided to hire two fund managers to invest her money in a variety of risk assets to earn promising returns as compared to the property market. On 1st September 2005, she placed part of her money with two fund managers. Manager A was given INR 80,000 and Manager B was given INR 140,000 to invest in a portfolio of assets. Both the managers received a net cashflow of INR 15,000 on 1st September 2006, bringing their total fund values to INR 103,000 and INR 183,000, respectively.

Mrs. Brown paid a further net cash flow of INR 20,000 to each manager on 1st September 2007. This brought their total fund values to INR 143,600 and INR 239,600, respectively. On 31st August 2008, the value of Manager A's fund was INR 172,320 and the value of Manager B's fund was INR 263,560.

Given that it has been 3 years since Mrs. Brown's first investment with the fund managers; she is looking to assess their performance and decide whether she would like to continue or withdraw her investments. Being an actuary, she decides to use multiple criteria to assess the performance of each of the fund managers.

- i) For the period from 1st September 2005 to 31st August 2008, calculate for each fund manager:
- a) The effective time weighted rate of return
- b) The effective money weighted rate of return. (8)
- ii) Although the calculations performed above, clearly indicate the better performing fund manager, Mrs. Brown is looking for further analysis. In addition to overall returns calculated above, she expects that the cash flow growth pattern for each fund manager will provide additional insight into their respective performance. By examining the growth factors between cashflows, analyse the performance of each manager over the three-year period. (3)
- iii) Taking into account both (i) and (ii) above, comment briefly on the relative performance of the two fund managers. (2)

[13]
