Institute of Actuaries of India

Subject ST6 – Finance and Investment B

May 2014 Examinations

INDICATIVE SOLUTIONS

Solution 1 :

i.

Call future options- The holder receives a long position in the underlying futures contract plus an amount of cash equal to the current futures price minus the strike price.

Put future options - The holder gets a short position in the underlying futures contract plus an amount of cash equal to the difference between the strike price and the current futures price.

[3]

ii.

Suppose there is constant risk-free interest rate *r* and the futures price *F(t)* of a particular underlying is log-normal with constant volatility *σ*. Then Black's formula states the price for a European call option of maturity *T* on a futures contract with strike price *K* and delivery date *T'* (with $T' \geq T$) is

$$c = e^{-rT}[FN(d_1) - KN(d_2)]$$

The corresponding put price is

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)]$$

Use Black's formula to calculate the value of option

d1 =	-0.578824
d2 =	-0.731576
N(d1) =	0.2813607
N(d2) =	0.2322192
Put =	6.9069

[4]
[Total Marks-7]

Solution 2 :

i. The term structure yield curve model is to be able to price options across the entire range of maturities in a yield curve. The exotic options depend not just on the evolution of forward rates along the curve, but on the correlation between changes.

Desirable features

- Arbitrage free to produce market consistent prices
- The current swap (or bond) curve should be reproduced by the model
- Flexible to cope with variety of derivative contracts
- Easy to specify and calculate
- Easy to calibrate
- Sufficient Parameters to cope with any yield curve shape, but not too many to avoid instability
- Volatility of rates of different maturity should be different, with shorter rates usually being more volatile
- Imperfect correlation between forward rates,
- Negative interest rates should not normally be allowed
- Reasonable dispersion of rates over time

[5]

ii. JC Hull 6th edition Pg 662 chapter 28 section 28.7 – General tree building process

General description of trinomial tree model
Stage 1:
Stage 2:

[9]
[Total Marks-14]

Solution 3 :

i.

Value at risk for a portfolio indicates the maximum loss which the portfolio can sustain over a given day for a given confidence level (say 95% or 99% certainty).

Under the variance-covariance approach VaR account for volatility of each constituent and interdependencies between them (covariance, or correlation).

VaR can be used for quantification of risk, management reporting of risk, allocating risk to P&L profit, hedging, stress testing

[4]

ii. One way of creating VaR is by mapping sensitivities of each instrument to interest rate changes to more basic instruments, e.g. 2-year, 5-year and 10-year discount bonds. Then they are combined into a theoretical portfolio which is assumed to having multi-nominal distribution.

- Obtain all the position for each instrument
- Create structure of the company
- Choose “synthetic securities” for mapping the exposure express all the securities in terms of these synthetic securities
- Obtain price history for the instruments – this is to calculate variance and covariance
- Select the confidence interval

Interest-rate futures – should be treated as loan - Then express this loan as a loan now up to expiry + three months, and a deposit starting now up to expiry.

Interest-rate swaps: These can be decomposed into a fixed-rate bond and a FRN including the margin on the floating side.

[7]
[Total Marks-11]

Solution 4 :

The stated instrument can be split into a fixed-floating swap plus a set of daily binary options (cash-or-nothing options) to cancel out the fixed payment for that day. Hence the valuation would be that of an ordinary swap, less the value of the binary options which have effectively been sold by the holder of the accrual swap.

In above instrument two options on each day: one to cancel the fixed Payment if rates go above 10%, the other to cancel it if rates go below 8%. Binary options are valued via the Black’s formula.

Theoretically, the binary option are valued daily and then summed over each accrual period, but in practice they are usually grouped weekly or fortnightly without any loss of accuracy.

The risk profile of the given accrual swap is that it behaves like an ordinary fixed-floating swap whilst rates lie well inside the 8% - 10% boundary, with small gamma. However, as soon as rates approach either boundary, the gamma increases dramatically and the option effect becomes very pronounced.

[6 Marks]

Solution 5 :

i.

Interest rate	12%
startint	4%
increase	0.50%
Recovery	40.00%

Year		Default per 100	Survival		Present value	
1	4%	4	96.00	0.88692	0.851444	
2	4.50%	4.32	91.68	0.786628	0.72118	
3	0.00%	0	91.68	0.697676	0.63963	
4	5.50%	5.0424	86.64	0.618783	0.536099	
					2.7484	

Periodic payment 2.7484S

Principal payment default take place mid of year

Year	Default per 100	Recovery	Pay off	Discount	Present value	
1	4	40.00%	2.4	0.941765	0.022602	
2	4.32	40.00%	2.592	0.83527	0.02165	
3	0	40.00%	0	0.740818	0	
4	5.0424	40.00%	3.02544	0.657047	0.019879	
					0.0641	

Total principal payment 0.0641

Accrual adjustment per 1 unit of spread

Year	Survival	Accrual	Discount	
1	96.00	2	0.941765	0.018835
2	91.68	2.16	0.83527	0.018042
3	91.68	0	0.740818	0
4	86.64	2.5212	0.657047	0.016565
			Sum	0.0534

Accrual payment			0.0534S	
Spread	0.022878	0.0641/(2.7484+0.0534)		
	228.7815			

[8]

ii.

Interest rate	12%				
startint	4%				
increase	0.50%				
Recovery	40.00%				
Year		Default per 100	Survival		Present value
1	4%	4	96.00	0.88692	0.851443619
2	4.50%	4.32	91.68	0.786628	0.721180423
3	5.00%	4.584	87.10	0.697676	0.607648173
4	5.50%	4.79028	82.31	0.618783	0.509294126
					2.6896

Periodic payment 2.6896S

Principal payment default take place mid of year

Year	Default per 100	Recovery	Pay off	Discount	Present value
1	4	40.00%	2.4	0.941765	0.022602349
2	4.32	40.00%	2.592	0.83527	0.021650204
3	4.584	40.00%	2.7504	0.740818	0.020375464
4	4.79028	40.00%	2.874168	0.657047	0.018884629
					0.0835

Total principal payment 0.0835

Accrual adjustment per 1 unit of spread

Year	Survival	Accrual	Discount	
1	96.00	2	0.941765	0.018835
2	91.68	2.16	0.83527	0.018042
3	87.10	2.292	0.740818	0.01698
4	82.31	2.39514	0.657047	0.015737
			Sum	0.0696

Accrual payment		0.0696S	
Spread	0.030262	= (0.0835/(.0693+2.6896)	
	302.6239		

Make loss of 78 bps

[6]

[Total Marks-14]

Solution 6 :

i. $1000 \exp(0.05) = [1250p + 800(1-p)]$

$\Rightarrow p = 0.55838$ and therefore $(1 - p) = 0.44162$

[2]

ii.

a. The tree is:

		1562.5
	1250	
1000		1000
	800	
		640

b. The minimum value is 640. The price will then be 420.9041p. **[3]**

[Total Marks-5]

Solution 7 :

i. Differentiate the Put – Call parity equation **[3]**

ii. "C" carries more rights than "A" since in "C" we can exercise two options separately. In terms of price, they will be the same since the two pieces will have the same optimal exercise time. Hence "C" is preferred to "A". **[3]**

iii. Gamma is zero **[2]**

iv. Call increases in value while the put decreases in value **[3]**

v. The price remains the same **[2]**

[Total Marks-13]

Solution 8 :

This follows from a backwards induction. In the final layer the values are the same. In each previous layer, we assume the result has already been proven for the next layer. At each node the discounted expectation of the next layer must then be at least as much for the American option as for the European option. Taking the maximum with the intrinsic value can only increase the value. So at each node the value is at least as much. The result now follows by inducting back to the initial point.

[5 Marks]

Solution 9 :

The AR(1) process

$X_t - b = a(X_{t-1} - b) + \varepsilon_t$

Can be rearranged to yield the following equation

$X_t - X_{t-1} = (1-a)(b - X_{t-1}) + \varepsilon_t$, This is nothing but the discrete time version of the Vasicek process. It may be recalled that the autocorrelation of X at lag one is a, which translates to (1 - β dt) in the Vasicek model.

[5 Marks]

Solution 10 :

i. Apply the Black Scholes formula given by $C = SN(d_1) - X e^{-rT}N(d_2)$
Substituting the numbers, this works out to 2.374 **[1]**

ii. Delta = $N(d_1)$

Gamma = $\frac{N'(d_1)}{S\sigma\sqrt{T}}$, where $N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$.

Substituting the values, we get Delta = 0.562 and Gamma = 0.0747 **[4]**

iii. To ensure delta neutrality, he needs to buy shares and thus borrow money. Let us say he buys "n" shares and borrows an amount B. The following two equations need to be satisfied

Portfolio has to be self-financing: 1000(2.374) – n(50) – B = 0
Portfolio has to be delta neutral: 1000(0.562) – n(1) = 0

a. This implies the number of shares to be bought is 562 **[2]**

b. The amount of money to be borrowed is this 25726 **[2]**

iv. If the stock price moves to 52, the delta will have to be recalculated. The new delta for the updated price is 0.7017.

This implies that the total number of shares required to be delta neutral on this day is 701.7 or approximately 702.

Thus the writer needs to buy an addition 139.7 or approximately 140 shares to remain delta neutral.

[3]

v. To ensure delta neutrality, he needs to buy shares, options and thus borrow money. Let us say he buys "n_1" shares, "n_2" options and borrows an amount B. The following two equations need to be satisfied

Portfolio has to be self-financing: $1000(2.374) - n_1(50) - n_2(1.1466) - B = 0$
Portfolio has to be delta neutral: $1000(0.562) - n(1) - n_2(0.2965) = 0$
Portfolio has to be gamma neutral: $1000(0.0747) - n_2(0.0529) = 0$

a. This implies that the number of new options (option 2) that he has to buy is 1412.10 or approximately 1412. **[2]**

b. This implies the number of shares to be bought is 143.31 or approximately 143 **[2]**

c. The amount of money to be borrowed is this 6410.75 **[2]**

d. The reason the amount of money required is less is because of the presence of a large number of options where the current outflow is the premium only. **[2]**

[Total Marks-20]

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