Institute of Actuaries of India

Subject CT8 – Financial Economics

May 2014 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1 :

(i) Three types of factors

1. Macroeconomic – the factors will include some macroeconomic variables such as interest rates, inflation, economic growth and exchange rates.

2. Fundamental – the factors will be company specifics such as P/E ratios, liquidity ratios and gearing levels.

3. Statistical – the factors do not necessarily have a meaningful interpretation. This is because they are derived from historical data, using techniques such as principal components analysis to identify the most appropriate factors.

[3]

(ii)

Consider any efficient portfolio of risky assets with mean *EP* , a variance *VP* and a portfolio composition *xP* .

The variance of a portfolio on the efficient frontier (where risk free assets sits on the efficient frontier) can be expressed as:

$$V = x_r^2\sigma_r^2 + x_P^2\sigma_P^2 + 2x_r x_P \sigma_r \sigma_P \rho$$

Since $\sigma_r = 0$, for risk free asset,

$$V = 0 + x_P^2\sigma_P^2 + 0 = x_P^2\sigma_P^2 \iff \sigma = x_P\sigma_P$$

----- (equation 1)

Also, we know that the expected return on a Portfolio is

$$E = x_r E_r + x_P E_P = (1 - x_P) E_r + x_P E_P$$

Which implies,

$$x_P = \frac{E - E_r}{E_P - E_r}$$

----- (equation 2)

Substitute equation 2 in 1, we get,

$$\sigma = \left(\frac{E - E_r}{E_P - E_r}\right)\sigma_P = \frac{\sigma_P}{E_P - E_r}E - \frac{E_r\sigma_P}{E_P - E_r}$$

Therefore the efficient frontier is a straight line in expected return-standard deviation space.

[3]

(iii)

$E_G = 5\%$, $E_E = 11\%$, $V_G = (6\%)^2$ and $V_E = (12\%)^2$

$E = x_G E_G + x_E E_E = 5x_G + 11(1- x_G)$

$\Rightarrow$ $x_G = (E-11) / (5-11) = (11-E) /6$

$\Rightarrow$ $x_E = 1- x_G = (E-5)/6$

Since the two assets are independent, the variance of the portfolio is

$V = x^2_G V_G + x^2_E V_E$

Substituting in the portfolio proportions:

$V = 5E^2 – 62E + 221$

$\sigma = (5E^2 – 62E + 221)^{0.5}$

The efficient frontier is the part of this line above the point at which the variance is minimised. To find this point we differentiate:

$dV/dE = 10E – 62 = 0$

So the efficient frontier is the part of the opportunity set where $E \geq 62/10 = 6.2\%$

[3]

[Total Marks-9]

Solution 2 :

(i) The required equation is $– Z'(w) > 0$ **[1]**

(ii) First order stochastic dominance holds if:

$F_x(a) \leq F_y(a)$, for all a , and

$F_x(a) < F_y(a)$, for some value of a

Description : This means that the probability of portfolio Y producing a return below a certain value is never less than the probability of portfolio X producing a return below the same value and exceeds it for at least some value of ‘a’ .

[2]

(iii) The expected utility of X is

$$E[Zx] = \int_x^y z(w)\, dF_x(w)$$

The expected utility of Y is

$$E[Zy] = \int_x^y z(w)\, dF_y(w)$$

Hence if, X is preferred over Y, then following holds true –

$$\int_{x}^{y} z(w)\, dF_x(w) - \int_{x}^{y} z(w)\, dF_y(w) > 0$$

This can be expressed as –

$$\int_{x}^{y} Z(w)\, [dF_x(w) - dF_y(w)] > 0$$

Solving the left side by Integration by parts gives the following result

$$[Z(w)\,(F_x(w) - F_y(w))] - \int_{x}^{y} Z'(w)\,[F_x(w) - F_y(w)]\, dw$$

Now, by definition F x (x)= Fy (x)= 0 and F x (y)= F y (y)= 1 and hence for the expression to be positive we need the value of the Integral to be negative.

Z′(w) > 0 by assumption, therefore Fx (w) - Fy (w) must be less than or equal to zero for all values of w with Fx < Fy for at least one value of w if the value is not to be zero.

[4]

[Total Marks-7]

<u>Solution 3</u> :

(i) Let n ex-dividend dates are anticipated for a stock and $t_1<t_2<....<t_n$ are the times before which the stock goes ex-dividend. Dividends are denoted by $d_{1....}d_{n.}$

If the option is exercised prior to the ex-dividend date then the investor receives $S(t_n) – K$.

If the option is not exercised, the price drops to $S(t_n) – d_n$.

The value of the american option is greater than $S(t_n) – d_n$- Kexp(-r(T-t_n)

It is never optional to exercise the option if $S(t_n) – d_n$- Kexp(-r(T-t_n) >= $S(t_n)$ – K i.e. d_n<=K*(1-exp(-r(T-t_n))

Using this equation: we have K*(1-exp(-r(T-t_n)) = 350*(1-exp(-0.95*(0.8333-0.25))=18.87 and 65*(1-exp(-0.95*(0.8333-0.25)) =10.91.Hence it is never optimal to exercise the american option on the two ex-dividend rates.

[3]

(ii) The required probability is the probability of the stock price being greater than Rs. 258 in 6 months time.

The stock price follows Geometric brownian motion i.e. $S_t = S_0 \exp(\mu - \sigma^2/2)t + \sigma W_t$

Therefore Ln (S_t) follows normal distribution with mean Ln (S_0) + $(\mu - \sigma^2/2)t$ and variance $\sigma^2 t$

Implies Ln (S_t) follows φ(Ln 254 + (0.16-0.35^2/2)*0.5, 0.35*0.5^(1/2))

= φ (5.59, 0.247)

This means [Ln (S_t) – μ (S_t)]/ σt^(1/2) follows standard normal distribution. Hence the probability that stock price will be higher than the strike price of Rs. 258 in 6 months time = 1- N(5.55-5.59)/0.247 = 1- N(-0.1364) =0.5542.

The put option is exercised if the stock price is less than Rs. 258 in 6 months time. The probability of this = 1- 0.5542= 0.4457

[4]

[Total Marks- 4]

Solution 4 :

(i) The value of the portfolio is 300,000 times the value of index. When the value of the portfolio falls by 10% (i.e. becomes 486 cr), the value of the index also falls by 10% to 16,200. The fund manager will therefore require 300,000 times put options with strike price 16,200 for portfolio insurance.

Value of one such put option is 16,200*exp(-rT)*N(-d2)-18,000*exp(-qT)*N(-d1),
where d1= [Ln(18,000/16,200)+(0.06-0.03+0.3^2)*0.5]/(0.3*0.5^(1/2))

Initial value of portfolio	5,400,000,000
Initial value of index	18,000
Reduction %	10%
Revised value of portfolio	4,860,000,000
Revised value of index	16200
Index Value	18,000
K	16200
Risk free rate	6%
Volatility	30%
T	0.5
Dividend yield	3%
d1	0.6735
d2	0.4613
N(d1)	0.749669771
N(d2)	0.677715085
1-N(d1)	0.250330229
1-N(d2)	0.322284915
Value of put option	627.85

Cost of buying 300,000 put options = 188,355,585 = 18.8.cr **[4]**

(ii) The delta of one put option = exp(-qT)*[N(d1)-1]
=-0.2466 which implies 24.66% i.e. 133 cr should be initially invested in risk free securities.

[2]

(iii) The delta of 9 month index future = exp((r-q)*0.75) = exp(0.03*.75)= 1.023

The spot short position required = (133*10^7)/18,000 times index futures which is (133*10^7)/(18,000*1.023*1000)= 72 (assumes future lot size of 1000). Please give credit for any reasonable assumption on lot size.

[2]

(iv) From put-call parity: S_{0*}exp(-qT)+P = C+K*exp(-rT) where P and C denote the values of put and call options.

Hence P = C- S_{0*}exp(-qT) + K*exp(-rT).
This shows that the put option can be created by shorting exp(-qT) of the index, buying a call option and investing the remaining in the risk free rate of interest.
Sell 540*exp(-0.03*.5) = 532 cr of stocks
Buy call options on 300,00 times the index at strike price 16,200 and maturity in 6 months.
Invest the remaining at 6% per annum risk free rate.

[4]
[Total Marks-12]

Solution 5 :

(i) **(a)** State-price deflator approach:

$$Y(t,T) = \frac{E_P\{A(T)|\,F_t\}}{A(t)} \times \text{Rs.}1000$$

Where $A(t)$ is the deflator.

(b) Risk-neutral approach:

$$Y(t,T) = \mathbb{E}_Q\left[\exp\left(-\int_t^T r_u\,du\right)|F_t\right] \times \text{Rs.}1000$$

[2]

(ii) The underlying's of the short rate *rt* under Q for the Vasicek model are:
$drt = \alpha(\mu - rt)dt + \sigma dZt,$

where Z is a Q-Brownian motion.

This is an Ornstein-Uhlenbeck process.

[3]

(iii) Consider $st = e\alpha trt$. Then
dst = αeαtrtdt + eαt drt
= αμeαtdt + σeαt dZt

Thus st = s0 + μ(eαt − 1) + $\sigma\int_0^t e^{\alpha u} dZ_u$
and hence,

rt = e−αtr0 + μ(1 − e−αt) + $\sigma\int_0^t e^{\alpha(u-t)} dZ_u$

[5]

(iv) From (iii) its clear that *rt* has a Normal distribution and hence from (i), *Y*(*t*, *T*) has a lognormal distribution. **[2]**

[Total Marks-12]

Solution 6 :

(i) According to the Efficient Markets Hypothesis, active investment management cannot be justified because it is impossible to exploit the mispricing of securities in order to generate higher expected returns. Even if price anomalies exist, then the costs of identifying them and then trading will outweigh the benefits arising from the additional investment returns.

[2]

(ii) If strong form EMH holds, then the current share price already reflects all relevant information, so there would be no advantage in using this approach.

If the market is inefficient or weak or semi-strong form efficient then this strategy may be beneficial (though it could be questionable on ethical grounds).

[2]
[Total Marks-4]

Solution 7 :

Variance
Rahul = 100^2 * 0.03 * 0.97 = 291
Sachin = 100 * 0.03 * 0.97 = 2.91

VAR @ 95%
Rahul = 0 as there is a less than 5% chance of any loss
Sachin
Using normal approximation, the mean is 0.03*100 =3 and std. deviation is 1.71 (square root of 2.91).
3 + 1.645* 1.71 = 5.81

[Total Marks- 4]

Solution 8 :

(i) Value of the market portfolio = 200(3) + 300(4) = 1,800.
Portfolio weights are $x_A = 1/3$ and $x_B = 2/3$.
$R_M = 1/3(16\%) + 2/3(10\%) = 12\%$ **[1]**

(ii) $\sigma_{AB} = \sigma_A \sigma_B \rho_{AB} = (0.3)(.15)(0.4) = 0.018$

$Cov(R_A, R_M) = Cov(R_A, 1/3R_A + 2/3R_B)$
$= 1/3(\sigma_A^2) + 2/3\sigma AB = 1/3(0.3)^2 + 2/3(0.018) = 0.042$
$Var(R_M) = (1/3)^2(0.3)^2 + (2/3)^2(0.15)^2 + 2(1/3)(2/3)(0.018) = 0.028$
$StDev(R_M) = 16.73\%$ **[1]**

(iii) $\beta_A = 0.042/0.028 = 1.5 \Rightarrow 16 = r_f + 1.5(12 - r_f) \Rightarrow r_f = 4\%$. $\beta_B = 0.75$ as market $\beta = 1 = 1/3 * \beta_A + 2/3 * \beta_B$ **[2]**

(iv) As derived above using CAPM **[1]**

(v) Limitations:

Unrealistic assumptions

Capital asset pricing model is based on a number of assumptions that are far from the reality. For example it is very difficult to find a risk free security. A short term highly liquid government security is considered as a risk free security. It is unlikely that the government will default, but inflation causes uncertain about the real rate of return. The assumption of the equality of the lending and borrowing rates is also not correct. In practice these rates differ. Further investors may not hold highly diversified portfolios or the market indices may not well diversify. Under these circumstances capital asset pricing model may not accurately explain the investment behavior of investors and beta may fail to capture the risk of investment.

Betas do not remain stable over time

Stability of beta, beta is a measure of a securities future risk. But investors do not further data to estimate beta. What they have are past data about the share prices and the market portfolio. Thus, they can only estimate beta based on historical data. Investors can use historical beta as the measure of future risk only if it is stable over time. Most research has shown that the betas of individual securities are not stable over time. This implies that historical betas are poor indicators of the future risk of securities.

Difficult to validate

Most of assumptions may not be very critical for its practical validity. Therefore is the empirical validity of capital asset pricing model. Need to establish that the beta is able to measure the risk of a security and that there is a significant correlation between beta and the expected return. The empirical results have given mixed results. The earlier tests showed that there was a positive relation between returns and betas. However the relationship was not as strong as predicted by capital asset pricing model.

[2]

[Total Marks-7]

Solution 9 :

(i) A discrete time stochastic process X_0, X_1, X_2 … is said to be a martingale if $E[|X_n|] < \infty$ for all n, and $E[X_n| X_0, X_1, X_2 \ldots, X_m] = X_m$ for all m < n.
In other words, the current value X_m of a martingale is the optimum estimator of its future value X_n. **[2]**

(ii) A counting process $\{N_t, t>=0\}$ is a Poisson process with rate λ if

i. $N_0 = 0$
ii. N_t has independent increments
iii. $N_t – N_s \sim \text{Poisson}(\lambda(t\text{-}s))$ for t > s

M(t)=N(t)-αt
We know $E[N_t – N_s \mid N_s] = \lambda(t\text{-}s)$
Hence $E[N_t – \lambda t|N_s]= N_s – \lambda s$. Hence N(t)- αt is a martingale for α = λ **[3]**

(iii) $Z_n = \sum_{i=1}^{n}(Xi - \mu)$, to prove $E(Z_n \mid Z_m) = Z_m$

E $[\sum_{i=1}^{n}(Xi - \mu) \mid X_{m,} m < n]$
= E $[\sum_{i=1}^{m}(Xi - \mu) \mid X_m]$ + E $[\sum_{i=m+1}^{n}(Xi - \mu) \mid X_m\ ]$
= $\sum_{i=1}^{m}(Xi - \mu) + 0 = Z_m$

[2]
[Total Marks-7]

Solution 10 :

(i)

Case 1	
Stock price	25
u	1.08
d	0.92
r	10%
T	0.0833
p = [exp(rT)-d]/(u-d)	0.5523
D	0.4477
Strike price	24

Payoff if stock moves to 27 = 27-K	3
Value = p*exp(-rT)*PAYOFF	1.6432

[2]

(ii)

Case 2 Payoff diagram

			Payoff	
		29	**841**	fuu
	27			
25		**25**	**625**	fud
	23			
		21	**441**	fdd

Case 2	
Stock price	25
u	1.08
d	0.92
r	10%
t	0.083333333
p	0.552300951
d	0.447699049
Value = exp(-r*2/12)[p^2fuu +2p(1-p)fud+(1-p)^2fdd]	643.198113

[3]

(iii)

Case 3

				Payoff
		29	28	4
	27		26	2
25		25		
	23		24	0
		21	22	0

Case 2	
Stock price	25
u	1.08
d	0.92
r	10%
t	0.083333333
p	0.552300951
d	0.447699049
Value	1.686333506

[4]
[Total Marks-9]

Solution 11 :

(i) The Wiener process W_t , t >=0 is characterized by three properties:

a) $W_0 = 0$
b) The function $t \to W_t$ is everywhere continuous with prob 1.
c) W_t has independent increments with $W_t - W_s \sim N(0, t-s)$ (for $0 \le s < t$), where $N(\mu, \sigma^2)$ denotes the Normal distribution with expected value μ and variance σ^2.

(ii) Ito's lemma states that for an Ito drift-diffusion process $dX_t = \mu dt + \sigma dW_t$ and any twice differentiable scalar function f(t,x) of two real variables t and x, one has

$$df(t, X_t) = \left(\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2}\right) dt + \sigma \frac{\partial f}{\partial x} dW_t$$ **[2]**

(iii) Using Ito's lemma for f(x) = 1/x, as in above equation, we can prove that

$$dZ_t = \sigma\, dW_t / X_t = -\sigma Z_t dW_t \;, t >= 0$$ **[4]**

(iv) From above differential equation for Z_t, we can claim that Z_t is a martingale and hence we have $E(Z_T \mid Z_0 = 1) = 1$. **[2]**

[Total Marks-10]

Solution 12 :

(i) The credit event is an event which will trigger the default of a bond and includes the following

a. failure to pay either capital or coupon
b. loss event
c. bankruptcy
d. rating downgrade of a bond by a rating agency

The outcome of a default may be that the contracted payment is

a. rescheduled
b. cancelled by the payment of an amount which is less than the default free value of the original contract
c. Cancelled and replaced with freshly issued equity in the company
d. Continued but at a reduced rate
e. Totally wiped out **[3]**

(ii) The risk neutral default probability is the probability that would exist in a world where all participants are risk neutral. It can be backed out from bond prices. The real world probability is the true probability. It can be calculated from historical data. The real world default probability is expected to be higher as risk adverse investors would require higher return to compensate. Hence risk neutral default probability would be recommended for valuation. The real probability of default should be used for scenario analysis. **[4]**

(iii) The probability of default in the first 3 years is 1- $e^{-0.005*3}$. The probability of default in the first 6 years = 1-$e^{-0.008*6}$. The probability of default between 3 and 6 years is $e^{-0.005*3}$ - $e^{-0.008*6}$ =0.0320 or 3.2% **[3]**

(iv) The probability of default in the first 3 years = (1- $e^{-0.005*3}$)/(1-0.25). The probability of default in the 6 years = (1-$e^{-0.008*6}$)/(1-0.30). The probability = 1-$e^{-0.008*6}$)/(1-0.30) - (1- $e^{-0.005*3}$)/(1-0.25) = 4.7%

[2]
[Total Marks-12]

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