

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

30th May 2014

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (10.15* – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1. Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
- 2. * You have 15 minutes at the start of the examination in which you are required to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*
- 3. You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*
- 4. The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
- 5. Attempt all questions, beginning your answer to each question on a separate sheet.*
- 6. Mark allocations are shown in brackets.*
- 7. Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q.1)**
- i) Describe briefly what is delivered upon the exercise of exchange traded put and call options on futures contracts. (3)
 - ii) How can you modify the Black-Scholes formula to calculate the value of options on futures? Calculate the price of a 7-month European put option on a futures contract when the futures price is Rs. 57,000, the risk-free rate of interest is 12% per annum, the strike price is Rs. 63,000 and the volatility of the futures price is 20% per annum. (4)
[7]
- Q. 2)**
- i) The Risk Audit of a derivative desk identifies the deficiency that both vanilla interest rate options (caps, floors and swaptions), and exotics interest-rate swaps were calculated using the Black model. The recommendation was to consider no-arbitrage term structure model. Outline the reasons for the above recommendation. Set out the main criteria the above suggested model must satisfy. (5)
 - ii) The Hull-White one-factor yield curve model projects the short-term rate r according to the formula:
$$dr = a(t)(b(t) - r)dt + \sigma(t)dz$$
where $\sigma(t)$ is the time-dependent short-rate volatility and z is a standard Brownian motion. Your bank has decided to use an implementation that sets parameters $a(t)$ and $\sigma(t)$ as constants a and σ respectively for all t .

Describe how you would construct a trinomial rate tree for this model, calibrated to the term structure of zero coupon bond prices and caplet volatilities. (9)
[14]
- Q. 3)** The recent ALCO (Asset liability committees) meeting for a large, multi-currency fixed-income trading company Chetan Private Ltd. requested implementation of Daily VaR calculation.
- i) Describe the concepts behind the variance-covariance approach to Daily Value at Risk (VaR) and explain the uses of Daily VaR reports for managing market risk specific to Chetan Private Ltd. (4)
 - ii) Outline how you would set up a Daily VaR for Chetan Private Ltd. and how you would cope with fixed and floating rate bonds, interest-rate swaps and interest-rate futures. (7)
[11]
- Q. 4)** As a member of the product desk in an investment bank you have been asked to structure a product which has following features
- i) One leg should have payments based on a floating rate
 - ii) Other leg should have payment based on a fixed interest rate but only for those days when the floating rate is between $\{F_m, F_n\}$ where $F_m < F_n$.
- Suggest a method of pricing the above product and describe the nature and risk of this product in terms of beta / gamma / vega. (6)

- Q. 5)** Your company is considering selling a 4 year credit default swap (CDS). The initial estimate of default for the company (who issued the bond underlying the CDS) is 4% for the first year but increases by 0.5% each year thereafter. You also assume a recovery rate of 40%, and that any default will take place at the mid-point of the year. The risk-free interest rate is 12%, continuously compounded.
- i) The Sales department to maintain the customer relationship assured the buyer that the pricing of the CDS would assume no default in between year 2-3. Estimate the theoretical price of the CDS using the information given. (Assume premiums paid annually in arrear, defaults take place mid of year) (8)
 - ii) How much loss would the company make from this deal (in spread terms)? (6)
- [14]**
- Q. 6)** A non-dividend paying stock has a current price of 1000p. In any unit of time the price either increases by 25% or decreases by 20%. The risk free interest rate is 5% per unit of time.
- i) Find the risk neutral probability measure for the model. (2)
 - ii) Denoting the stock price after t units of time by S_t :
 - a) Draw the stock price tree for this stock up to $t = 2$; i.e. 2 periods (1)
 - b) Find the price of a path dependent option on the stock with expiry date $t = 2$ which pays $S_2 - M_2$, where $M_2 = \min_{0 \leq t \leq 2} S_t$. (2)
- [5]**
- Q. 7)** Answer the following:
- i) Without using the Black-Scholes formula, show that European call and put options with the same strike and expiry have the same vega. (3)
 - ii) Suppose there are two American options, "A" and "B". Further, let us assume that "B" has half the notional of "A" but is otherwise identical. Let us now consider a portfolio "C" consisting of two contracts of type "B". Show "C" is preferable to "A". (3)
 - iii) What happens to the gamma of a vega neutral plain vanilla option (2)
 - iv) In the Black-Scholes model, as the risk free interest rate increases, what happens to the European Call and Put values? (3)
 - v) What happens to the price of an American digital option when the volatility is increased and the barrier has been breached? (2)
- [13]**
- Q. 8)** Prove that the price of an American option implied by a tree will always be as much as the price of a European option with the same parameters priced on the same tree. (5)
- Q. 9)** Connect the Vasicek model with the AR (1) process? (5)

Q. 10) Let the following hold for an asset: Share Price, $S = \text{Rs. } 50$, Strike Price, $X = \text{Rs. } 50$, Time to Maturity, $T = (65/365)$ days. Also, let volatility be 25% and the risk free interest rate be 6%. You are required to price a European call option using this data.

Answer the following questions:

- i) Calculate the value of the European call option using the Black-Scholes formula. (1)
- ii) Calculate the value of Delta and Gamma for the above mentioned option. (4)
- iii) The writer has sold 1000 calls and wants to have a delta neutral portfolio at the current time. To ensure that,
 - a) How many shares should the writer buy? (2)
 - b) How much additional money does the writer need to invest / borrow? (2)
- iv) Suppose the next day the share price moves to Rs. 52 with the other terms remaining constant. How will the hedge portfolio need to be changed to ensure delta neutrality for the 1000 calls he has written? (3)
- v) On the day the option is written, the writer wants to be both Delta and Gamma neutral for the 1000 calls he has written. To do this, he finds that another option (option 2) on the same is available in the market for 1.1466 and the delta and gamma for option 2 are 0.2965 and 0.0529 respectively. In that case,
 - a) How many new options (option 2) should the writer buy? (2)
 - b) How many shares should the writer buy? (2)
 - c) How much additional money does the writer need to invest / borrow? (2)
 - d) Comment on why the cash requirement is lower in this case as compared to the delta neutral case (2)

[20]
