INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st May 2014

Subject CT6 – Statistical Models

Time allowed: Three Hours (10.30 – 13.30 Hrs.)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) **i)** State Lundberg's inequality and explain the significance and nature of the adjustment coefficient. (4)

ii) The daily number of road accidents in a small city follows a Poisson distribution with mean 0.5. The road clearance cost incurred by the traffic authority, after each accident has the following distribution:

P [clearance cost = Rs 1000] = 0.5

P [clearance cost = Rs 2000] = 0.25

P [clearance cost = Rs 3000] = 0.25

Every day morning the traffic authority receives a grant of Rs 1000 from the municipality to its road clearance fund.

The traffic authority starts a week with Rs 500 in the road clearance fund.

Calculate the probability that the traffic authority will fall short of the road clearance fund before the end of the second day ignoring any interest or any other expenses. You may not use any approximation. (8)

[12]

Q. 2) **i)** A random variable X has Pareto distribution with density function f(x) and parameters α and λ. Show that for R > 0

$$\int_R^\infty x f(x)dx = \frac{\lambda^\alpha(\alpha R + \lambda)}{(\alpha-1)(\lambda+R)^\alpha}$$

(5)

ii) Claims under a particular class of insurance follow Pareto distribution with mean 4.5 and variance 60.75 (figures in INR 000s). In any one year 15% of policies are expected to give rise to a claim.

An insurance company has 500 policies on its books and wishes to take out individual excess of loss reinsurance to cover all the policies in the portfolio. The reinsurer has quoted premiums for two levels of reinsurance as follows (figures in INR 000s):

Retention Limit	**Premium**
25	22
30	16

a) Calculate the probability, under each reinsurance arrangement, that a claim arising will involve the reinsurer.

b) By investigating the average amount of each claim ceded to the reinsurer, calculate which of the retention levels gives more profit to the insurer (ignoring the insurer’s attitude to risk). (10)

[15]

Q. 3) The total number of claims *N* on a portfolio of insurance policies has a Poisson distribution with mean λ. Individual claim amounts are independent of *N* and each other, and follow a distribution *X* with mean μ and variance σ^2. The total aggregate claim in the year is denoted by *S*. The random variable S therefore has a compound Poisson distribution.

i) Derive an expression for the moment generating function of *S* in terms of the moment generating function of *X*. (4)

ii) Derive expressions for the mean and variance of *S* in terms of λ, μ and σ. (6)

For a particular type of policy, individual losses are exponentially distributed with mean 100. For losses above 200 the insurer incurs an additional expense of 50 per claim.

iii) Calculate the mean and variance of *S* for a portfolio of such policies with λ = 500. (10)

[20]

Q. 4) An insurer believes claims amounts (in thousands of INR) from it's property portfolio follow a Pareto distribution with parameters α=3 and λ=300. The insurer wishes to introduce a deductible such that 20% of the losses result in no claim for the insurer.

i) Calculate the size of the deductible. (4)

ii) Calculate the average claim amount net of deductible. (5)

[9]

Q. 5) A direct insurance sales office of a motor insurance company receives random number of enquiry calls every day. The number of calls each day follows a Poisson distribution with unknown mean β.

Prior beliefs about β are specified by a Gamma distribution with mean of 200 and standard deviation of 50. The sales team has received 240 calls daily on average recently.

Calculate the Bayesian estimate of β under Quadratic loss. **[7]**

Q. 6) A general insurer believes that claims in the motor insurance portfolio arise as an *Exp* (λ) distribution. There is a retention limit of Rs. 1,00,000 in force, and claims in excess of Rs 1,00,000 are paid by the reinsurer.

The insurer, wishing to estimate λ, observes the last year claims and finds that out of total 250 claims, that the average amount of the 226 claims that did not exceed Rs 1,00,000 was Rs 540. The each of the remaining 24 claims were above Rs. 1,00,000 and are yet to be settled by the reinsurer.

Write down the likelihood function clearly and find the MLE estimate of λ. **[6]**

Q. 7) **i)** Explain the advantages of using pseudo-random numbers over truly random numbers. (2)

ii) Explain how would you determine the number of simulations to carry out in order to estimate a quantity of interest? Also, specify the two common ways of measuring the discrepancy in doing the same. (4)

[6]

Q. 8) **i)** Define, giving relevant equations, the three main linear models used for modeling stationary time series. (6)

ii) Classify each of the following processes as *ARIMA*(*p*,*d*,*q*), if possible:

a) $X_t = 0.6\,\varepsilon_{t-1} + \varepsilon_t$

b) $X_t = 1.4X_{t-2} + \varepsilon_t + 0.5\,\varepsilon_{t-3}$

c) $X_t = 1.4X_{t-1} - 0.4X_{t-2} + \varepsilon_t + \varepsilon_{t-1}$

In each case ε_t denotes white noise with mean 0 and variance σ^2. (4)

iii) A moving average time series is defined by the relationship

$X_t = 3.1 + \varepsilon_t + 0.25\,\varepsilon_{t-1} + 0.5\,\varepsilon_{t-2} + 0.25\,\varepsilon_{t-3}$

where $\varepsilon_t \sim N(0, \sigma^2)$ denotes white noise.

Calculate the autocorrelation function ρ_k, at all lags k. (5)

[15]

Q. 9) **i)** Explain the Bornhuetter-Ferguson method mentioning its steps and its advantage(s) over the basic chain ladder method. (4)

ii) An insurance company has paid the following claim amounts (in Rs.000s):

		Development Year				
		1	2	3	4	5
Accident Year	1	4900	2450	1727	564	100
	2	5705	3507	1780	737	
	3	6745	3462	1500		
	4	6514	3700			
	5	8097				

The earned premiums are 11,100 for Accident Year 1, 13,677 for Accident Year 2, 15,585 for Accident Year 3, 16,381 for Accident Year 4 and 16,507 for Accident Year 5 (premiums are in Rs.000s).

Apply the Bornhuetter Ferguson method to estimate the amount of claims yet to be paid, stating any assumptions that you make. (6)

[10]

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