INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd May 2014

Subject CT4 – Models

Time allowed: Three Hours (10.30 – 13.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Australia and Ireland are playing an international T20 cricket match. At the end of the 20-overs played by each team the scores are tied. The two teams have further played the Super Over and the scores are level again at the end of the Super Over. To determine the eventual winner, the captain of each team nominates a bowler to bowl in a one over shoot-out. In the shoot-out, the bowlers of the two teams bowl alternately. The team whose bowler strikes the stumps the most times within the one over will win the match. If the scores are still level after one over, the shoot-out continues until one team has a clear one point lead over the other.

Australia wins the toss and decides to bowl first.

Five deliveries have been bowled by each team and the current score is tied at 2-2. The tie-breaker will continue until one team takes a clear lead of one strike over the opponent at the end of their turns.

- The Australian bowler goes next and if he strikes the stumps Australia gains lead and the score would become 3-2 in Australia's favor. If Ireland's bowler fails to strike on his next turn, the score would be 3-2 and Australia would win the match. However, if Ireland's bowler does strike the stumps on his next attempt, the score would become 3-3 and the tie-breaker would continue.

- Alternatively, during Australia's strike, if their bowler fails to strike the stumps the score remains at 2-2. If Ireland's bowler strikes the stumps on his next turn the score would be 2-3 and Ireland would win the match. However, if Ireland's bowler fails to strike the stumps on his next attempt, the score would remain at 2-2 and tie-breaker would continue.

The probability of Australia's bowler striking the stumps is 90% and the probability of Ireland's bowler striking the stumps is 55%.

i) Describe how you can model the tie-breaker in this case as a Markov chain, clearly specifying the states. (5)

ii) Write down the transition matrix of this Markov chain. (2)

iii) State, with reasons, whether the chain is reducible: (1)

iv) Discuss whether each of the states in the Markov Chain is periodic or aperiodic. (1)

v) Calculate the total number of balls that must be bowled (from both teams) in the tie-breaker before there is more than a 95% chance of the game being completed. (3)

vi) Calculate the probability that:

a) Australia eventually wins the match.

b) Ireland eventually wins the match.

and comment on your answers. (4)

[16]

Q. 2) **i)** Describe the Poisson process and its properties.

Suppose that N_t is a Poisson process with rate 2.5. (3)

ii) Compute Pr ($N_5 = 8, N_9 = 12$).

Suppose that N_t is a Poisson process with rate λ. (3)

iii) Show that

$\Pr(N_{k1} = r | N_{k2} = n) = \binom{n}{r} p^r (1-p)^{(n-r)}$

where $n > r$, $k2 > k1$ and $p = k1/k2$ (4)

Suppose that N_t is a Poisson process with rate λ, and that X_k are independent, identically distributed random variables with mean m and variance s^2. Let $Y_t = \sum_{k=1}^{Nt} \mathrm{X_k}$

iv) Find E(Y_t) and Var (Y_t) in terms of m and s^2. (3)

[13]

Q. 3) Consider the following transition diagram for Cargo Ships with transition monthly rates as shown in the diagram:

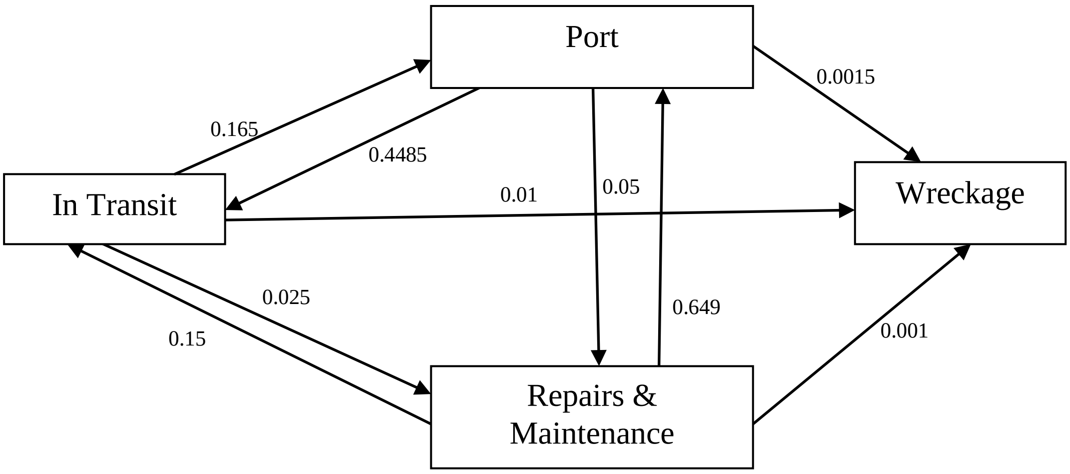
i) Calculate the probability that a Ship positioned at Port sick goes to a Wreckage site when it leaves the port. (1)

ii) Calculate the expected holding time for a Ship in the Repairs and Maintenance State. (1)

iii) Calculate the expected future lifetime for:

a) A Ship in Transit

b) A Ship parked at Port (6)



[8]

Q. 4) **i)** Describe the requirements of a good model. (4)

ii) A large betting house is worried about an unusual number of large bets going wrong. He has approached you as an actuary to model his betting payouts above INR 1mn. Comment on the data required to model the payouts and steps one should undertake in the development of model to model payouts in excess of INR 1mn.

Comment whether a deterministic or stochastic model is suitable. (8)

[12]

Q. 5) **i)** Explain in brief what is meant by graduation of mortality rates and the main aims of graduation. (3)

ii) Describe the advantages and disadvantages of graduating a set of observed mortality rates using a parametric formula. (2)

[5]

Q. 6) Information regarding the survival times (in weeks) of 20 patients with suffering from leukaemia is provided below:

6, 6, 6, 6*, 7, 9*, 10, 10*, 11*, 13, 16, 17*, 19*, 20*, 22, 23, 25*, 32*, 32*, 34*

where * represents the censored time.

i) Calculate the Kaplan-Meier estimate of the survival function assuming censoring occurs just after death at the respective duration. (4)

ii) Construct an approximate 95% confidence interval for the probability that a patient survives for at least 8 weeks after the start of the drug treatment. (3)

iii) What types of censoring are present in the data provided for the leukaemia patients? (2)

[9]

Q. 7) A life insurance company is performing the mortality experience for the calendar year 2013 for its traditional business. The investigation is to be done using the two standard tables IALM 94-96 and the new IALM 06-08.

Based on the past experience it has been suggested that only 77% of the total claims have been reported as at the date of investigation for calendar year 2013 for each of the ages.

The data is provided below:

Age	Initial exposed to risk	Standard Mortality rates (IALM 94-96)	Standard Mortality rates (IALM 06-08)	Actual reported deaths at date of investigation
x	Ex	qx(1)	qx(2)	A
40	1,00,000	0.002053	0.001803	130
41	98,927	0.002247	0.001959	142
42	98,100	0.002418	0.002140	155
43	97,450	0.002602	0.002350	170
44	96,600	0.002832	0.002593	185
45	95,800	0.003110	0.002874	205

i) Why a Life Insurance Company would like to perform mortality investigation? (3)

ii) Use the chi square tests to determine which table is a better fit as compared to own mortality experience. (5)

iii) What is the main limitation of performing the chi square test as inferred from the data above? (1)

iv) Perform sign test using the IALM 06-08 standard table against our own mortality experience. (2)

v) State the key observation on the comparison tests performed above. (2)

[13]

Q. 8) Following data is collected in a mortality investigation:

dx = total number of policies under which death claims are made when the policyholder is aged x last birthday in each calendar year

Px(t) = number of in-force policies where the policyholder was aged x nearest birthday on 1^{st} January in year t

i) State the principle of correspondence. (1)

ii) Obtain Derive a formula for the central exposed to risk that corresponds to the death data, stating any assumptions that you make. (4)

[5]

Q. 9) What are the main limitations of performing mortality investigation by cause of claim? **[3]**

Q. 10) **i)** What type of hazards are modelled by a decreasing Weibull distribution? (1)

ii) Prove memory less property of constant hazard Weibull distribution. (2)

iii) The time to fail for implanted heart membrane in a patient follows the Weibull distribution with failure rate b = 2 and c = 240 months. Using the information provided and taking 2 different time points show that as time elapses the probability of survival reduces and plot a rough sketch of the distribution? After how many months is 50% survival achieved? (4)

[7]

Q. 11) **i)** What do you understand by Maximum Likelihood Estimator? Give an example. (2)

ii) A small survey was conducted on the number of life threatening vehicle accidents individuals have had in their lifetime. A random sample of 5 observation is provided:

Number of accidents	**Age of the individual in years**
0	18
1	25
2	34
2	44
4	80

It is given that the number of accidents by individuals follows Poisson distribution with mean λ. Assuming that λ is a function of age (i.e. $\lambda = \lambda_0$ * age (i)), derive the MLE for λ. (5)

iii) Arrive at the probability that only 1 accident would be observed at the following ages and comment on your results:

- Age 25
- Age 80 (2)

[9]

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