INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

30th May 2013

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. ** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You have then three hours to complete the paper.*
3. *You must not start writing your answers in the answer sheet unless instructed to do so by the supervisor.*
4. *The answers are not expected to be any country or jurisdication specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*
5. *Attempt all questions, beginning your answer to each question on a separate sheet.*
6. *Mark allocations are shown in brackets.*
7. *Please check if you have received complete Question paper and no page is missing. If so, kindly get a new set of Question paper from the Invigilator.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q.1) **i)** Prove that forward and futures prices are equal when the risk free rate is deterministic? (5)

ii) Argue why they would not be equal if the risk free rate is not deterministic? (3)

[8]

Q. 2) **i)** Suppose we have two processes X, Y as given below where $W_t$is the Brownian motion

$$dX_t = \mu_t dt + \sigma_t dW_t$$

$$dY_t = \nu_t dt + \rho_t dW_t$$

Use Ito's lemma to show that $d(X_t Y_t) = X_t dY_t + Y_t dX_t + \sigma_t \rho_t dt$

(3)

ii) Are the following Brownian motion?

a) $X_t = \frac{1}{\sqrt{\alpha}} W_{\alpha t} \quad where\ W_t\ is\ Brownian\ motion$

b) $X_t = \rho W_t + \sqrt{1-\rho^2}\widetilde{W}_t\ where\ W_t and\ \widetilde{W}_t are\ independent\ brownian\ motion$

(3)

iii) Evaluate the mean and variance of the random variable of X_t where

$$X_t = \int_0^t W_s^2 dW_s$$

(2)

[8]

Q. 3) **i)** State the general form of Girsanov's theorem. (1)

ii) Let g be the value of a numeraire asset with volatility σ_g. If f is the price of any security and the market price of risk is equal to σ_g, then prove that f/g is a martingale under the real world probability measure. (5)

iii) Hence show that $f_t = g_t E_g\left[(\frac{f_T}{g_T})|F_t\right]$. Where E_g denotes the expected value in a world that is forward risk neutral with respect to g. f_t (f_T) and g_t (g_T) are the prices of f and g respectively at time t (T). (1)

iv) Define Quantos. (1)

v) The rupee price of a stock $S_t$and dollar value of one rupee $C_t$follow the process given below:

$$S_t = S_0 \exp\left(\sigma_1 W_1(t) + \mu t\right)$$

$$C_t = C_0 \exp(\rho\sigma_2 W_1(t) + \bar{\rho}\sigma_2 W_2(t) + \nu t)\quad where\ \bar{\rho} = \sqrt{1-\rho^2}\ .$$

(5)

The prices of zero- coupon Dollar bond and zero-coupon Rupee bond (D_t) follow the process given below:

$$B_t = \exp{(rt)}$$

$$D_t = exp(ut)$$

Where r and u are interest rates in zero-coupon dollar bond and zero-coupon rupee bond respectively and W_1 and $W_2$are independent Brownian motions.

Please find out the market price of risk which makes the discounted dollar tradables namely $(B_t^{-1}C_tD_t\ , B_t^{-1}C_tS_t)$ into martingales given that $\mu = \nu = 10\%, \sigma_1 = 30\%,\ \sigma_2 = 10\%\ , \rho = 60\%\ ,\ r = 0\%\ and\ u = 10\%$

vi) Which is higher, the expected return on a stock or that of a call option on a stock? Assume the CAPM model governs returns in the real world. (4)

[17]

Q. 4) You are given the following node values on an explicit finite-difference lattice.

V (i + 1; j + 1) = 4.5; V (i + 1; j) = 5.2; V (i + 1; j - 1) = 6.3. The time step (increasing in index i) is denoted h =1/12, and the log stock price step (increasing in j) is denoted k = 0.10.

i) Is this more likely to be a call or a put option? (1)

ii) The simple risk-free rate is 10% per annum and is constant for all maturities. If the stock return volatility is 30%, what is the value of the derivative security at node V (i; j)? (4)

[5]

Q. 5) Quoted price of a T-Bond futures is 114.81. Which of the following is cheapest to deliver?

Bond	**Price**	**Conversion Factor**
1	162.63	1.3987
2	138.97	1.2820
3	131.06	1.1273

[3]

Q. 6) **i)** Outline strengths and weaknesses of Hull & White, Cox –Ingersoll-Ross and Vasicek interest rate models. (6)

ii) A proposal is made that call and put options to be used to replicate equity cash flows rather than holding direct equity. Evaluate the risks associated with this proposal. (6)

[12]

Q. 7) **i)** What is an nth to default contract? How does credit correlation impact this contract? (3)

ii) In a CDO suppose there are three tranches: A gets first claim to all cash flows from the collateral, B gets second claim, and there is a residual equity tranche E. Suppose if the level of default risk in the economy declines but the correlations of default increase, what would be the likely impact of this on the values of the three tranches? (3)

[6]

Q. 8) Suppose you have a model for pricing convertible bonds that accounts for equity risk, interest-rate risk and credit risk and is calibrated using observable stock prices, bonds, and credit default swaps. If the model price of the convertible bond exceeds that of the market and you believe the model is accurate, what broad strategy will you adopt to construct an arbitrage portfolio?

[4]

Q. 9) **i)** What is the "forward measure"? Explain its importance in the pricing of interest rate derivatives. (3)

ii) You have a Forward Rate Agreement (FRA) to borrow at 5% that has six months to run until maturity and is for the period (6,12) containing 183 days. The current forward rate for the period (6,12) is 5.2%. What is the mark-to-market value of the FRA? What is the Price Value of a Basis Point (PVBP) of this contract? Explain the sign of the PVBP. The yield curve is flat. Assume the standard Actual/360 money market convention. The PVBP of an FRA is the change in value of the FRA for a one basis point change in yield. (5)

[8]

Q. 10) **i)** If the value of a firm is $100 million, the value of equity in the firm is $40 million, the risk-free rate is 4%, and the firm has outstanding debt with a face value of $70 million with zero coupons and a maturity of three years, what is the firm's volatility of returns on its assets? What is the risk-neutral probability of the firm becoming insolvent in three years if we assume that the Merton (1974) model applies? (4)

ii) Calculate the distance to default based on Moody's KMV model for the above firm given in part a. (2)

iii) Explain why the spread curve in the Merton model is hump-shaped. (3)

[9]

Q. 11) We extend the Vasicek model to allow the mean rate θ to become stochastic. Think of consider a situation in which the Reserve Bank makes minor adjustments to short-term market rates to manage the growth/inflation of the economy. The model comprises the following two equations:

$$dr = k(\theta - r)dt + \sigma dB$$

$$d\theta = \eta dB$$

The Brownian motion dB is the same for both the interest rate r and its mean level θ.

i) Given the bond price function P(r; θ ; T), write down the process for dP using Ito's lemma, Where T denotes the time to maturity (t may be used to denote current time). (3)

ii) Suppose the market price of risk is zero for both stochastic variables r and θ. Then the bond's instantaneous return will be given by E(dP) = rPdt. Using this identity, derive the partial differential equation (pde) for the price of the discount bond, stating clearly the boundary condition for the bond price. (2)

iii) Assume the following functional form for the solution of the pde.

$$P(r, \theta, T) = A(T)exp[-rB(T) - \theta C(T)]$$

Use this to derive a closed-form expression for the price of the bond. (7)

iv) Will bond prices be higher or lower in this model versus a model in which $\eta = 0$, i.e. where the mean rate is constant? (2)

[14]

Q. 12) Explain carefully the distinction between real world and risk neutral probability of default. Which is higher?

A bank enters into a credit derivative where it agrees to pay Rs 100 Cr at the end of one year if a certain company's credit rating falls from AA to A or lower during the year. The one year risk free rate is 8% continuously compounded. Using the rating transition matrix (1 year) given below estimate the value of the derivative. What assumptions are you making? Do they tend to overstate or understate the value of the derivative?

Rating	AAA	AA	A	BBB	BB	B	C	D
AAA	97.00%	3.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
AA	1.59%	92.33%	5.06%	0.68%	0.24%	0.04%	0.03%	0.03%
A	0.00%	3.58%	86.72%	5.98%	2.43%	0.20%	0.35%	0.74%

[6]

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