INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

29th May 2013

Subject CT8 – Financial Economics

Time allowed: Three Hours (10.00 – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q.1) A trader sells a 30-day European call option on a non-dividend paying stock S, priced as per the Garman-Kolhagen model. Initially (at day 0), he had:

i. Stock price $S_0 = 100$
ii. Risk free interest rate r = 0.05 (per annum with continuous compounding)
iii. Volatility σ = 0.4 (per annum)
iv. Strike price K = 100

a) Calculate the price of the call option at day 0 (3)

b) Find the delta (Δ) of the call option (1)

c) If the investor (call writer) wishes to delta-hedge his position on day 0, how can he do so by trading the stock S? (1)

d) Assume that the investor (call writer) has to borrow the money from the market at risk free rate to cover for the net expenses of his hedged position. What will be his cost of borrowing? (2)

e) Suppose the price of the stock moves to 105 the next day (day 1). What will be the trader's profit/loss? (2)

[9]

Q. 2) An investment market has only two securities A and B such that
E_A (Expected return on Security A) = 2% V_A (Variance of returns on security A)= 4%%
E_B (Expected return on Security B)= 5% V_B (Variance of returns on Security B)= 9%%

i. Write down the Lagrangian function W when the coefficient of correlation $\rho_{AB} = 0.25$ (1)

ii. Write down the first order conditions and solve them to determine the proportions of the securities in the global minimum variance portfolio (6)

iii. Comment on where the efficient frontier lies in expected return-standard deviation space. (2)

[9]

Q. 3) Let $B_t\,, t \geq 0$ be standard Brownian motion with $B_0 = 0$

i. Show that $\frac{1}{2}(B_t^2 - t)$ is a martingale with respect to F_t the filtration associated with B_t (3)

ii. Show that $\exp(23B_t - \frac{529}{2}t)$ is also a martingale. (4)

iii. State the scaling property and the time inversion property of Brownian motion. (2)

[9]

Q. 4) **i.** Use stochastic dominance to choose between the two assets offering the following returns

Return	Asset A probability	Asset B probability
7%	0.2	0
6%	0.3	0.6
5%	0.5	0.4

(2)

ii. What are the main advantage and disadvantage of using stochastic dominance to make investment decisions? (2)

[4]

Q. 5) The price of a non-dividend paying share goes up 5% or down 3% over each month. Assume a monthly risk-free rate (continuously compounded) of 1.5%. Using the two-period binomial tree model calculate each of the following:

i. Value of a two month European call with a strike price of Rs 700 and current share price of Rs 665 (4)

ii. Value of a two month European put with the same strike price (3)

[7]

Q. 6) **i.** Consider an asset returns on which are uniformly distributed over -1.5% to 6.5% per annum. Ravi has a choice to invest either in this asset or in a risk-free asset offering 1% return per annum. What investment would Ravi choose to minimize his expected shortfall assuming his benchmark return is 3% per annum. (2)

ii. Comment on your answer should Ravi's benchmark level be 0% instead. (3)

[5]

Q. 7) Stock prices in 'Wienerland' are modelled as per $S_t = S_0.\exp(\mu t + \sigma B_t)$ where S_t is the stock price after t years, B_t is standard Brownian motion, μ = 0.05 and σ^2 = 0.16

i. find the probability that $S_4 > 175$, given $S_3 = 125$ (3)

ii. calculate the variance at 4 years, given $S_3 = 125$ (3)

[6]

Q. 8) You are the buyer of a European put option on a share with one month to expiry and an exercise price of Rs 18.00. You believe the share price at expiry would either be Rs 22. 50 or Rs 13.50. The risk-free force of interest is 1.5% per month.

i. Outlining how you could create a hedged position, evaluate the price of the put if the share is currently priced at Rs 20.44 (3)

ii. Value the put using risk-neutral valuation if the current share price is Rs 20.46 (2)

iii. Use your answers above to estimate the option's delta when the share is priced at Rs 20.45 and comment on your answer. (2)

[7]

Q. 9) **i.** Define a martingale in discrete time (2)

ii. Let $\{X_t\}$ be a stochastic process defined on the integers with the following transition probabilities:

$P[X_{i+1}=j+1|X_i=j] = p$, $P[X_{i+1}=j-1|X_i=j] = 1-p$

Show that $\{X_i\}$ is a martingale if and only if p = 0.5 (2)

iii. Suppose $X_0=0$ and {-3,3} are absorbing states, i.e., $P[X_{i+1}=3|X_i=3] = 1$, $P[X_{i+1}=-3|X_i=-3] = 1$. Generate the transition probability matrix for the restricted state space [-3,3] (3)

[7]

Q. 10) X_t follows an Ornstein-Uhlenbeck process defined by $dX_t = -3X_t dt + dB_t$ where B_t is a standard Brownian motion. Let $m_t = E[X_t \mid X_0 = x]$ and $s_t = E[X_t^2 \mid X_0 = x]$.

a) Show that $dm_t = -3m_t dt$ and hence prove $m_t = xe^{-3t}$ (2)

b) Use Ito's lemma to obtain the stochastic differential equation for X_t^2 (1)

c) Show that $ds_t = -6s_t dt + dt$ and hence prove $s_t = x^2e^{-6t} + \frac{1}{6}(1 - e^{-6t})$ (3)

d) Using m_t and s_t derived above write down an expression for $v_t = Var[X_t \mid X_0 = x]$ (1)

[7]

Q. 11) An investor is faced with two portfolios:

<u>Portfolio 1</u>

Rs. 4,000 in stock M and Rs. 6,000 in stock N

State of Economy	Probability of state of Economy	Returns Stock M	Returns Stock N
Boom	25%	18%	10%
Normal	75%	7%	8%

<u>Portfolio 2</u>

30% Invested in stock S and 70% in stock T

State of Economy	Probability of state of Economy	Returns Stock S	Returns Stock T
Boom	40%	12%	20%
Normal	60%	6%	4%

Stock	β
M	0.64
N	1.04
S	0.36
T	1.48

a) What can you say about the risk free return and the expected return on the market which would be consistent with CAPM? (9)

b) State the assumptions for CAPM to hold good (3)

c) In order to overcome the limitations of CAPM, the investor looked at an alternate method for asset pricing. Suggest and briefly describe the possible method he can use. (3)

[15]

Q. 12) An investor is faced with a choice of 3 states: I = investment grade, J = junk grade and D = default (absorbing). The one year risk free zero coupon rate is 5% and the spreads for the investment and junk grades are 130 and 170 basis points respectively.

a) List the possible credit events and outcomes of a default on a corporate bond (4)

b) Describe how the Jarrow-Lando-Turnbull (J-L-T) credit risk model is an example of a "reduced form model" (2)

c) How might J-L-T model be generalized to allow for changes in credit spreads under different economic conditions? Give a practical example of such a change. (3)

d) Estimate the risk-neutral probabilities of the two types of corporate bonds (2)

e) Construct the 3x3 ratings transition matrix assuming the investment grade I has 90% chance of retaining the same rating over the year whereas the junk bond has 80% chance for the same situation. (2)

f) Suppose the recovery rate for the two corporate bonds are 40% and 30% for I and J. Reconstruct the ratings transition matrix (2)

[15]

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