INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

21st May 2013

Subject CT6 – Statistical Models

Time allowed: Three Hours (10.00 – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) An insurer issues insurance contracts under only yearly premium mode, with various terms. X is a random variable denoting the term (in years) of the contract, then

$P(X=x) = e^{-b}\ b^{x-1} / (x-1)!$ for x = 1,2,3, ...

= 0 otherwise.

Let K be the random variable denoting the number of years for which the premiums are paid before the policy finally terminates by way of cancellation, death, maturity etc. K can take the values 1,2, 3,..., x with equal probability for a given x.

Let R be the number of years for which the premiums remain unpaid.

Find the mean and variance of R.

[9]

Q. 2) **a.** The incremental claims paid each year under a certain cohort of insurance policies are recorded in the table below, for accident years Y1, Y2, Y3 and Y4:

Development Year				
Accident Year	0	1	2	3
Y1	2,000	2,400	600	250
Y2	2,500	2,500	810	
Y3	3,000	2,600		
Y4	3,500			

Calculate the development factors under the basic chain ladder technique and estimate total outstanding claims

(6)

b. Explain in words the assumptions underlying the Chain Ladder Technique for completing the delay triangle? (3)

c. The rate of claims inflation over these years, measured over the 12 months to the middle of each year are given below:

Annual claim inflation rate (past)		Estimated annual claim inflation rate (future)	
Y2	2.00%	Y5	2.5%
Y3	2.50%	Y6	2.5%
Y4	2.75%	Y7	2.5%

Use the inflation adjusted chain ladder method to estimate the total amount outstanding for future claims arising from accident years 2006 and 2007. (12)

[21]

Q. 3) A General insurer has prepared its following commercial property data in two states to analyse using models of Empirical Bayes Credibility theory. Claim payments (in lakhs of INR) and the number of policy sold in each state are as given below:

Claim Payments:

	Year1	Year2	Year3
State A	3112	2124	1106
State B	5129	4116	3154

Policies Sold:

	Year1	Year2	Year3
State A	332	242	125
State B	427	326	198

a. Analyze the data using EBCT model1 and calculate the total expected claim payments to be made in coming year in each state. (6)

b. Analyze the data using EBCT model 2 and calculate the total expected claim payouts in each state in coming year assuming 100 policies and 200 policies are sold in state A and B respectively. (8)

[14]

Q. 4) **a.** (x_1 , x_2 , ..., x_n) is a random sample from Gamma (α,λ) distribution, where λ is an unknown parameter. Find the posterior distribution of λ assuming its prior distribution as Gamma (α',λ'). (5)

b. Write down the Bayesian point estimator of λ under quadratic loss. (2)

[7]

Q. 5) Aggregate claims on a portfolio of large commercial insurance policies follow Pareto distribution with mean 600 and std. Deviation 1200. The company has an XOL reinsurance arrangement on this group of policies with retention level of 1600. Calculate the mean claim amount paid by the reinsurer on this portfolio given the maximum amount paid by the reinsurer on a claim is 1200. (All figures are in lakhs of INR)

[9]

Q. 6) Aggregate claims on a general insurance company's portfolio form a compound Poisson process with parameter λ. Individual claims have a Gamma distribution with mean 200. The company applies a 40% premium loading. The insurer effects proportional reinsurance with a retained proportion of β. The reinsurer applies a 50% premium loading.

a. State the important assumptions underlying a compound Poisson process. (2)

b. Calculate the minimum value of β such that the insurer's net income is greater than the expected net claims.
Assuming reinsurer applies only a 20% premium loading (instead of 50%), again calculate the value of β and comment on your results. (5)

c. Assuming reinsurers' profit loading is 50% and that the first parameter (α) of Gamma distribution is 1, show that the direct insurer's adjustment coefficient, R , satisfies: (4)

$$R = (1 - 5\beta) / (200\beta - 3000\,\beta^2)$$

d. By differentiating the result from (c), show that $\beta = 0.3633$ maximizes the adjustment coefficient and calculate the corresponding optimal value of R. (4)

[15]

Q. 7) **a.** State and briefly explain the important components which need to be specified in order to define a Generalized Linear Model (GLM)? (3)

b. In context of a GLM, the covariates (or explanatory variable) enter the model through the linear predictor. Write the form the linear predictor could take in case:

1. Only age is the explanatory variable. (1)
2. Age and gender are the explanatory variables (but the effect of the age is the same for both male and female). (1)
3. Age and gender are the explanatory variables and the effect of the age is different for males and females. (1)

c. Claim amounts for motor insurance claims are believed to have an Poisson distribution with parameter λ:

$$f(y_i) = \frac{\lambda^{y} e^{-y}}{y!}$$

The insurer believes that a linear function of age affects the claim amount:

$$\eta_i = \alpha + \beta x_i$$

Using the canonical link function, find the equations satisfied by the maximum likelihood estimates for a and b , based on a random sample of claim amounts $y_1, \ldots, y_n$. (4)

d. What is scaled deviance? How is the concept of deviance used in context of GLM? (3)

[13]

Q. 8) An insurance company issues 1000 policies to professional cyclists, where the probability of claim in one year is p for each policy. The cyclists participate in cycle races held at either mountain area or plain area. The value of p is 0.03 in mountain area and it is 0.01 in plain area. The probability of being in mountain or plain area in the next year is 50%. Let N be the aggregate number of claims from the portfolio in the coming year.

a. Calculate the mean and variance of N. (4)

b. Calculate an alternative approximation for mean and variance of N with approximate common value of p as 0.02. (2)

An Actuary relooks the portfolio and segregates the portfolio in 2 groups as high and low risk category. The aggregate claim from each of the categories follows compound Poisson distribution. The high risk category includes 400 policies with Poisson parameter 0.2 and fixed individual claim amount of INR 4000. The low risk category includes 400 policies with Poisson parameter 0.1 and individual claim amounts are either of INR 8000 with probability 0.6 or of INR 10000 with probability 0.4. All policies are assumed to be independent and let S be the random variable denoting the aggregate claim amount from the portfolio over the next year.

c. Calculate the mean and variance of S. (3)

d. Using normal approximation of S calculate the value of y where $P(S > y) = 0.2$. (3)

[12]

****