INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

22nd May 2013

Subject CT4 – Models

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet. <u>However, answers to objective type questions could be written on the same sheet</u>.***

4. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) **i)** State the factors you would consider for assessing the suitability of a model. (3)

ii) Briefly describe the task of simulating a time-homogeneous Markov jump process. (3)

[6]

Q. 2) **i)** Explain the importance of dividing the data into homogeneous classes, including subdivision by age and sex in the context of mortality investigations. (2)

ii) A recent European ruling on gender has made it illegal for insurance companies to discriminate policyholders based on their gender. Therefore, insurance providers may no longer charge differential premium rates to males and females. Consequently, the Appointed Actuary of a large insurance company has suggested that having separate mortality tables for males and females is now pointless and proposed constructing a single combined life table to be used in all subsequent actuarial investigations. Do you agree? Explain your answer. (4)

[6]

Q. 3) **i)** Write down an expression for ${}_t\boldsymbol{q}_{\boldsymbol{x}}$, where $\boldsymbol{q}_{\boldsymbol{x}}$is the probability of death between integer ages $\boldsymbol{x}$ and $\boldsymbol{x+1}$; and$\boldsymbol{0 \leq t \leq 1}$ under the following assumptions:

- Deaths occur uniformly between integer ages
- The force of mortality is constant between integer ages
- The Balducci assumption holds

(2)

ii) A study of infant mortality in rural India found that the number of children that die within the first year of birth is close to 67 deaths per 1000 live births. Estimate the probability of death within the first three months of birth under each of the above assumptions. (2)

iii) State which assumption you think is most appropriate for the study of infant mortality and why? (2)

[6]

Q. 4) **i)** State the precise mathematical formula representing the complete expectation of life, $\dot{\boldsymbol{e}}_{\boldsymbol{x}}$in terms of probability of survival, ${}_t\boldsymbol{p}_{\boldsymbol{x}}$. Explain in words what this represents. (2)

You are given that the force of mortality, $\boldsymbol{\mu}_{\boldsymbol{x}}$ is constant 0.0325 at all ages. Calculate:

ii) The curtate expectation of life (2)

iii) The probability that a life aged exactly 36 will survive to age 45. (2)

iv) The exact age $\boldsymbol{x}$ representing the median of the life-time $\boldsymbol{T}$ of a new born baby. (2)

[8]

Q. 5) A tablet device can be either in an active mode (state 1) or in an inactive mode (state 2). If it is in an active mode, the probability that it will move to the inactive mode by the end of next minute is 0.25. If it is in the inactive mode, the probability that it will move to the active mode by the end of next minute is 0.20. The initial state is inactive. Let X_n be the state of the device after n minutes.

i) Find the distribution of X_2. (2)

ii) Find the steady-state distribution of X_n. (4)

[6]

Q. 6) Company A specializes in manufacturing mobile semiconductors. In the production of mobile semiconductors, two types of defects may occur: Type I and Type II. It is possible that a particular semiconductor may suffer from both the defects simultaneously.

It has been found from testing that for every 100 such semiconductors there are, on an average, 3 defects of type I and 5 defects of type II. Each of these types of defects follows a Poisson process. The company supplies semiconductor to the clients in packets, with each packet containing 20 semiconductors.

i) For a lot of 20 semiconductors, what is the chance of finding exactly 2 Type I defects? (2)

ii) For the same 20 semiconductors, what is the chance of finding one or more Type II defects? (2)

If the pack of 20 semiconductors contained more than 2 defects, it would be rejected.

iii) What is the probability that a pack will be rejected? (3)

iv) Company A had decided to supply four such packs to a mobile manufacture this year. Assume the defect conditions between the four such packs are statistically independent. What is the probability that only one of the packs would be rejected? (3)

[10]

Q. 7) A shopkeeper runs his shop in an area that typically gets heavy rains. He has three umbrellas.

Every day, he goes to his shop in the morning and comes back home in the evening. If it is raining in the morning, he would carry an umbrella on the way to the shop (unless if, unfortunately, all three umbrellas happen to be at the shop). Likewise, if it is raining in the evening, he would carry an umbrella on his way back home (unless if, unfortunately, all three umbrellas happen to be at his home). If it is raining and he doesn't have an umbrella, he would still need to go to his shop (or come back home) and will unfortunately end up getting wet.

If it is not raining in the morning, and all three umbrellas are at home, he would nevertheless carry an umbrella to the shop just in case it rains in the evening. However, if he has goods to carry to his shop that day, he would not be able to carry an umbrella and would run the risk of getting wet in the evening.

Likewise, if it is not raining in the evening, and all three umbrellas are at shop, he would nevertheless carry an umbrella back home just in case it rains next morning. However, if he has goods to carry back home that evening; he would not be able to carry an umbrella and would run the risk of getting wet the next day.

In other words, he would always carry an umbrella if:

- It is raining and he has an umbrella to carry from his starting place; or
- It is not raining, he has all three umbrellas at his starting place and he does not have any goods to carry.

The probability that he has goods to carry from one place to the other (home to shop or shop to home) is 75%.

The probability that it's raining in the morning is *p*. Likewise, the probability that it's raining in the evening is also *p*. Rains in the morning and in the evening are events independent of each other.

i) Draw a transition diagram and write down the transition matrix for this Markov chain.

[Hint: You may define X_i to represent the number of umbrellas at a place where the shopkeeper presently is. X_i can take the values 0, 1, 2 and 3.]

(5)

ii) Using the stationary distribution derived from the transition probabilities, ascertain the steady state probability that the shopkeeper gets wet.

(5)

iii) Current estimates show that $p = 0.7$ in the area. How many umbrellas should the shopkeeper have so as to reduce the probability of getting wet to less than 2%?

(5)
[15]

Q. 8) The generator matrix of a continuous time Markov jump process with states A, B, C and D is shown below(transition rate in unit of years):

$$\begin{array}{cccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{array}$$
$$\begin{pmatrix} -.08 & .05 & 0 & .03 \\ 1.2 & -1.36 & .12 & .04 \\ 0 & 0 & -.35 & .35 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

i) Draw the transition diagram of the process.

(2)

ii) Calculate the probability that, having started in state A, the process hasn't visited state B and C by 4 years' time.

(3)

iii) Calculate the expected holding time in state B

(1.5)

iv) Calculate the probability of landing directly into state D when the process moves from state B

(1.5)

v) What is expected time to reach state D from State B? (5)
[13]

Q. 9) A large life insurance company with a long and credible history of mortality experience has undertaken a recent mortality investigation. The Appointed Actuary wishes to use the results from this investigation to form the basis for the best estimate assumptions to be used in the economic valuation of the company that is publicly disclosed.

A senior student has carried out the mortality investigation and graduated the crude rates from the experience using a graphical approach.

i) Clearly explain the risks and limitations of using the graduated rates obtained from the above investigation for the purpose of public disclosures of the economic value of the company. (6)

The Appointed Actuary has suggested using an alternative method to graduation whereby the graduated rates are derived using the formula:

$\boldsymbol{\alpha_1 + \alpha_2 exp(\alpha_3 x)}$; where

$\boldsymbol{x}$ is the crude rate of mortality from the experience investigation; and

$\boldsymbol{\alpha_i}$ are the fitted parameters.

ii) Describe how the alternative method proposed by the Appointed Actuary overcomes the risks and limitations of the graduation method adopted by the student.

What additional challenges or risks are posed by the alternative method? (4)

You have been appointed as an independent expert and asked to decide on which set of graduated rates should be used as the final best estimate assumption.

iii) Describe what tests you would carry out on the graduated rates to determine the most appropriate assumption to use. Clearly explain what conclusion you expect to draw from each of the tests.

What additional considerations would you take into account before making your final recommendation?

(6)
[16]

Q. 10) A new cancer treatment has been developed by researchers whereby the patient is hospitalized immediately upon diagnosis of a particular type of cancer and kept under strict medical supervision for one month. After one month, the patient is discharged from the hospital but required to take prescribed medication over a long period of time.

The medication is revised after check-ups at specified intervals, depending upon the patient's progress. The intervals are: 2 months after discharge; 6 months, 1 year, 2 years and 5 years after discharge from the hospital.
A study has been undertaken to determine the efficacy of this treatment. Data is available for patients admitted by quarter over a period of one year and this is summarized below:

	Q1	**Q2**	**Q3**	**Q4**
No. of new patients admitted	553	454	773	897
No. of deaths within the first month, whilst still in hospital	4	3	8	10
No. of deaths reported within first 2 months after discharge	21	14	42	40
No. of deaths reported within first 6 months after discharge	33	38	76	100
No. of deaths reported within 1 year after discharge	67	83	130	169
No. of deaths reported within 2 years after discharge	132	130	218	285
No. of deaths reported within 5 years after discharge	275	262	397	495

In addition to the data above, the study has highlighted that:

- Nearly 10% of all deaths go unreported after the patient has been discharged
- Approximately 8% of patients discontinue their treatment immediately after being discharged and therefore should be excluded from the study. A further 5% of the surviving patients discontinue treatment at the time of the first check-up after 2 months and another 2% discontinue at the subsequent 6 months post-discharge check-up. However it is rare that a patient would discontinue medication if he or she has been under treatment for one year or more. You can assume that once a patient commits to taking revised medication, he or she continues to do so at least until the next check-up.

Construct a model of survival for the patients undergoing treatment to determine the probability of death for each time interval between subsequent check-ups. Hence, calculate the probability of survival for 4 years following initial diagnosis for a patient undergoing this treatment.

You may assume a uniform distribution of deaths for the years 3-5 after discharge.

[14]

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