

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

27th May 2013

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception.*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1) i)** A zero-coupon bond has the price at time $t = 0$ as per the following equation:
- $$P_t = (100 - 1.5t^2)\% \quad \text{where } 2 \leq t \leq 4$$
- Calculate the instantaneous forward rate F_3 . (2)
- ii)** The spot rate at time $t = 0$ is expressed as $(3.05 + 0.35n) \%$ per annum effective for $1 \leq n \leq 5$.
- a)** Calculate the three year forward rate at time $t = 2$ expressed as an effective annual rate of interest. (2)
- b)** What does a par yield represent? (1)
- c)** Calculate the four year par yield. (1)
- [6]
- Q. 2)** An actuarial student has been offered an Educational Loan at a rate of interest of r per annum effective. The loan is repayable through 25 equal yearly installments in arrears. The capital component of the 21st installment is equal to the interest component of the 16th installment. Calculate the interest rate r . (5)
- Q. 3)** In order to accumulate to a sum of ` 24000 at the end of $5n$ years in a bank account, the accountholder needs to invest ` 275 p.a. payable quarterly at the end of each quarter of the first n years and ` 450 p.a. payable quarterly at the end of each quarter for the next $4n$ years. The annual effective rate of interest is i .
- Given that $(1+i)^n = 1.5$ calculate i . (4)
- Q. 4) i)** Give two examples of Fixed Interest Government Borrowings and list four characteristics of each. (5)
- ii)** Describe the cash flows to a borrower of a fixed interest repayment loan. (2)
- iii)** State the relationship between the money rate of interest and the real rate of interest under different inflation conditions. (2)
- iv)** Under a share, dividends of ` 0.30 per share are expected on 30th June 2013 and 31st December 2013. The 6-month and 12-month spot rates are 7% and 7.5% per annum effective respectively on 1st January 2013. If the share price on 1st January 2013 is ` 12/-, calculate the value of a one-year forward contract to be issued on 1st January 2013 (2)

- v) A 15-year fixed interest bond with a coupon of 6% per annum payable half yearly was issued on 1st January 2010. The bond is redeemable at 130% on the maturity date. An investor who pays income tax at the rate of 20% and capital gains tax at the rate of 30% has purchased the bond on 1st Jan 2014. Calculate the purchase price that the investor should pay if he requires a net yield to redemption of 8% per annum effective on date of purchase. (5)
[16]
- Q. 5)** The force of interest, $\delta(t)$, can be expressed as a function of time t (where t is measured in years) as under: $\delta(t) = 0.05$ for $0 < t \leq 10$,
 $= 0.006t$ for $10 < t \leq 20$
 $= 0.003t + 0.0002t^2$ for $20 < t$
- i) Calculate the present value of ` 100 at time $t = 25$. (4)
- ii) Calculate the effective rate of interest per unit time from time $t = 19$ to time $t = 20$. (1)
- iii) A continuous payment stream is paid at the rate of $e^{-0.03t}$ per unit time between time $t = 0$ and time $t = 5$. Calculate the present value of that payment stream. (2)
[7]
- Q. 6)** A fund has a liability to make two payments in future, the first one due at the end of t years from now and the second one 3 years after the first. Further, the second payment is 2.5 times the first payment.
- i) If the present value of these payments and discounted mean term of the liabilities at 8% p.a. effective are ` 7107.77 and 5 years respectively. Determine the timing (in complete years) and the amount of both payments. (7)
- ii) State the Redington's conditions for immunization of a fund against small changes in interest rates. (3)
- iii) The asset backing this instrument is to be invested in a zero coupon bond maturing after 5 years giving maturity value of ` 10443.65. Investigate whether the fund satisfies the necessary conditions to be immunized against small interest changes in the rate of interest. (3)
[13]
- Q. 7)** i) Prove from first principles that :
- $$(I \bar{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{\delta} \quad (4)$$

- ii)** A man deposits ` 6000/- in a bank account which earns interest of 4% per annum effective. He withdraws ` 500/- at the end of the 51st month, ` 490/- at the end of the 54th month, ` 480/- at the end of the 57th month and so on until he withdraws a payment of ` 390/- at the end of the 84th month.
- a)** Find the present value of the series of withdrawals at the end of the 48th month. (5)
- b)** Find the amount available in the deposit at the end of 72nd month, immediately after the withdrawal then due. (5)
- [14]**
- Q. 8)** An investment project gives rise to the following cash flows. At the beginning of each of the first three years ` 180,000 will be invested in the project. From the beginning of the first year until the end of the twenty-fifth year, net revenue will be received continuously. The initial rate of payment of net revenue will begin at ` 25,000 per annum. The rate of payment is assumed to grow continuously at a rate of 6% per annum effective.
- i)** In respect of an investment project, define:
- a)** the discounted payback period
- b)** the payback period (2)
- ii)** Discuss why both the discounted payback period and the payback period are inferior measures compared to the net present value for determining whether to proceed with an investment project or not. (3)
- iii)** Calculate the net present value of the project at an effective rate of interest of 7% per annum. (6)
- iv)** Calculate the discounted payback period of the project at an effective rate of interest of 7% per annum (4)
- [15]**

Q. 9) An investor has the following two options for a lump sum investment:

Option A Fixed rate of return of 5% per annum effective for 5 years.

After 5 years the accumulated value will be invested for a further 5 years. The expected annual return at that time will be 7.5% with standard deviation of annual return 3%. The annual effective rates of return will be independent and $(1+i_t)$ is log-normally distributed, where i_t is the rate of return in the year t .

Option B Fixed rate of return of 5% per annum effective for 7 years.

After 7 years the accumulated value will be invested for further 3 years at prevailing three-year spot rates. The three-year spot rates per annum effective at that time will be 4%, 5%, 6% with probabilities 30%, 60% and 10% respectively.

The investor wants to receive the accumulated investment at the end of ten years.

i) Calculate the expected value of the accumulated value and standard deviation of the investment at the end of 10 years for Options A and B of a unit sum of money invested now. (15)

ii) Determine the probability that the investment will be less than ` 165/- at end of 10 years for each ` 100/- invested at start for Option A and B. (5)

[20]
