Institute of Actuaries of India

Subject ST6 – Finance and Investment B

May 2009 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable

1 a With CBI being liability sensitive, it would prefer fixed-rate funding to reduce risk. . If CBI further issues debt at floating rate, it will further increase interest cost for CBI in case interest rate rises. Thus, CBI should issue debt at fixed rate to reduce interest rate risk.

b Borrowing Alternatives: Cash Market

	CBI	IOB	Difference
6-month floating	MIBOR plus 0.80%	MIBOR+0.55%	0.25%
fixed-rate	9%	8.45%	0.55%
Difference			0.30%

Consider the above borrowing costs. The difference in quality spreads for floating rate and fixed-rate debt indicates that a negotiated swap is feasible because IOB can effectively rent its higher quality to CBI and provide savings for both groups.

<u>CBI</u>

a. Cash market: issue floating rate debt at MIBOR + 0.80%

b. Swap Transaction: Pay fixed-rate to swap counterparty @ 8.40%, Receive floating rate from swap counterparty @ 6-month MIBOR + 0.30%

<u>IOB</u>

a. Cash market: issue fixed-rate debt at 8.45%

b. Swap Transaction: Pay floating rate to counterparty @ 6-month MIBOR+ 0.30%, Receive fixed-rate from counterparty @ 8.33%

Net cost:

CBI: MIBOR + 0.80% + 8.40% -MIBOR-0.30% = 8.90% (Save 0.10%)

IOB: 8.45% + MIBOR+0.30% – 8.33% = MIBOR + 0.42% (Save 0.13%)

The swap dealer (counterparty) earns the spread of 0.07%. Together, the savings amount to the 0.30% difference in the spreads between CBI's and IOB's fixed-rate and floating-rate borrowing costs.

c On the face of it, this looks like an arbitrage type situation where the valuation between two markets is not consistent. It is not purely against the arbitrage theory since this does not give rise to pure arbitrage. However, if a company is interested in borrowing then it may be able to do so cheaply using the stated mechanism. The floating rate market is not asking for the level of premium from CBI as is being asked by fixed rate market (or vice versa for IOB). It may be possible due to

i) liquidity premium due to one market being more liquid than the other

ii) demand supply gap in different markets due to investor preferences.

iii) there may be other clauses which are not mentioned – for example the credit spread under floating contract being linked to credit worthiness of the issuer

iv) the intermediary bank may be able to leverage its relationships to get a better deal for the party

v) Alternatively, such situation may arise in the market temporarily due to information flow problems and this does not last long

vi) Some government incentives like different tax savings on different interest components

[13]

2 a Change in market value of assets:

Bonds: -2.0[(-0.02)/(1.075)]1,20,000 = +4,465.12
Loans: -1.75[(-0.02)/(1.115)]4,50,000 = +14,125.56
Total: +18,590.68

Change in market value of liabilities:

Small TDs: -4.20[(-0.02)/(1.04)]1,50,000 = + 12115.38
Large CDs: -1[(-0.02)/(1.065)]90,000 = + 1690.14
Trans. Accts: -3.75[(-0.02)/(1.03)]3,00,000 = +21,844.66
Total: +35650.19

Change in Networth = 18,590.68 - 35,650.19 = -17,059.50

b Here the average duration of liabilities exceeds the average duration of assets. Objective: to reduce (immunize against) risk, the bank needs to move change in market value of equity closer to zero. This involves increasing the average duration of assets relative to the average duration of liabilities. One alternative might be to issue lower duration liabilities and use the proceeds to buy higher duration assets. This can also be achieved by usage of derivative transactions like swaps, forwards, FRAs, swaptions etc.

[5]

3 a Yes. The theoretical future price should be higher by the interest amount on the current stock price where as the listed future price is lower than the current stock price. [Assumption is that no dividend is due on the stock since it is not stated in the question]

b To carry this arbitrage one would have to sell the stock and if short sales are blocked then those who own the shares can buy futures and sell their holding. At maturity the futures would be close to the cash price and he can reverse the trades to lock the profit and reduce their holding costs.

The markets may remain in such a situation for long period for one or more of the following reasons:

i) It is possible that traders use the futures contract to hedge the expected downfall in the market by selling futures and hence as the market nears the maturity of the contract the traders then roll-over these contracts where in the next month contract will then be in the discount.

ii) It is possible that the market is expecting some corporate actions which are not allowed for in futures price– for example an announcement of dividends

iii) Institutional share holders would be holding large blocks and they may not find the depth in the futures/cash to carry out large amount of trade and they may not be interested in small trades. Small shareholders may not be active.

iv) There may be large bid/offer spread which is not reflected in the closing price data which may means that such arbitrage does not exist

v) There may be differences in tax treatments giving rise to higher taxation for existing share holders rendering the gains meaningless.

vi) The free floating shares may be low for this stock and hence this is not widely held by large number of share holders.

vii) Existing institutional share holders are barred from trading in derivatives and hence they are forced to buy the cash and hold equity.

c The trade is long March future@148 and short March call of 120 @32. The call option is in the money by 33 rupees. The future would coincide with the stock price at expiry say S. The total payoff at expiry is 32+S-148-Max(S-120, 0). If S is above 120, then the gain is 4. If the stock price goes below 116 the trader starts making a loss as if he is long at 116.

There exists a buyer for such a call option since such a call behaves like a future for the gains with low time value with a protection at the downside. Besides, the margin money is limited to the premium amount paid to buy the call where as for future contract the margin money would be high. Also, to be noted that this being an American option the seller would not be willing to sell below the intrinsic value.

d In a pure arbitrage situation he would have locked in the gain of 4 under all situations. The obvious risk is that the stock may crash and go below 116. The call options may not be protected for some type of corporate actions for example exercise price may not be reduced for dividend payouts. The call option being American may be exercised with future discount widening and you may not be able to enter into a new call trade. Yes, April call is available with exercise price of 140 and hence downside protection is limited to 122. However, there is more profit in it with having higher probability of occurrence of other risks mentioned above.

e For profit, the stock price should be higher than 116 i.e. P(S>116) = P(lnS > ln(116)) = P(Z > (ln 116 – (ln 153 + 0.2 – 0.5^2 /2) * (13/365)) /(0.5*(13/365)^0.5)) = P(Z > -2.9622) =99.8% where Z follows standard normal distribution.

f Because put-call parity holds for the Black-Scholes model, we must have

$$p_{BS} + S_0 e^{-qT} = c_{BS} + Ke^{-rT}$$

In the absence of arbitrage opportunities, put-call parity holds for the market prices, so that

$$p_{Mkt} + S_0 e^{-qT} = c_{Mkt} + Ke^{-rT}$$

Subtracting these two equations, we get

$$p_{BS} - p_{Mkt} = c_{BS} - c_{Mkt}$$

$$19 - 18 = 5 - p_{mkt}$$

$$p_{Mkt} = Rs.4$$

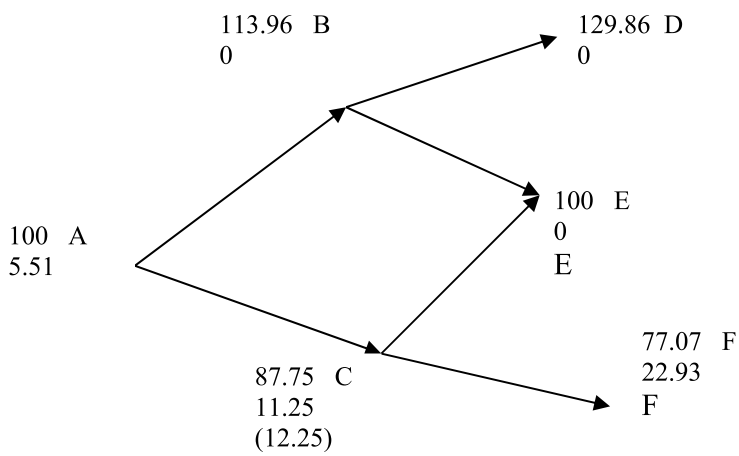
[14]

4 a In this case $\Delta t = 2/12$, $u = e^{0.32(2/12)^{\wedge}0.5} = 1.1396$, $d = 1/u = 0.8775$, and

$$p = \frac{e^{0.06x(2/12)} - 0.8775}{1.1396 - 0.8775} = 0.5057$$

b

At node B, the value of derivative is: $[0.5057x0 + (1-0.5057)x0]e^{-0.06x(2/12)} = 0$
At node C, the value of derivative is: $[0.5057x0 + (1-0.5057)x22.93]e^{-.06x(2/12)} = 11.25$
Thus the value of derivatives is (at node A): $[0.5808x0 + (1-0.5808)x11.25]e^{-0.06*(2/12)} = 5.51$



The actual market put premium (1.50) is less than theoretical put premium (5.51). Thus in the market put is relatively cheaper. Thus, there exists an arbitrage opportunity

The delta for stock price movements over the first step is:

$$\frac{0-11.25}{113.96-87.75} = -0.4294$$

The delta for stock price movements over the second step if there is an upward movement over the first step:

$$\frac{0-0}{129.86-100} = 0$$

The delta for stock price movements over the second step if there is a downward movement over the first step:

$$\frac{0-22.93}{100-77.07} = -1$$

Arbitrage Profit; Cash Flow

Position	T =0	T = 2 months (u)	T=2 months (d)	T = 4 months (uu)	T = 4monts (ud)	T = 4 months (du)	T = 4months (dd)
Buy Put at T=0	-1.50			0.00	0.00	0.00	22.99
Buy 0.4294 shares at T = 0	-42.94	48.93				42.94	33.07
Borrow from Risk-free market at T=0	48.45	-48.93				-49.43	-49.43
Buy additional shares (0.5706) at T=2 for downward movement			-50.07			57.06	43.94
Borrow money to purchase share in case of downward movement at T=2			50.07			-50.57	-50.57
Total	4.01	0.00	0.00	0.00	0.00	0.00	0.00

The arbitrageur will earn expected profit of Rs. 4 at T = 0 months.

c Early exercise at node C give 12.25 which is more than 11.25. Thus in case of downward movement over the first step, the option should be exercised if it is American.

[8]

5 a On the first day of the delivery month the bond has 20 years and 7 months to maturity. The value of the bond assuming it lasts for 20.5 years and all rates are 6% per annum with semiannual compounding is:

$$\sum_{t=1}^{41}\frac{6}{1.03^t}+\frac{100}{1.03^{41}}=170.2372$$

Therefore the conversion factor is 1.7024

b On the first day of the delivery month the bond has 26 years and 4 months to maturity. The value of the bond assuming it lasts for 26.25 years and all rates are 6% per annum with semiannual compounding is:

$$\frac{1}{\sqrt{1.03}}\left[4.5+\sum_{t=1}^{52}\frac{4.5}{1.03^t}+\frac{100}{1.03^{52}}\right]=141.6405$$

Subtracting accrued interest of 2.25 from 141.6405, this becomes 139.3905.
Therefore the conversion factor is 1.3939

c For the first bond, the quoted futures price time the conversion factor is:
120.50x1.7024 = 205.1358
This is 2.8642 less than the quoted bond price.
For the first bond, the quoted futures price time the conversion factor is:
120.50x1.3939 = 167.9656
This is 6.0344 less than the quoted bond price.
The first bond is therefore the cheapest to deliver.

d The clean price received for the bond is 205.1358. There are 178 days between February 1, 2009 and July 28, 2009. There are 181 days from February 1, 2009 to August 1, 2009. The accrued interest is therefore

$6x\frac{178}{181}$ = 5.9006 giving the dirty price to be 211.0364.

[7]

6 a The company should short

$$\frac{0.80x800{,}000{,}000}{2700x50} = 4741 \text{ contracts (approx)}$$

b The company should short

$$\frac{(0.80-0.50)x800{,}000{,}000}{2700x50} = 1778 \text{ contracts (approx)}$$

[4]

7 a Define:
y(T): Yield on a T-year corporate zero coupon bond
y*(T): Yield on a T-year risk free zero coupon bond
Q(T): probability that corporation will default between time zero and time T

The value of T-year risk-free zero-coupon bond with a principal of 100 is $100e^{-y^*(T)T}$ while the value of a similar corporate bond is $100e^{-y(T)T}$. The expected loss from default is therefore $100(e^{-y^*(T)T}-e^{-y(T)T})$.

Assume the recovery rate of R in the event of default. There is a probability Q(T) that the corporate bond will be worth 100R at maturity and a probability 1 – Q(T) that it will be worth

100. The value of bond therefore is:
{Q(T)x100R + [1 – Q(T)]x100} $e^{-y^*(T)T}$

Thus, $100e^{-y(T)T}$ = {Q(T)x100R + [1 – Q(T)]x100} $e^{-y^*(T)T}$

$$Q(T) = \frac{1-e^{-[y(T)-y^*(T)]T}}{1-R}$$

Year	Cumulative Default Probability (%)	Unconditional Default Probability in year (%)
1	0.4161	0.4161
2	1.6584	1.2422
3	3.4635	1.8051
4	5.2489	1.7854
5	8.1284	2.8795

b

Year	Conditional Default Probability in year (%)
1	0.4161
2	1.2474
3	1.8356
4	1.8495
5	3.0390

[7]

8 The seller receives
500,00,00,000 x 0.0080 x 0.25 = 1,00,00,000
at times 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75 and 3.00 years. The seller also receives final accrual payment of about 33,33,333 (=500,00,00,000 x .008 x 1/12) at the time of the default (3 years and 1 month). The seller pays
500,00,00,000 x 0.70 = Rs. 350,00,00,000

[3]

9 a When the value of the portfolio goes down by 4% in three months, the total return from the portfolio, including dividends, in the three months is:
-4 + 0.75 = -3.25%
i.e. -13% per annum (with quarterly compounding). This is 18% per annum (with quarterly compounding) less than the risk-free rate of interest. Since the portfolio has a beta of 1.25 we would expect the market to provide a rate of return of 14.4% per annum (with quarterly compounding) less than the risk-free rate i.e. we would expect the market to provide a rate of return of -9.4% per annum (with quarterly compounding). Since dividends on the market index is 2% per annum (with quarterly compounding), we would expect the market index to have dropped at the rate of 11.4% per annum (with quarterly compounding) or 2.85% per three months: i.e. we would expect the market index to have dropped to 2914.50. A total of 50,000

(600 Cr/150,000 (3000 x 50)*1.25) put contracts on NSE Nifty with exercise price of 2914.50 and expiration date in three months are therefore required.

S_0 = 3000, K = 2914, r (with continuous compounding) = 0.04969, σ = 0.25, T = 0.25 and q (with continuous compounding) = 0.01995. Hence

$$d_1 = \frac{\ln(3000/2914.5) + (0.05 - 0.02 + 0.0625/2)x0.25}{0.25\sqrt{0.25}} = 0.353292$$

$$d_2 = 0.35381 - 0.125 = 0.228292$$

The value of one put option is

$$2914.5e^{-0.05x0.25}\Phi(-0.35381) - 3000e^{-0.02x0.25}\Phi(-0.22881) = \text{Rs. } 98.8764$$

The total cost of insurance is therefore

50,000 x 50 x 98.8764 = Rs. 24.72 Cr or around 4.12% of the portfolio.

b When stock price is likely to make a large jump however the direction is not known. This may be based either on a systemic event or may be fund specific event. For example, outcome of general election, outcome of a budget announcements, release of data on economic events like GDP growth, etc which may make the market move in either direction depending on the surprise element. The fund specific events may be a mandate to shift in asset allocation, reduction of beta, redemption due in near future where the fund manager may want to protect the downside while carrying out the trades.

c The alternative strategy is derived from the put call parity equation. The fund manager may buy call options and invest the fund into debt assets. This would be equivalent to have an exposure to upside with limited downside (lose of the premium paid for call options).

Possible reasons are listed below :

- i) It is not easy to liquidate large portfolio and then build again without incurring substantial trading related costs (brokerage, stamp duties, bid-offer spread etc)
- ii) Liquidity or market depth in stocks may be an issue
- iii) It may lead to crystallization of tax or give rise to un-necessary tax due to short term and long term differentiation
- iv) The fund manager may also be relying on specific alpha factors which are neglected in the whole analysis

Another alternative strategy would be to use index futures to create a portfolio with the same delta as that of put and portfolio.

[17]

10 a. In the following equation:

$$IV_{iT,t} = 0.50 + 0.05\left(\frac{S_t}{K_{iT,t}}\right)^2 - 0.10\left(\frac{S_t}{K_{iT,t}}\right)$$

Implied volatility is minimum at S/K = 1. This indicates that deeply in the money and deeply out of the money options have higher implied volatility than at the money options. This relationship is termed as volatility smile. This kind of relationship between implied volatility and degree of moneyness generally hold in case of foreign currency options.

b. In case of $IV_{iT,t} = 0.50 + 0.01\left(\frac{S_t}{K_{iT,t}}\right)$, the volatility smile tends to be downward sloping. This means that out of the money puts and in the money calls tend to have high implied volatilities whereas out of the money calls and in the money puts tend to have low implies volatilities. This kind of relationship between implied volatility and degree of moneyness generally hold in case of equity (both stock and index) options.

[6]

11 The process followed by B, the bond price, is from Ito's lemma:

$$dB = \left[\frac{\partial B}{\partial y}\mu(\alpha - y) + \frac{\partial B}{\partial t} + \frac{1}{2}\frac{\partial^2 B}{\partial y^2}\sigma^2 y^2\right]dt + \frac{\partial B}{\partial y}\sigma y dZ$$

$$B = \int_0^\infty 2e^{-yt}dt = \frac{2}{y}$$

$$\frac{\partial B}{\partial t} = 0; \frac{\partial B}{\partial y} = -\frac{2}{y^2}; \frac{\partial^2 B}{\partial y^2} = \frac{4}{y^3}$$

$$dB = \left[-\mu(\alpha - y)\frac{2}{y^2} + \frac{2\sigma^2}{y}\right]dt - \frac{2\sigma}{y}dZ$$

The expected instantaneous rate at which capital gains are earned from the bond is:

$$\left[-\mu(\alpha - y)\frac{2}{y^2} + \frac{2\sigma^2}{y}\right]$$

The expected interest per unit time is Rs. 2. The total expected instantaneous return is:

$$\left[2-\mu(\alpha - y)\frac{2}{y^2} + \frac{2\sigma^2}{y}\right]$$

When expressed as a percentage of the bond price this is:

$$\left[\frac{2-\mu(\alpha - y)\frac{2}{y^2} + \frac{2\sigma^2}{y}}{\left(\frac{2}{y}\right)}\right] = y - \frac{\mu}{y}(\alpha - y) + \sigma^2$$

[8]

12 a. If $G(S,t) = S^n$, then from Ito's lemma, we have

$$dG = \left[\frac{\partial G}{\partial S}\mu S + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S^2}\sigma^2 S^2\right]dt + \frac{\partial G}{\partial S}\sigma S dZ$$

$$\frac{\partial G}{\partial t} = 0; \frac{\partial G}{\partial S} = nS^{n-1}; \frac{\partial^2 G}{\partial S^2} = n(n-1)S^{n-2}$$

$$dG = \left[n\mu S^n + \frac{1}{2}n(n-1)\sigma^2 S^n\right]dt + n\sigma S^n dZ$$

$$dG = \left[n\mu + \frac{1}{2}n(n-1)\sigma^2 \right] Gdt + n\sigma GdZ$$

This shows that $G = S^n$ follows geometric Brownian motion with the expected return of $n\mu + \frac{1}{2}n(n-1)\sigma^2$ and volatility of $n\sigma$.

b.

Define S_X, μ_X and σ_X as the stock price, expected return and volatility for stock X. Define S_Y, μ_Y and σ_Y as the stock price, expected return and volatility for stock Y. Define dS_X and dS_Y as the change in S_X and S_Y in short interval of time. Since each of the two stocks follows geometric Brownian motion,

$$dS_X = \mu_X S_X dt + \sigma_X S_X dZ$$

$$dS_Y = \mu_Y S_Y dt + \sigma_Y S_Y dZ$$

$$dS_X + dS_Y = (\mu_X S_X + \mu_Y S_Y)dt + (\sigma_X S_X + \sigma_Y S_Y)dZ$$

This can be written as:

$$dS_X + dS_Y = \mu(S_X + S_Y)dt + \sigma(S_X + S_Y)dZ$$

if $\mu_X = \mu_Y = \mu$ and $\sigma_X = \sigma_Y = \sigma$. Hence the value of the portfolio follows geometric Brownian motion if the expected return and volatility for the stocks are the same.

****

[8]