INSTITUTE OF ACTUARIES OF INDIA

CT8 – Financial Economics

MAY 2009 EXAMINATION

INDICATIVE SOLUTION

1.

(a)

Efficient markets hypothesis claims that market prices already incorporate the relevant information. The market price mechanism is such that the trading pattern of a small number of informed analysts can have a large impact on the market price. Lazy (or cost conscious) investors can then take a free ride, in the knowledge that the research of others is keeping the market efficient.

If we assume that there are no arbitrage opportunities in a market, then it follows that any two securities or combinations of securities that give exactly the same payments must have the same price. This is sometimes called the "Law of One Price".

Arbitrage-free markets can be inefficient as not having an arbitrage opportunity does not mean that all the information is reflected in the market price of securities.

(b)

Shiller found strong evidence that the observed level of volatility in the markets contradicted the EMH.

Numerous criticisms were subsequently made of Shiller's methodology, these criticisms covered

• the choice of terminal value for the stock price

• the use of a constant discount rate

• bias in estimates of the variances because of autocorrelation

• possible non-stationarity of the series, i.e. the series may have stochastic trends which invalidate the measurements obtained for the variance of the stock price

[5]

2.

(a)

(i) Expected return = $(0.6 \times 20\%) + (0.4 \times 5\%) = 14\%$

Standard deviation = $0.6 \times 25\% = 15\%$

(ii) With 60% of his money invested in my fund's portfolio, the client's expected return is 14% per year and standard deviation is 15% per year. If he shifts that money to the passive portfolio (which has an expected return of 15% and standard deviation of 20%), his overall expected return becomes:

$$E(r_C) = r_f + 0.7[E(r_M) - r_f] = 5 + [0.6 \times (15 - 5)] = 11\%$$

The standard deviation of the complete portfolio using the passive portfolio would be:

$$\sigma_C = 0.6 \times \sigma_M = 0.6 \times 20\% = 12\%$$

Therefore, the shift entails a decrease in mean from 14% to 11% and a decrease in standard deviation from 15% to 12%. Since both mean return *and* standard deviation decrease, it is not yet clear whether the move is beneficial. The disadvantage of the shift is that, if the client is willing to accept a mean return on his total portfolio of 11%, he can achieve it with a lower standard deviation using my fund rather than the passive portfolio.

To achieve a target mean of 11%, we first write the mean of the complete portfolio as a function of the proportion invested in my fund (y):

$$E(r_C) = 5 + y(20 - 5) = 5 + 15y$$

Our target is: $E(r_C) = 11\%$. Therefore, the proportion that must be invested in my fund is determined as follows:

$$11 = 5 + 15y \Rightarrow y = \frac{11-5}{15} = 0.40$$

The standard deviation of this portfolio would be:

$$\sigma_C = y \times 25\% = 0.40 \times 25\% = 10\%$$

Thus, by using my portfolio, the same 11% expected return can be achieved with a standard deviation of only 10% as opposed to the standard deviation of 12% using the passive portfolio.

The fee would reduce the reward-to-variability ratio, i.e., the slope of the CML (using my portfolio as a proxy for market portfolio). The client will be indifferent between my fund and the passive portfolio if the slope of the after-fee CML (using my portfolio as a proxy for market portfolio) and the CML (using passive portfolio) are equal. Let f denote the fee:

$$\text{Slope of CML (using my portfolio) with fee } = \frac{20-5-f}{25} = \frac{15-f}{25}$$

$$\text{Slope of CML (using passive portfolio, which requires no fee)} = \frac{15-5}{20} = 0.50$$

Setting these slopes equal we have:

$$\frac{15-f}{25} = 0.50 \Rightarrow 15 - f = 25 \times 0.20 = 12.5 \Rightarrow f = 15 - 12.5 = 2.5\% \text{ per year}$$

(b)

Advantage: normal distribution is easy to manipulate to calculate VaRs based on only two parameters.
Disadvantage: results may be misleading with skewed or fat tailed distribution.

(c) Since Stock A and Stock B are perfectly negatively correlated, a risk-free portfolio can be created and the rate of return for this portfolio, in equilibrium, will be the risk-free rate. To find the proportions of this portfolio [with the proportion w_A invested in Stock A and $w_B = (1 - w_A)$ invested in Stock B], set the standard deviation equal to zero.

With perfect negative correlation, the portfolio standard deviation is:

$$\sigma_P = \text{Absolute value } [w_A\sigma_A - w_B\sigma_B]$$

$$0 = 18w_A - [15 \times (1 - w_A)] \Rightarrow w_A = 0.4545$$

The expected rate of return for this risk-free portfolio is:

$$E(r) = (0.4545 \times 9) + (0.5454 \times 8) = 8.455\%$$

Therefore, the risk-free rate is 8.455%.

[11]

3.

(a) $E(R_{ACC}) - 5 + 5 = 2.20\% + 1.20\,[E(R_M) - 5] + 6$

$E(R_{ACC}) - 5 = 3.20\% + 1.20\,[E(R_M) - 5]$

$E(R_{Wipro}) - 5 + 5 = -2.80\% + 1.30\,[E(R_M) - 5] + 6.5$

$E(R_{Wipro}) - 5 = -1.30\% + 1.30\,[E(R_M) - 5]$

The intercepts of the two regressions are not consistent with the CAPM. Actual expected rate of return on ACC is 2.20% more than the CAPM return.
Thus, ACC is underpriced. Actual expected rate of return on Wipro is 1.30% less than the CAPM return. Thus, Wipro is overpriced

(b) $\dfrac{1.20^2 x 22^2}{31.55^2} = 70\%$ of return variability of ACC is explained by market movement.

$\dfrac{1.30^2 x 22^2}{31.98^2} = 80\%$ of return variability of Wipro is explained by market movement.

For Wipro stock market movement explain a greater fraction of return vriability

(C) The covariance between the returns of A and B is (since the residuals are assumed to be uncorrelated):

$$Cov(R_{ACC}, R_{Wipro}) = \beta_{ACC}\beta_{Wipro}\sigma_M^2 = 1.20 \times 1.30 \times 484 = 755.04\%^2$$

The correlation coefficient between the returns of A and B is:

$$\rho_{AB} = \frac{Cov(R_{ACC}, r_{Wipro})}{\sigma_{ACC}\sigma_{Wipro}} = \frac{755.04}{31.55 \times 31.98} = 0.748$$

(d)

For portfolio P we can compute:

(i) $\sigma_P = [(0.4^2 \times 31.55^2) + (0.6^2 \times 31.98^2) + (2 \times 0.4 \times 0.6 \times 755.04]^{1/2} = [889.8629]^{1/2} = 29.83\%$

(ii) $\beta_P = (0.4 \times 1.20) + (0.6 \times 1.30) = 1.26$

$\sigma^2(e_P) = \sigma_P^2 - \beta_P^2\sigma_M^2 = 889.8629 - (1.26^2 \times 484) = 121.4645\%^2$

(iii) $Cov(R_P, R_M) = = \beta_P\sigma_M^2 = 1.26 \times 484 = 609.84\%^2$

This same result can also be attained using the covariances of the individual stocks with the market:

$Cov(R_P, R_M) = Cov(0.4R_{ACC} + 0.6R_{Wipro}, R_M) = 0.4Cov(R_{ACC}, R_M) + 0.6Cov(R_{Wipro}, R_M)$

$= (0.4 \times 1.20 \text{ x } 484) + (0.6 \times 1.30 \text{ x } 484) = 609.84\%^2$

(e)

(i) Note that the variance of T-bills is zero, and the covariance of T-bills with any asset is zero.

Therefore, for portfolio Q:

$$\sigma_Q = \left[w_P^2\sigma_P^2 + w_M^2\sigma_M^2 + 2\times w_P \times w_M \times Cov(R_P, R_M)\right]^{1/2}$$
$$= \left[(0.4^2 \times 889.8629) + (0.5^2 \times 484) + (2\times 0.5\times 0.4\times 609.84)\right]^{1/2} = 507.3141^{1/2} = 22.52\%$$

[2]

(ii) $\beta_Q = w_P\beta_P + w_M\beta_M = (0.4\times 1.26) + (0.5\times 1) + 0 = 1.004$

$$\sigma^2(e_Q) = \sigma_Q^2 - \beta_Q^2\sigma_M^2 = 507.3141 - (1.004^2 \times 484) = 19.4343$$

[13]

4.

(a) The market portfolio is (2/7, 3/7, 2/7), so

$R_M = (2R_A + 3R_B + 2R_C) / 7$.

Thus, $Cov(R_i, R_M) = [2\,Cov(R_i, R_A) + 3Cov(R_i, R_B) + 2\,Cov(R_i, R_C)] / 7$.
So,
$Cov(R_A, R_M) = [.32 + .12 + .04] / 7 = .06857$
$Cov(R_B, R_M) = 0.22/7 = .03143$,
$Cov(R_C, R_M) = .09/7 = .01286$,

$\sigma_M^2 = [2\,Cov(R_M, R_A) + 3\,Cov(R_M, R_B) + 2\,Cov(R_M, R_C)] / 7 = .03674$.
We conclude that $\beta_A = 1.8664$, $\beta_B = 0.8555$ and $\beta_C = 0.35$.
Finally, solving
$E_i = r + \beta_i(E_m - r)$

We get $E_A = 0.4$, $E_B = 0.2$ and $E_C = 0.1$.

(b)

The market price of risk is $(E_m - r) / \sigma_M$ and risk free rate of return is 3.077%

Em= (22 / 77) x (.2 x 40% + .3 x 20% + .5 x 10%)
+ (32 / 77) x (.2 x 20% + .3 x 5% + .5 x 10%)
+ (22 / 77) x (.2 x 10% + .3 x 20% + .5 x 7%)

Em = 13.2143%

$\sigma_M^2 =$ .2 x (22 / 77 x 40% + 33 / 77 x 20% + 22 / 77 x 10% - 13.2143%)^2
+ .3 x (22 / 77 x 20% + 33 / 77 x 5% + 22 / 77 x 20% - 13.2143%)^2
+ .5 x (22 / 77 x 10% + 33 / 77 x 10% + 22 / 77 x 7% - 13.2143%)^2

$\sigma_M^2 = .002692 = (.0519)^2$

Therefore, market price of risk is
(13.2143% - 3.077%) / 5.19% = 1.9537 = 195%

(C)

As a portfolio becomes very well-diversified:

- The systematic risk of the portfolio tends towards a weighted average of the systematic risks of the constituent securities

- The non-systematic or specific risk tends to zero.

[11]

5.

(a) The continuous-time lognormal model may be inappropriate for modelling investment returns because:

- the volatility parameter σ may not be constant over time. Estimates of volatility from past data are critically dependent on the time period chosen for the data and how often the estimate is re-parameterised.
- the drift parameter μ may not be constant over time. In particular, interest rates will influence the drift.
- there is evidence in real markets of mean-reverting behaviour, which is inconsistent with the independent increments assumption.
- there is evidence in real markets of momentum effects, which is inconsistent with the independent increments assumption.
- the distribution of security returns log $\left(S_t / S_u\right)$ has a taller peak in reality than that implied by the normal distribution. This is because there are more days of little or no movement in financial markets.
- the distribution of security returns log $\left(S_t / S_u\right)$ has fatter tails in/ reality than that implied by the normal distribution. This is because there are big "jumps" in security prices.

(b)

The lognormal model of security prices is consistent with weak form market efficiency because log returns over non-overlapping time intervals are assumed to be independent in the model and knowing the past patterns of returns cannot help you predict future returns.

In contrast, the Wilkie model is not consistent with weak form market efficiency. This can be shown by using the model to project the equity risk premium – the excess expected total return on equities compared to index-linked government bonds.

[5]

6. In:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

the expected increase in the stock price and the variability of the stock price are constant when both are expressed as a proportion (or as a percentage of the stock price).

In:

$$dS_t = \mu dt + \sigma dZ_t$$

The expected increase in the stock price and the variability of the stock price are constant in absolute terms.

In:

$$dS_t = \mu S_t dt + \sigma dZ_t$$

The expected increase in the stock price is a constant proportion of the stock price while the variability is constant in absolute terms.

In:

$$dS_t = \mu dt + \sigma S_t dZ_t$$

The expected increase in the stock price is constant in absolute terms while the variability of the proportional stock price change is constant.

The model:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

is the most appropriate one since it is most realistic to assume that the expected percentage return and the variability of the percentage return in a short interval are constant.

[8]

7.

a) The price of a forward contract is given by
$K = S_0 e^{rt}$, where S_0 is the share price, r is the continuously compounded risk free rate and t is the time to expiry.
Therefore, the price of the forward contract is
Rs. $2000 \times e^{0.1}$
= Rs. 2,210.3

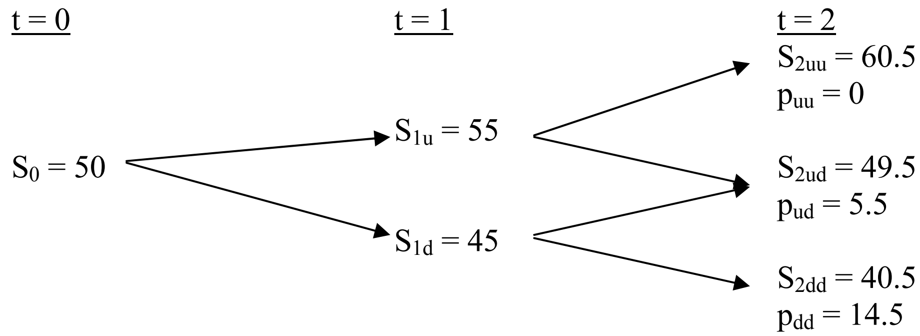
b) The guarantee is in effect a put option that is sold to the customer.
i) The price of a put option increases with increasing term. Therefore, the charge for the guarantee will be higher for longer term contracts.
ii) The price of a put option increases with decreasing interest rates. Therefore, the charge will increase with falling interest rates and decrease with increasing interest rates.

c) Consider a portfolio consisting of the European call option and cash equal to $Ke^{-r(T-t)}$, where K is the strike price, r is the continuously compounded risk free rate and (T-t) is the time to maturity at time t.
If at time T, the price of the stock S_T is greater than K, the option will be exercised using the cash and the value of the portfolio will be equal to S_T.
However, if $S_T < K$, the option will not be exercised and the value of the portfolio will be K.
Since the portfolio produces a payoff that is at least equal to the payoff from holding a share, it must have a value at time t of greater than the share price.
Therefore,
$c_t + Ke^{-r(T-t)} \geq S_t$

i.e. $c_t \geq S_t - Ke^{-r(T-t)}$

d)

i)



Risk neutral probability of up move = $q = \dfrac{e^{rt} - d}{u - d} = \dfrac{1.008 - 0.9}{1.1 - 0.9} = 0.542$

Therefore,

Price of the European put option = $e^{-rT}(q^2 p_{uu} + 2q(1-q)p_{uu} + (1-q)^2 p_{dd})$

$= e^{-0.1\times 2/12}(.542^2 \times 0 + 0.542 \times 0.458 \times 5.5 + 0.458^2 \times 14.5)$

= 5.68

(Please give due credit if calculations are done with different levels of accuracy in terms of number of decimal places)

ii)

For an American option, we have to evaluate if it would be beneficial to exercise the option after the first month.

IF the stock price goes up, the options payout on exercise will be 0, therefore it is not beneficial to exercise the option.

If the stock price goes down, the option payoff will be Rs. 10, and the value of holding on to the option is $e^{-rt}(q \times 5.5 + (1-q) \times 14.5) = 9.54$

Therefore, it will be beneficial to exercise the option and the American option is more valuable. The extra value at time 0 of this is $e^{-rt} \times (10 - 9.54) = 0.45$

Therefore, the price of the American option is 5.68+0.45 = 6.13

(Please give due credit if calculations are done with different levels of accuracy in terms of number of decimal places)

[11]

8.

i) For the put option,

$S_t = 1{,}000$
$K = 1{,}000$
$\sigma = 35\%$
$T - t = 1$
$r = 8\%$

Therefore,
$d_1 = \{\log(S_t/K) + (r + (\tfrac{1}{2}\sigma^2)) \times (T-t)\} / (\sigma \times \sqrt{(T-t)})$
$= \{\log(1{,}000/1{,}000) + (.08 + (\tfrac{1}{2}\times 0.35^2)) \times 1\} / (.35 \times \sqrt{1})$
$= 0.4036$

$d_2 = d_1 - (\sigma \times \sqrt{(T-t)})$
$= 0.4036 - (.35\times\sqrt{1})$
$= 0.0536$

Therefore,
$\Phi(-d_1) = 0.3433$
$\Phi(-d_2) = 0.4786$

$p_t = 1{,}000\times (\exp(-0.08)) \times 0.4786 - 1{,}000 \times .3433$
$= 98.57$

ii) We need to calculate the strike price for which the price of the call option will be 98.57.

Let us start with $K = 1{,}100$.

Therefore,
$d_1 = \{\log(S_t/K) + (r + (\tfrac{1}{2}\sigma^2)) \times (T-t)\} / (\sigma \times \sqrt{(T-t)})$
$= \{\log(1{,}000/1{,}100) + (.08 + (\tfrac{1}{2}\times 0.35^2)) \times 1\} / (.35 \times \sqrt{1})$
$= 0.1313$

$d_2 = d_1 - (\sigma \times \sqrt{(T-t)})$
$= 0.1313 - (.35\times\sqrt{1})$
$= -0.2187$

Therefore,
$\Phi(d_1) = 0.5522$
$\Phi(d_2) = 0.4134$

$c_t = 132.41$, which is greater than the target price and we need to increase the strike price.

For $K = 1{,}200$

Therefore,
$d_1 = -0.1173$
$d_2 = -0.4673$

Therefore,
$\Phi(d_1) = 0.4533$
$\Phi(d_2) = 0.3201$
$c_t = 98.68$

This is close to the price of the put option. Therefore, the fund manager should sell call options with a strike price of 1,200.

(Please give credit for getting close to Rs. 1,200 by using other strike prices in the trial and error and interpolating to get to the strike price)

iii) The implications of selling the call options:

- If the stock price at the end of 1 year is greater than 1,200, the option will be exercised.
- Therefore, the fund manager's maximum payout is restricted to Rs. 1,200 per share, i.e. the manager foregoes any returns beyond 20% to protect downside.
- The benefit of this is that the downside is protected without any upfront payment.

iv) For every underlying share the fund is long 1 put option with strike price 1,000 and short 1 call option with strike price 1,200.

Delta of underlying stock = 1
Delta of put option = $-\Phi(-d_1)$
Delta of call option = $\Phi(d_1)$

For the put option with strike price 1,000,
Delta = $-\Phi(-d_1)$ = -0.3433 (from (i) above)

For the call option with strike price 1,200,
Delta = $\Phi(d_1)$ = 0.4533 (from (ii) above)

Therefore, delta of the portfolio = 1-0.3433-0.4533
= 0.2034

[12]

9.

i) Let X_t and Y_t be Q-martingales and let X_t have non-zero volatility. Then there exists a unique process ϕ_t such that $dY_t = \phi_t dX_t$. Furthermore, ϕ_t is previsible.

ii) Consider a derivative with a random payoff X at time T. A self-financing portfolio V_t is a replicating portfolio for the derivative if $V_T = X$.

iii) If t > u, then

$$E_Q[D_t|F_u] = E_Q[e^{-rt}S_t|F_u] = e^{-rT}E_Q[S_t|F_u] = e^{-rt}S_u e^{r(t-u)}$$

The last part of the equation is derived from the fact that Q is risk-neutral. This is now equal to:

$e^{-ru}S_u = D_u \Rightarrow D_t$ is a Q-martingale.

[6]

10.
a).

(i) Forward Rate for the second year = 11 x 2 – 10 x 1 = 12%

Forward Rate for the third year = 12 x 3 – 11 x 2 = 14%

Forward Rate for the fourth year = 13 x 4 – 12 x 3 = 16%

(ii) Price of the bonds = $100e^{-0.10} + 1100e^{-0.11x2}$ = Rs. 973.25

Next year, the price of the bond will be:

(iii) $1100e^{-0.12}$ = Rs. 975.61

Therefore, there will be a capital gain equal to: 975.61 – 973.25 = Rs. 2.36

The holding period return is: $\frac{100+2.36}{973.25} = 0.1052 = 10.52\%$

b) The price of zero-coupon bond under a Vasicek model is given by:

$$B(t,T) = e^{a(\tau)-b(\tau)\times r(t)}$$

$$\tau = T - t$$

where, $b(\tau) = \frac{1-e^{-\alpha\tau}}{\alpha}$

$$a(\tau) = (b(\tau)-\tau)(\mu - \frac{\sigma^2}{2\alpha^2}) - \frac{\sigma^2}{4\alpha}b(\tau)^2$$

We know the price of a 1-year and a 2-year bond, the short rate and the value of α. We can use the price of the two bonds to set up 2 equations to solve for the unknown values - μ and σ.

$$b(1) = \frac{1-e^{-0.25}}{0.25} = 0.884797$$

$$b(2) = \frac{1-e^{-0.5}}{0.25} = 1.573877$$

$$\ln B(1) = [(b(1)-1)(\mu - 8\times\sigma^2) - \sigma^2 \times b(1)^2] - b(1)\times 0.085$$

$$or, -0.086381 = (.884797-1)(\mu - 8\sigma^2) - .782865\sigma^2 - .884797\times .085$$

$$or, -0.115203\mu + 0.138759\sigma^2 = -0.011173$$

$$\ln B(2) = [(b(2)-2)(\mu - 8\times\sigma^2) - \sigma^2 \times b(2)^2] - b(2)\times 0.085$$
$$or, -0.174062 = (1.573877-2)(\mu - 8\sigma^2) - 2.477090\sigma^2 - 1.573877\times .085$$
$$or, -0.426123\mu + 0.931894\sigma^2 = -0.040282$$

Therefore,
$$0.931894\times 0.115203\mu - 0.138759\times 0.426123\mu = 0.931894\times 0.011173 - 0.138759\times 0.04028$$
$$\therefore \mu = 10.0\%$$
Therefore,
$$\sigma^2 = (-.011173 + 0.011520)/0.138759$$
$$\sigma = 5.00\%$$

(Please give due credit if calculations are done with different levels of accuracy in terms of number of decimal places)

[14]

11.

a) Credit event:
- Failure to pay either capital or coupon
- Loss event
- Bankruptcy
- Rating downgrade by a rating agency

b) Outcome of a default may be that the contracted payment stream is:
- Rescheduled
- Cancelled by the payment of an amount which is less than the default-free value of the original contract
- Continued but at a reduced rate
- Totally wiped out.

c) In the event of a default, the fraction of the defaulted amount that can be recovered through bankruptcy proceedings or some other form of settlement is known as the recovery rate.

[4]
[Total 100 Marks]

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