INSTITUTE OF ACTUARIES OF INDIA

CT5 – General Insurance, Life and Health Contingencies

MAY 2009 EXAMINATION

INDICATIVE SOLUTION

Question 1:

i. When the reserve is negative, it means that the policyholder "owes" that amount to the company.

In other words the present value of future benefits plus expenses is less than the present value of future premiums.

ii. The regulators ask the insurance companies to set up the reserve to zero in such a case because if the policy lapses at such a time then the debt becomes unrecoverable and the company makes a loss.

iii. Since the economic scenario has changed significantly an increase in the valuation interest rate will cause the reserve to be negative if the other parameters are unchanged.

The premiums are calculated at best estimate assumptions using the following formula:

PV of Premiums = PV of Benefits + PV of Expenses

The reserves are calculated at slightly prudent assumptions using the following formula:

Reserves = PV of Benefits + PV of expenses – PV of Premiums

The above result will be zero if we use the same assumptions used to calculate the premium rates.

Now if the interest rates are increased such that they are now more than the interest rate used to calculate premiums, the first two factors will reduce by a bigger amount than the third factor in the reserve formula leading to negative reserves.

[8]

Question 2:

$$P_x \ddot{a}_{x:n} = A_{x:n}$$

$$P_x [\ddot{a}_{x:t} + V^t\ {}^tp_x\ \ddot{a}_{x+t:n-t}] = A^1_{x:t} + V^t\ {}^tp_x\ A_{x+t:n-t}$$

Rearranging

$$P_x \ddot{a}_{x:t} - A^1_{x:t} = V^t\ {}^tp_x\ [A_{x+t:n-t} - P_x \ddot{a}_{x+t:n-t}]$$

Dividing both sides by $V^t\ {}^tp_x$

$$(P_x \ddot{a}_{x:t} - A^1_{x:t})(1+i)^t / {}^tp_x = A_{x+t:n-t} - P_x \ddot{a}_{x+t:n-t}$$

$${}_tV^{retro} = {}_tV^{pro}$$

[4]

Question 3:

An independent rate of decrement assumes that there are no other decrement operating on the population. For example in a single premium whole life assurance contract, the policy condition may define that the contract may terminate on death only.

A dependent rate of decrement takes into account the other decrements operating on the population. For example, there might be multiple decrements viz. death, lapse, maturity in life assurance contracts.

Let us assume that the decrements α and μ occur uniformly distributed over the year of age from x to x+1.

$(aq)_x^{\alpha} = q_x^{\alpha} * (1 - 0.5*q_x^{\mu}) = 0.05 * (1 - 0.5* 0.02) = 0.0495$

$(aq)_x^{\mu} = q_x^{\mu} * (1 - 0.5*q_x^{\alpha}) = 0.02 * (1 - 0.5* 0.05) = 0.0195$

[5]

Question 4 :

i. The two main objectives a company must consider before declaring bonuses are:

- To distribute surplus no faster than it arises
- To defer the distribution of some surplus, since this will allow the company to use the non distributed surplus as a cushion, which allows a riskier investment strategy and consequent greater eventual expected investment returns.

Examples of valid points could be PRE, competition, smoothing policy etc.

ii.

The three methods of adding bonuses are:

- Simple Bonus
- Compound Bonus
- Super Compound Bonus
- The simple bonus does not satisfy the first objective. Most of the surplus which arises comes from investment return, and the company will get this compound; if it distributes its as "simple " interest there is a mismatch between incidence and outgo. It does not satisfy the second objective which is distributing earlier than the other two methods.
- The compound bonus satisfies the first objective but distributes the surplus as it arises rather than deferring it.
- The super compound bonus system defers the distribution of surplus, thereby creating a cushion of undistributed capital that will permit the company to pursue a higher risk higher return investment strategy. However, we have the disadvantage of increased complexity, both in administering the increases and in communicating them to policyholders.

[7]

Question 5:

- The risk discount rate (RDR) can be defined as;

RDR = Risk free rate + margin for risks in the product cash flows;

Where margin for risks reflects the degree of riskiness associated with the product.

Some of the factors explaining the reasons for the RDR being higher than risk free rate are given below:

- Shareholders supply the capital to make good the shortfall between the Income (premium) and the Outgo (expenses and the money for setting up of reserves).
- Thus writing a policy can be thought of as an investment by the shareholders of the company and shareholders would like to be compensated for the riskier investment.
- If the shareholders are providing capital, then they expect a return on that capital appropriate to the riskiness of their investment. It is much riskier investing by writing life insurance business than by buying government bonds, because more things could go wrong.
- So to quantify the extra risk, a margin is added to the investment returns on risk-free assets such as government bonds, and then price the product using the resultant risk discount rate.

(5)

Question 6:

i.

Unit Account:

Policy Year	Premium	Allocated Premium	Cost of Allocation	Fund b/f	Fund before FMC	FMC	Fund c/f
1	100000	80000	76000	0	82080.00	1026.00	81,054.00
2	100000	90000	85500	81054.00	179878.32	2248.48	177,629.84
3	100000	90000	85500	177629.84	284180.23	3552.25	**280,627.98**

(ii) Non Unit Fund:

Policy Year	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	FMC	Maturity	Profit
1	24000	2000	5000	1189.46	680.00	1026.00	0.00	17516.54
2	14500	200	2000	223.70	492.00	2248.48	0.00	14816.78
3	14500	200	2000	0.00	492.00	3552.25	29700.00	-13355.75

Policy Year	NU Reserve at end	Increase in Reserve	Investment Income on Reserve	Revised Profit Vector	Present value of Profits	PV of Premiums
1	0.00	0.00	0	17516.54	15231.77	100000.00
2	12842.06	12713.64	0	2103.13	1574.37	86086.96
3	0.00	-12842.06	513.68	0.00	0.00	74109.64
					16806.14	260196.60
				Profit Margin --->		**6.46%**

[15]

Question 7:

i. (a) $LHS = A_{x:\,t} + V^t\ {}_tp_x\ A_{x+t:\,n-t}$

$$= A^{\,1}_{x:t} + \frac{D_{x+t}}{D_x} + V^t\ {}_tp_x\ A_{x+t:\,n-t}$$

$$= A^{\,1}_{x:t} + \frac{D_{x+t}}{D_x} + V^t\ {}_tp_x\ A^{\,1}_{x+t:n-t} + \frac{D_{x+n}}{D_x}$$

$$= A^{\,1}_{x:n} + \frac{D_{x+t}}{D_x} + \frac{D_{x+n}}{D_x} = RHS$$

(b)

ii. A whole life benefit is equal to a term assurance for n years (which pays out on death in the first n years) plus a benefit covering the whole of the remainder of the policyholder’s life, provided (s)he survives for n years.

iii. The deferred whole life assurance is paid on death, but only of death happens after n years. Therefore the benefit is equal to a whole life assurance paid to a life n years older, but only if the life aged x survives for n years (and discounted to allow for interest)

(c)

The identity $a_{x:n} = \ddot{a}_{\text{x:n}} - 1$ works only for whole life annuities.

From first principle we get:

$$a_{x:n} = vp_x + v^2\ {}_2p_x + \ldots\ldots\ldots + v^n\ {}_np_x$$

$$\ddot{a}_{\text{x:n}} = 1 + vp_x + v^2\ {}_2p_x + \ldots\ldots\ldots + v^{n-1}\ {}_{n-1}p_x$$

Equating these two equations we get:

$$a_{x:n} - \ddot{a}_{\text{x:n}} = v^n\ {}_np_x - 1$$

So, $a_{x:n} = \ddot{a}_{\text{x:n}} - 1 + v^n\ {}_np_x$

[6]

ii.

(a) PV of profit = EPV of premiums – EPV of benefits – EPV of Increase in reserves + EPV of interest earned on reserves

Reserves do not reduce profit but only defer the release of profits.

Therefore the EPV of increase in reserves less the EPV of interest earned on reserves will be equal to zero if the interest earned on reserves and the interest rate used to discount the reserves are same.

Hence the above formula reduces to:

PV of profit = EPV of premiums – EPV of benefits

PV of profit = $18000 * \ddot{a}_{[55]:5} - 100000\ A_{[55]:5}$

$= 18000 * 4.590 - 100000*0.82348$

$= 272$

(b) The reasons for element of caution are:

- To allow a contingency margin, to ensure a high probability that the premiums plus investment income meet the cost of the benefits, allowing for random variations.
- To allow for uncertainty in the estimates itself

[14]

Question 8:

i. $${}_t\overline{V} = {}_{n-t}p_{x+t} \exp(-\delta(n-t))$$

$$\frac{\partial}{\partial t}\,{}_t\overline{V} = \frac{\partial}{\partial t}\left[{}_{n-t}p_{x+t} \exp(-\delta(n-t))\right]$$

$$= \exp(-\delta(n-t))\ \frac{\partial}{\partial t}\left({}_{n-t}p_{x+t}\right) + {}_{n-t}p_{x+t}\frac{\partial}{\partial t}\left(\exp(-\delta(n-t))\right)$$

$$\frac{1}{{}_{n-t}p_{x+t}}\frac{\partial}{\partial t}({}_{n-t}\,p_{x+t}) = \frac{\partial}{\partial t}\mathrm{Ln}({}_{n-t}\,p_{x+t}) = \frac{\partial}{\partial t}\mathrm{Ln}(\frac{l_{x+n}}{l_{x+t}})$$

$$= \frac{\partial}{\partial t}\{\mathrm{Ln}(l_{x+n}) - \mathrm{Ln}(l_{x+t})\} = \mu_{x+t}$$

Therefore ;

$$\frac{\partial}{\partial t}({}_{n-t}\,p_{x+t}) = {}_{n-t}\,p_{x+t}\,\mu_{x+t}$$

$$\frac{\partial}{\partial t}(\exp(-\delta(n-t))) = \delta\exp(-\delta(n-t))$$

$$\Rightarrow \frac{\partial}{\partial t}\,{}_t\overline{V} = \{\exp(-\delta(n-t))\;{}_{n-t}\,p_{x+t}\,\mu_{x+t}\} + \{{}_{n-t}\,p_{x+t}\,\delta\exp(-\delta(n-t))\}$$

$$= {}_{n-t}\,p_{x+t}\exp(-\delta(n-t))\,(\mu_{x+t}+\delta)$$

$$\Rightarrow \frac{\partial}{\partial t}\,{}_t\overline{V} = (\mu_{x+t}+\delta)\;{}_t\overline{V}$$

ii.

The present value random variable for a deferred annuity with a deferment period of n years payable annually in advance to a life aged x is:

$= 0$ If $K_x < n$

$= V^n\ \ddot{a}_{Kx+1-n}$ If $K_x >= n$

The present value random variable can also be written as:

$V^n\ \ddot{a}_{\max(Kx+1-n,0)}$

And its variance is:

$$\mathrm{Var}(V^n\ \ddot{a}_{\max(Kx+1-n,0)}) = V^{2n}\ \mathrm{Var}(\ddot{a}_{\max(Kx+1-n,0)})$$

$$= V^{2n}\ \mathrm{Var}\left(\frac{1-v^{\max\{Kx+1-n,0\}}}{d}\right)$$

$$= \frac{v^{2n}}{d^2}\mathrm{var}\,(v^{\max\{Kx+1-n,0\}})$$

$$= \frac{v^{2n}}{d^2}\mathrm{var}\left(\frac{v^{\max\{Kx+1,n\}}}{v^n}\right)$$

$$= \frac{1}{d^2}\mathrm{var}\,(v^{\max\{Kx+1,n\}})$$

$$= \frac{1}{d^2}[\,E\,(v^{2\max\{Kx+1,n\}}) - (E\,(v^{\max\{Kx+1,n\}}))^2\,]$$

Now :

$$v^{\max\{Kx+1,n\}} = \begin{cases} v^n & \text{If } K_x < n \\ v^{Kx+1} & \text{If } K_x >= n \end{cases}$$

In other words, 1 is paid at time n if the life dies before time n and is paid at the end of the year of death if the life dies after time n.

So:

$$E\,(v^{\max\{Kx+1,n\}}) = v^n \; {}_nq_x + v^n \; {}_np_x \; A_{x+n}$$

And :

$$E\,(v^{2\max\{Kx+1,n\}}) = v^{2n} \; {}_nq_x + v^{2n} \; {}_np_x \; {}^2A_{x+n}$$

Hence, the variance of the present value of the deferred annuity is:

$$Var(V^n \; \ddot{a}_{\max\,(Kx\,+\,1\,-\,n\,,0)})$$

$$= \frac{1}{d^2}\,[v^{2n} \; {}_nq_x + v^{2n} \; {}_np_x \; {}^2A_{x+n} - (v^n \; {}_nq_x + v^n \; {}_np_x \; A_{x+n})^2\,]$$

$$= \frac{v^{2n}}{d^2}\,[\,{}_nq_x + {}_np_x \; {}^2A_{x+n} - (\,{}_nq_x + {}_np_x \; A_{x+n})^2\,]$$

[14]

Question 9 :

- Climate and geographical locations are closely linked. On some locations, the climatic conditions might be better than other locations and hence the mortality and morbidity risk might be different from one location to other.
- Levels and patterns of rainfall and temperature lead to an environment that is amicable to certain kinds of diseases, *eg* those associated with tropical regions.
- The differences in mortality and morbidity risks could be significant between rural and urban areas in a geographical region. For example, the risk in urban areas could be lower due to
  - o The availability of readily accessible modern medical facilities nearby can reduce the delay in receiving effective medical treatment.
  - o Preventative screening can identify some conditions at an early stage.
  - o Immunisation programmes can control epidemics.
- The accident risk could be different in urban and rural areas due to;
  - o Risk in urban areas could be higher due to individuals living in cities are more likely to be involved in motor accidents,
  - o Although the traffic speed may be lower in urban areas than rural areas, so that urban people are less likely to be fatally injured.
- Mortality rates in geographical locations with high level of violence and social unrest will be higher. For example, there may be increased risk of death or injury to the people living in locations with higher level of terrorist activities.
- Natural disasters (such as tidal waves and famines) will also affect mortality and morbidity rates, and may be correlated to particular climates and geographical locations

[5]

Question 10 :

Pensionable Salary:
Pensionable Salary is the salary on which the pension will be based at the time of retirement. It is usually defined (in the scheme rules) in one of three ways:

- annual rate of salary at retirement, termed as final salary
- average annual salary in the last few years or few months (usually 3 to 5 years) before retirement, termed as final average salary
- average annual salary during scheme membership, termed as career average salary or lifetime earnings

Ill-health Retirement:

The retirement based on the ill health reasons is called Ill health retirement provided that the member has completed a minimum years of service.

To prevent selection against scheme rules, following cares are taken before granting the ill health retirements:

- the entitlement of the benefit is based on evidence on illness
- the pensionable salary is based on the salary at the time of ill health retirement

Accrued Benefit:

An accrued benefit is a benefit that has been earned as a result of pensionable service (or credited service) prior to the valuation date eg a pension of (n / 60th) of final average salary where n is the number of years of past pensionable service at the valuation date.

Future Service Benefit:
A future service benefit is a benefit that is expected to be earned as a result of pensionable service after the valuation date e.g. a pension of (60 - x)/ 60th of final average salary on age retirement at normal pension age of 60 for a member aged x at the valuation date.

[8]

Question 11 :

EPV = 10000 * [$a_{65(m):15}$ + ${}_{15}p_{65(m)}$ v^{15} $a_{60(f):80(m)}$ – $a_{60(f):65(m)}$]

Where $a_{65(m):15}$ = $a_{65(m)}$ – ($l_{80(m)}/l_{65(m)}$) v^{15} $a_{80(m)}$
= (13.666 – 1) – (6953.526 / 9647.797) * $(1.04)^{(-15)}$ (7.506 -1)
= 10.0623

${}_{15}p_{65(m)}$ v^{15} $a_{60(f):80(m)}$ = ($l_{80(m)}/l_{65(m)}$)*$(1.04)^{(-15)}$ * $a_{60(f):80(m)}$
= (6953.526 / 9647.797) * $(1.04)^{(-15)}$ * (7.335 -1)
= 2.53527

$a_{60(f):65(m)}$ = 12.682 – 1 = 11.682

EPV = 10,000 * [10.0623 + 2.53527 – 11.682] = 9155.70

[7]

Question 12

Age	Industry Data			Life Insurer A			Life Insurer B		
	Population (in thousand)	No of deaths	Death Rate(per thousand)	Population (in thousand)	No of deaths	Death Rate(per thousand)	Population (in thousand)	No of deaths	Death Rate (per thousand)
18 to 30	2000	800	0.400	200	100	0.500	100	36	0.360
31 to 40	4500	3600	0.800	400	400	1.000	200	88	0.440
41 to 50	1500	4500	3.000	150	750	5.000	50	150	3.000
51 & above	1000	6000	6.000	100	800	8.000	40	200	5.000
	9000	14900	1.656	850	2050	2.412	390	474	1.215
Crude Rate (per thousand) -->					**2.41**			**1.22**	
Standard Mortality Rate (per thousand) -->					**2.33**			**1.36**	
Standardised Mortality Ratio->(Actual/ expected deaths)					**1.414**			**0.803**	

[8]

[Total 100 Marks]

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