INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

25th May 2009

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q 1) A bank offers two types of Fixed Deposit Accounts :

Account A: The amount invested will earn 12% p.a. simple interest.
Account B: The amount invested will earn 12% effective p.a. compound interest

The interest along with the amount invested will be paid to the investor at the end of the chosen term of the fixed deposit.

Which Account will give a higher maturity amount if the term of the deposit is :

(i) 6 months (1)
(ii) 12 months (1)
(iii) 18 months (1)
[3]

Q 2)
(i) Define a real rate of interest. (1)
(ii) Define a money rate of interest. (1)
(iii) Comment on which will be higher and under what circumstances (1)
[3]

Q 3) The force of interest is given by

$$\delta(t) = \begin{cases} 0.09 + 0.0006\,t^2 & 0 \le t < 9 \\ 0.1836 - 0.005\,t & 9 \le t < 15 \\ 0.1086 & t \ge 15 \end{cases}$$

where t is measured in years.

(i) Find a general expression for the accumulation factor A(0,t) from time 0 to time t. (7)

(ii) Calculate the accumulated amount at the end of 17 years of Rs. 5000 invested at time 0. (2)

(iii) Hence calculate the equivalent effective annual rate of interest over the 17 years period. (1)

(iv) How much money needs to be invested at time 0 to get an accumulated value of Rs. 6000 at the end of 18 years? (2)
[12]

Q 4)
(i) Given i = 0.08, Calculate $d^{(12)}$, $i^{(365)}$, δ and $i^{(1/2)}$ (4)

(ii) From a corpus of Rs.10,000/- in a bank account, the bank makes quarterly payments in arrears for 5 years at the rate of Rs. 300/- p.a., the payment increasing by Rs.100/- p.a. The payments are deferred for one year, i.e., the first quarterly payment is at the end of 15 months from now.

Calculate the amount remaining in the corpus at the end of 6 years from now if the account earns interest at the rate of 8% p.a. convertible monthly for the first 3 years and 12% p.a. convertible half-yearly for the next 3 years. (8)
[12]

Q 5) A life insurance company holds a fund whose market value has been moving as under :-

Date	Market Value (in Crores)
01 April 2003	Rs.45,283
01 April 2004	Rs.48,492
01 April 2005	Rs.52,706
01 October 2005	Rs.59,251
01 April 2006	Rs.63,677

New Cashflows occurred during the period in the following pattern :

Date	Amount (in crores) Inflow	Amount (in crores) Outflow
31 March 2004	Rs.4,500	
31 March 2005	Rs.3,247	
30 September 2005		Rs.2,884
31 March 2006	Rs.1,321	

Calculate

(i) The money weighted rate of return (4)

(ii) The time weighted rate of return (3)

Express your answers as annual rates rounded to the nearest 0.01%.

(iii) Which is a better measure of fund performance – TWRR or MWRR? Why? (2)

[9]

Q 6) **(i)** Outline the differences between government bonds and unsecured loan stocks. (4)

Investor A purchased a bond with exactly 8 years to redemption at a gross redemption yield of 6.25% pa effective. The bond is redeemable at 110% of its face value. It pays coupon of 4.5% p.a. half-yearly in arrear. He pays tax at 30% on the coupons and is also subject to 10% surcharge on the tax.

(ii) Calculate the price paid by Investor A for the bond. (3)

(iii) After exactly 2 years, immediately after the payment of the coupon then due, Investor A sells this bond to Investor B who pays income tax at the rate of 25% (with no surcharge) and capital gains tax at the rate of 32%. The bond is purchased by Investor B to provide a net return of 6% p.a. effective. Calculate the price paid by Investor B. (5)

[12]

Q 7) **(i)** List 3 types of derivatives. (1)

(ii) Define arbitrage and explain why arbitrage may be considered impossible in many markets. (4)

(iii) The price of a given share is Rs. 120. The risk free rate of interest is 6% p.a. convertible quarterly. Assuming no arbitrage and that the share will not pay any income, calculate the forward price for the share, for settlement in exactly 3 months. (2)

[7]

Q 8) Co. X has been given a project of airport development in City A.

The company has to make investment in the project as follows:

- Rs. 1000 crores on 01.01.2008 (i.e., at the beginning of the project)
- Rs. 750 crores on 01.07.2008
- Rs. 750 crores on 30.11.2008
- Rs. 5 crores p.m. at the beginning of each month from 1.01.2009 for a period of 18 months.

Revenue from the project will be as follows :-

(i) An Airport Development Fee (ADF) is to be charged from passengers with effect from 01.03.2009 for a period of 3 years. ADF is Rs.200 per domestic passenger & Rs.1000 per international passenger. The passenger traffic for 3 years is expected as

Period	Total expected number of passengers	
	Domestic	International
01.03.2009 to 28.02.2010	1.5 crores	0.5 crores
01.03.2010 to 28.02.2011	5% more than previous year	20% more than previous year
01.03.2011 to 29.02.2012	same as preceding year	same as preceding year

(ii) Revenue from additional parking slots for aircrafts with effect from 01.09.2010 of Rs. 10 crores p.a. payable annually in advance increasing by Rs. 1 crore p.a. at the end of each year, for a period of 9 years.

(iii) Rent from new duty free shops with effect from 01.01.2011 of Rs. 1 crore p.m. payable every month in advance for 10 years increasing by 5% at the end of each year.

Show, using NPV criterion, that the project is not viable at a risk discount rate of 10% p.a. effective. Assume that passengers travel uniformly throughout the year, and there is continuous flow of ADF.

[17]

Q 9) **(i)** You are given the following term structure of spot interest rates:

Term (in years)	Spot Interest rate
1	6.50%
2	7.50%
3	8.00%
4	8.25%

A three-year immediate annuity will be issued a year from now with annual payments of 5000. Using the forward rates, calculate the present value of this annuity a year from now. (4)

(ii) State Redington's theory of immunization. (3)

(iii) A bond pays coupon half yearly in arrears of Rs.16 p.a. It is to be redeemed at par exactly in 8 years. The gross redemption yield from the bond is 7% p.a. effective.

a) Calculate the duration of the bond. (8)

b) Without calculating, explain how the duration of the bond would vary if the coupon rate were Rs. 20 p.a. instead of Rs. 16 p.a. (2)

[17]

Q 10) Let i_t be the effective rate of interest in year t. Find the expected value and standard deviation of the accumulated value at time 3 of investments Rs.500 at time 1, Rs.1000 at time 2 and Rs.1500 at time 3, given the following information :

$E(i_1) = 6\%$	Standard deviation of $i_1 = 0.75\%$
$E(i_2) = 7\%$	Standard deviation of $i_2 = 0.5\%$
$E(i_3) = 8.5\%$	Standard deviation of $i_3 = 1\%$

Assume that Interest rates in different years are independent.

[8]

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