Actuarial Society of India

EXAMINATIONS

17th May 2006

Subject CT4 (104) – Models (104 Part)

Time allowed: One and a Half Hours (12.00 am – 13.30 noon)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q1) You are doing a mortality study and have obtained the following data on the observed deaths at each age and you wish to test whether these observed deaths were consistent with the graduated rates. The graduated rates follow a(55) Ultimate Mortality Table for Male Annuitants

Age (x)	Exposed to Risk (Ex)	Observed Deaths (èx)
70	600	23
71	750	31
72	725	33
73	650	29
74	700	35
75	675	39

i) State the null hypothesis and use Chi-squared goodness of fit test to compare the observed mortality with the graduated rates. Comment on the results (8)
ii) What are the strengths and weaknesses of chi-square test? (3)
iii) State the tests you perform to overcome these deficiencies (2)

[13]

Q2) State and prove Gompertz Law **[3]**

Q3) (a) How does the central exposed to risk differ from initial exposed to risk? (2)
(b) A life insurance company has some mortality data relating to its temporary assurance policyholders, which are to be collated in select form. The following is an extract from the data:

Life	Date of Birth	Date of Entry	Date of Exit	Reason for Exit
A	14 August 1959	1 April 1990	6 June 1998	Death
B	2 May 1960	1 October 1990	30 November 1998	Withdrawal
C	19 July 1960	1 August 1998	-	-

i) Calculate the contributions made by each life to the central exposed to risk during the calendar year 1998. The contributions should be grouped by curtate duration and age last birthday, and the particular age and duration for each contribution should be clearly stated. The contributions should be given in days. You should assume that the day of exit does not count in the calculation of the exposed to risk, but that the day of entry does count. (5)
ii) Repeat part (i) when contributions are grouped by age last birthday at entry and by curtate duration. (3)

[10]

Q4) A life insurance company has carried out a mortality investigation. It followed a sample of independent policyholders aged between 40 and 45 years. Policyholders were followed from their 40^{th} birthday until either they died, or they withdrew from the investigation while still alive or they celebrated their 45^{th} birthday (whichever of

these events occurred first).

(a) Describe the types of censoring present in this investigation (2)

(b) An extract from the data for 20 policyholders is shown in the table below. Use these data to calculate the Kaplan-Meier estimate of the survival function. Determine an approximate 95% confidence interval for your estimate

Person Number	Last age at which person was observed (Years and Months)	Outcome
1	40 yrs 6 months	Died
2	40 yrs 6 months	Withdrew
3	41 yrs 0 months	Died
4	41 yrs 0 month	Died
5	41 yrs 6 months	Withdrew
6	42 yrs 3 months	Died
7	42 yrs 3 months	Withdrew
8	42 yrs 3 months	Died
9	42 yrs 6 months	Withdrew
10	43 yrs 0 months	Withdrew
11	43 yrs 3 months	Died
12	43 yrs 3 months	Withdrew
13	44 yrs 3 months	Withdrew
14	44 yrs 6 months	Withdrew
15	44 yrs 9 months	Died
16	45 yrs 0 months	Survived
17	45 yrs 0 months	Survived
18	45 yrs 0 months	Survived
19	45 yrs 0 months	Survived
20	45 yrs 0 months	Survived

(10)
[12]

Q5) The following model for the force of mortality for a life insurance company's annuitants has been proposed:

$\mu(t,i) = (0.015-0.0001t).\exp[\alpha(x_i - 70) + \beta.y_i + \gamma. z_i]$

Where $\mu(t,i)$ = force of mortality for the ith life, in calendar year 2000+t;

x_i = age of the ith life;

y_i = 1 if the ith life is a smoker, or $y_i = 0$ if a non-smoker;

z_i = 1 if the ith life is male , or $z_i = 0$ if female; and

α,β,γ = are the parameters of the model

The following data have been observed over the calendar year 2003:

Risk characteristics	Number of annuitants	Number dying
Male non-smoker, average age 65	800	6
Male smoker, average age 60	200	5
Female non-smoker, average 70	450	2
Female smoker , average age 65	150	1

You can assume the numbers of annuitants in each class remained constant throughout the investigation period, and that the average age for each class can be treated as representing the value of x_i for each individual in that class.

i) Explain why this model is a proportional hazards model (2)

ii) Explain the importance of subdividing the data by age, sex and smoking status and explain whether you think each of the parameters á, â and ã would be likely to be positive or negative. (3)

iii) Calculate the force of mortality for female non-smokers with average age 70 in 2007, according to this model. (1)

iv) (a) Obtain an expression for the partial likelihood based on the given data, expressing your answer in terms of á, â and ã only.

(b) State how you would estimate the parameters of the model using the partial likelihood (6)

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