Actuarial Society of India

EXAMINATIONS

17th May 2006

Subject CT4 (103) – Models (103 Part)

Time allowed: One and a Half Hours (10.30 am – 12.00 noon)

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor

Q1) The levels of discount in a car insurance no claims discount scheme are given by 0%, 10%, 20% and 40%. If the driver does not make a claim in any one year he moves up a level (e.g. from 10% to 20%); if he is already at 40% he stays there. If he makes a claim he moves down a level; if he is at 0% he stays there. A somewhat reckless driver has a probability of 0.5 of making a claim i any year. The full premium is Rs. 10,000 and the driver is currently at the 0% level. Calculate

(i) The average premium he pays per year in the long run (4)

(ii) The expected time it will take him to reach the 40% level for the first time. (3)

(iii) The expected value of the total premiums he will pay till he reaches 40% level for the first time. (3)

[10]

Q2) The ticket office at a metro station has a single ticket machine that is used by the passengers to buy the tickets. The machine has a tendency to break down and then has to be repaired. The time till the machine breaks down and the time required to repair the machine both follow the exponential distribution.

Let $P_{1i}(t), i = 0,1,$ be the probability that at time (t > 0) there are i ticket machines working at the metro station, given that the ticket machine is working at time t = 0.

(i) Derive the Kolmogrov forward equations for P_{1i}, i = 0, 1 in terms of (4)

- s, where $1/s$ is the mean time to breakdown for a machine
- r, where $1/r$ is the mean time repair a machine

(ii) Show that $P_{10}(t) = \frac{s}{s + r}\left(1 - e^{-(s+r)t}\right)$ deduces the value of $P_{11}(t)$ (6)

(iii) The station manager is considering adding an addition (identical) machine though there is only one repairman to work on both machines if both are out of action simultaneously. Assuming that the second machine is added and operates independently of the original one: (8)

(a) State the generator matrix of the Markov jump process X_t which counts the number of working ticket machines at time t

(b) Derive the Kolmogrov forward differential equations for $p_i(t), i = 0,1,2,$ the probability that i ticket machines are working

(c) Given that, for some

$$t,\ p_0(t) = \frac{2s^2}{2s^2 + 2rs + r^2},\ p_1(t) = \frac{2s^2}{2s^2 + 2rs + r^2},\ \text{and}\ p_2(t) = \frac{s^2}{2s^2 + 2rs + r^2},$$

show that

$$\frac{d}{dt} p_i(t) = 0 \text{ for i=0,1,2}$$

(d) State what conclusions you can draw from part (c)

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Q3) Let the rows and columns of a finite non-negative square matrix be such that each of it's row sums and each of it's column sums are equal to 1. Show by using equilibrium equations that if a Markov chain has a transition matrix as defined in the earlier sentence, then the distribution that gives equal probabilities to all states is an equilibrium distribution. **[7]**

Q4) Suppose that N (t) is a Poisson process with rate l . Show that for s < t and n ≥ k,

$$P(N(s)=k \mid N(t)=n)=\binom{n}{k}p^{k}(1-p)^{n-k}$$

where p is to be determined. **[8]**

Q5) Suppose that N(t) and M(t) are two independent Poisson processes with rates l and m respectively. Show that N(t) + M(t) is also a Poisson process with rate $(l+m)$. **[7]**

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