Actuarial Society of India

EXAMINATIONS

16th May 2006

Subject CT3 – Probability and Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 pm)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. **Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.**
2. **Mark allocations are shown in brackets.**
3. **Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.**
4. **Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.**
5. **In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.**

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q1) The following table gives the one-way commuting distance (in nearest kms) of 30 working women in an Insurance company.

Commuting distance (kms)				
13	47	10	3	16
7	25	8	21	19
12	45	1	8	4
6	2	14	13	7
34	13	41	28	50
14	26	10	24	36

(a) Make a stem-leaf display of the data. (1)

(b) Taking class intervals of the form 1-5, 6-10, 11-15 and so on construct a frequency distribution and using this draw Histogram. (2)

(c) Find the mean and variance of the commuting distances based on the frequency distribution (3)

[6]

Q2) A card is drawn at random from a pack of cards. Let A be the event that the card is red and B be the event that the card is king. Does the colour of card affect its probability of being a king? **[2]**

Q3) In an entrance examination in Mathematics and Statistics, of the 120 students appeared for the examination, 65 passed in Mathematics, 75 passed in Statistics and 35 passed in both the tests. A student is selected at random. What is the probability that the student has

(a) failed in both the tests (2)

(b) passed in Mathematics given that the student has passed in one test atleast. (3)

[5]

Q4) The joint cdf of (X,Y) is

$$F_{X,Y}(x,y) = 1 - e^{-2x} - e^{-3y} + e^{-(2x+3y)} \text{ ; } x,y > 0$$
$$= 0 \quad \text{elsewhere}$$

Find

(a) Joint pdf of (X,Y) (1)

(b) Marginal pdf of X and Y (2)

(c) $P[(X \le 1) \cap (Y \le 1)]$ (1)

(d) $P[(1 < X < 3) \cap (1 < Y < 2)]$ (2)

[6]

Q5) A random variable X has the following pdf

$$f(x) = \frac{1}{2a} e^{-\frac{|x-a|}{a}} \quad ; \quad a > 0, - ¥ < x < ¥$$
$$= 0 \quad elsewhere$$

Find the cumulant generating function and find mean and variance. **[5]**

Q6) In a certain Metropolitan city the daily consumption of electric power (in Million Kilowatt Hour (MKH)) may be regarded as a random variable having Gamma distribution with parameter (3,2). If the power plant has a daily capacity of 12 MKH, what is the probability that this power supply will be inadequate on any given day. **[3]**

Q7) During a promotional campaign of a new soft drink the company places prize winning caps on one of every ten bottles. Hoping to win a prize, a child decides to try a bottle of new drink each day for one full week. What is the probability that the child will win prize(s)

(a) atleast one day (1)

(b) none of the days? (1)

(c) all the days? (1)

[3]

Q8) On the average 8 calls per hour are received in a telephone board. Assuming that the number of calls received in the board in a given length of time is a Poisson process, find the probability that

(i) 6 calls received in 2 Hours (1)

(ii) atleast 2 calls in the next 20 minutes (2)

[3]

Q9)

(a) What is central limit theorem? (1)

(b) Over the years, it has been observed that of all the undergraduate students, in Mathematics who take a Society's examination, only 57% pass . Suppose that this year 950 graduate students in Mathematics are taking examination. What is the probability that

i) 565 or more pass (2)

ii) between 535 and 575 pass? (2)

[5]

Q10) Let X be a random variable following exponential distribution with density

$$f(x|m) = \begin{cases} me^{-mx} & if\, 0 < x < \infty, m > 0 \\ 0 & otherwise \end{cases}$$

For testing $H_0 : \mu = 20$ against $H_1 : \mu = 30$, a single value is observed from this distribution. If this value is less than 28, H_0 is accepted otherwise rejected. Find the probabilities of Type I error and Type II error **[2]**

Q11) Two thousand individuals were chosen at random by a researcher and cross classified according to gender and colour blindness as given below:

Description	Male	Female
Normal	904	998
Colour Blind	91	7

(a) Apply an appropriate test to conclude that there is overwhelming evidence against the hypothesis that there is no association between gender and colour blindness. (4)

(b) A genetic model states that the human population is split in the proportions as illustrated in the following table where *q* $(0<q<1)$ is a parameter related to the distribution of the colour blindness

Description	Male	Female
Normal	$(1-q)/2$	$(1-q^2)/2$
Colour Blind	$q/2$	$q^2/2$

Using the data in a):

i) Write down the likelihood function for the above model. (3)

ii) Determine the maximum likelihood estimate of q (8)

[15]

Q12) Eight pairs of slow learners with similar reading capabilities are identified in a third grade class. One member of each pair is randomly assigned to the standard teaching method, while the other is assigned to a new teaching method. The scores are as given below.

Pair	1	2	3	4	5	6	7	8
New Method	77	74	82	73	87	69	66	80
Old Method	72	68	76	68	84	68	64	76

(a) Test for the difference between mean scores for the two methods (6)

(b) Test for the equality of variances for these two methods (4)

(c) Obtain the 95% of confidence interval for the difference in means. (2)

[12]

Q13) A survey was conducted to investigate whether people tend to marry partners of about the same age. This question was addressed to 12 married couples and their ages were given in the following table.

Couple No.	1	2	3	4	5	6	7	8	9	10	11	12
Husband's age (x)	30	29	36	72	37	36	51	48	37	50	51	36
Wife's age (y)	27	20	34	67	35	37	50	46	36	42	46	35

(a) Draw the scatter plot and comment on it (1)

(b) Find the correlation coefficient and interpret it (4)

(c) If ρ represents the population correlation coefficient between the ages of partners, test the significance of ρ at 5% level (3)

[8]

Q14) A study was made on the effect of temperature on the yield of a chemical process. The following data (in coded form) were collected.

x :	-5	-4	-3	-2	-1	0	1	2	3	4	5
y :	1	5	4	7	10	8	9	13	14	13	8

Linear regression model $y = \beta_0 + \beta_1 x + e$ is fitted for the above data.

(a) Describe the parameters and variables in the model (1)

(b) Obtain the least squares estimates of β_0 and β_1 and hence the prediction equation. (3)

(c) Construct ANOVA table and test $H_0 : \beta_1 = 0$ at 5% level of significance (3)

(d) What are 95% confidence limits for β_1? (2)

(e) What are the confidence limits for the true mean value of y when x = 3 at 95% confidence level? (2)

(f) Are there any indications that a better model can be tried? (2)

[13]

Q15) The joint probability distribution of the amounts X and Y of two commodities supplied to a market has the probability density function (*pdf*)

$$f(x, y) = kxy(4x + 9y)e^{-(x+y)}; x, y > 0, k \text{ is a constant}$$

(a) Determine the constant *k* (3)

(b) Find the conditional *pdf* of X given Y=y and that of Y given X=x. (4)

(c) Find the conditional variance of X given Y=y. (3)

[10]

Q16) Let $X_1, X_2, \ldots$ be *i.i.d.* random variables from a Poisson distribution with mean 5 and let $S = \sum_{i=1}^{N} X_i$, where *N* follows Binomial with *n = 10, p = 0.2.* Obtain the mean and variance of S. **[2]**

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