Actuarial Society of India

Examinations

May 2006

CT5 – General Insurance, Health and Life Contingencies

Indicative Solutions

Que 1)

Since decrements a and b are uniformly distributed,

$${}_tp_x^a = 1 - tq_x^a$$

and

$${}_tp_x^b = 1 - tq_x^b$$

Also, ${}_tp_x^a\ \mu_{x+t}^a$ = constant = q_x^a for $0 \le t \le 1$

The dependent rate of decrement due to decrement a can be expressed as:

$$(aq)_x^a = \int_0^1 {}_t(ap)_x\,(a\mu)_{x+t}^a\,dt = \int_0^1 {}_tp_x^a\ {}_tp_x^b\ {}_tp_x^c\ \mu_{x+t}^a\,dt$$

so, $$(aq)_x^a = q_x^a \int_0^1 (1 - tq_x^b)\ {}_tp_x^c\,dt$$

decrement c takes place at time ¼ only, so

${}_tp_x^c = 1\ (0 < t < \tfrac{1}{4})$ and ${}_tp_x^c = p_x^c\ (\tfrac{1}{4} \le t < 1)$

Substituting this into the integral above:

$$(aq)_x^a = q_x^a \int_0^{1/4} (1 - tq_x^b)\,dt + q_x^a \int_{1/4}^1 (1 - tq_x^b)\,p_x^c\,dt$$

$$= q_x^a \left[t - \tfrac{1}{2}t^2 q_x^b\right]_0^{1/4} + q_x^a (1 - q_x^c)\left[t - \tfrac{1}{2}t^2 q_x^b\right]_{1/4}^1$$

$$= q_x^a \left[\tfrac{1}{4} - (1/32)\,q_x^b\right] + q_x^a (1 - q_x^c)\left(1 - \tfrac{1}{2} q_x^b - \tfrac{1}{4} + (1/32)\,q_x^b\right)$$

$$= q_x^a \left(1 - \tfrac{1}{2} q_x^b - \tfrac{3}{4} q_x^c + (15/32)\,q_x^b q_x^c\right)$$

[Total 6]

Que 2)

(i) General reasoning approach:

We must assume here that t< 20 since the term of the annuity is 20 years. Consider a small time period (t, t+h). The reserve, ${}_tV$, at the start of this time period will accumulate with interest to ${}_tVe^{\delta h}$ by time t+h.

This must provide both the annuity payment and a reserve for those that survive to time t+h. The combined annuity payment and the reserve is given by:

${}_{t+h}V + 5000h + o(h)$

where o(h) is any function such that $\dfrac{o(h)}{h} \to 0$ as $h \to 0$

The probability of surviving is given by:

$_{h}p_{x+t} = 1 - {}_{h}q_{x+t} = 1 - \text{ì}_{x+t}\, h + o(h)$

So:

$_{t}V e^{\text{äh}} = (1 - \text{ì}_{x+t}\, h)\,({}_{t+h}V + 5000h) + o(h)$

Expanding and rearranging:

$_{t}V\,(1+\text{ä}h + o(h)) = {}_{t+h}V + 5000\,h - \text{ì}_{x+t}\, h\; {}_{t+h}V + o(h)$

$\Rightarrow {}_{t+h}V - {}_{t}V = {}_{t}V\,\text{ä}h - 5000\,h + \text{ì}_{x+t}\, h\; {}_{t+h}V + o(h)$

$$\Rightarrow \frac{t+h\text{V} - t\text{V}}{h} = {}_{t}V\,\text{-}5000 + \text{ì}_{x+t}\; {}_{t+h}V + \frac{o(h)}{h}$$

Taking the limit as $h \to 0$, we get:

$$\frac{{}_{t}V}{t} = {}_{t}V\text{ä} - 5000 + \text{ì}_{x+t}\; {}_{t}V = (\text{ä} + \text{ì}_{x+t})\; {}_{t}V - 5000 \quad (t<20)$$

(3)

(ii) Algebraic derivation:

We know that the reserve at any time is the expected present value of the future benefits (as there is only a single premium at the start for this type of policy). Here we have:

$$_{t}V = 5000\; \bar{a}_{x+t\,:\,\overline{20-t|}}$$

Expressing the annuity as an integral we get:

$$_{t}V = 5000 \int_{0}^{20-t} e^{-\text{d}s}\; {}_{s}p_{x+t}\; ds$$

We then need to differentiate this with respect to t:

$$\frac{\partial\, {}_{t}V}{\partial t} = \frac{\partial}{\partial t}\left(5000 \int_{0}^{20-t} e^{-ds}\; {}_{s}p_{x+t}\, ds\right)$$

Using the formula on page 3 of the Tables, the derivative is:

$$\frac{\partial\, {}_{t}V}{\partial t} = -5000 e^{-d(20-t)}\; {}_{20-t}p_{x+t} + 5000 \int_{0}^{20-t} e^{-ds}\, \frac{\partial\, {}_{s}p_{x+t}}{\partial t}\, ds$$

We now need to differentiate $_{s}p_{x+t}$ with respect to t. Since $_{s}p_{x+t} = \dfrac{\lambda_{x+t+s}}{\lambda_{x+t}}$, it follows that:

$$\ln({}_{s}p_{x+t}) = \ln(\lambda_{x+t+s}) - \ln(\lambda_{x+t})$$

Differentiating with respect to t gives:

$$\frac{\partial}{\partial t}\left(\ln {}_s p_{x+t}\right) = \frac{1}{{}_s p_{x+t}} \frac{\partial\, {}_s p_{x+t}}{\partial t}$$

$$= \frac{1}{l_{x+t+s}} \frac{\partial \lambda_{x+t+s}}{\partial t} - \frac{1}{\lambda_{x+t}} \frac{\partial \lambda_{x+t}}{\partial t}$$

$$= -m_{x+t+s} + m_{x+t}$$

Hence:

$$\frac{\partial\, {}_s p_{x+t}}{\partial t} = {}_s p_{x+t} m_{x+t} - {}_s p_{x+t} m_{x+t+s}$$

Substituting this into the integral, we get:

$$\frac{\partial_t V}{\partial t} = -5000 e^{-d(20-t)} {}_{20-t} p_{x+t} + 5000 \int e^{-ds} \left({}_s p_{x+t} m_{x+t} - {}_s p_{x+t} m_{x+t+s}\right) ds$$

$$= -5000\ e^{-d(20-t)} {}_{20-t} p_{x+t} + 5000 \left(\int_0^{20-t} e^{-ds} {}_s p_{x+t} m_{x+t} ds - \int_0^{20-t} e^{-ds} {}_s p_{x+t} m_{x+t+s} ds \right)$$

$$= -5000\ e^{-d(20-t)} {}_{20-t} p_{x+t} + 5000 \left(m_{x+t} \int_0^{20-t} e^{-ds} {}_s p_{x+t} ds - \int_0^{20-t} e^{-ds} {}_s p_{x+t} m_{x+t+s} ds \right)$$

The first integral is an annuity and the second is a term assurance, and we can write:

$$\frac{\partial_t V}{\partial t} = 5000 \left(-e^{-d(20-t)} {}_{20-t} p_{x+t} + m_{x+t} \bar{a}_{x+t:\overline{20-t}|} - \bar{A}^{1}_{x+t:\overline{20-t}|} \right)$$

Note that the first and last term in the brackets can be combined to give an endowment assurance. So:

$$\frac{\partial_t V}{\partial t} = 5000 \left(m_{x+t} \bar{a}_{x+t:\overline{20-t}|} - \bar{A}_{x+t:\overline{20-t}|} \right)$$

Using the premium conversion formula, $\bar{A}_{x+t:\overline{20-t}|} = 1 - d\bar{a}_{x+t:\overline{20-t}|}$, we have:

$$\frac{\partial_t V}{\partial t} = 5000 m_{x+t} \bar{a}_{x+t:\overline{20-t}|} - 5000\left(1 - d\bar{a}_{x+t:\overline{20-t}|}\right)$$

$$= 5000\left(m_{x+t} + d\right)\bar{a}_{x+t:\overline{20-t}|} - 5000$$

Finally, noting that ${}_tV = 5000\bar{a}_{x+t:\overline{20-t|}}$, we obtain the result:

$$\frac{\partial\, {}_tV}{\partial t} = 5000(m_{x+t} + d)\,{}_tV - 5000 \qquad \text{for } t<20$$

(7)

[Total 10]

Que 3)

(i) ${}_{15}p_{45} = l_{60}/l_{45} = 40/55 = 8/11$ (1)

(ii) We can write:

$$\mu_x = -\frac{\partial}{\partial x}\ln l_x$$

$$= -\frac{\partial}{\partial x}\ln(100-x)$$

$$= \frac{1}{100-x}$$

So: $\mu_{40} = 1/60$ (2)

(iii) $P(T_{42}<25) = {}_{25}q_{42} = 1 - (l_{67}/l_{42}) = 1 - (33/58) = 25/58$ (2)

(iv) $P(K_{53} = 10) = {}_{10}p_{53} * q_{63} = \dfrac{l_{63} - l_{64}}{l_{53}} = 1/47$ (2)

(v) $\mathring{e}_{33} = \int_0^{67} {}_tp_{33}\, dt = \int_0^{67} (l_{33+t})/\, l_{33} = (1/67)\int_0^{67} (67-t)\, dt$

$= (1/67)\left(67t - ½t^2\right)_0^{67} = 33.5$ (2)

[Total 9]

Que 4)

Ignore the positive cashflow in year 5.

To zeroise the -15 at the end of year 4, we need a provision in place at the start of the year 4 i.e. at the end of year 3, equal to

${}_3V = (15/\ 1.06) = 14.15$

Since no provision is required at the end of year 4 we do not need to consider people surviving year 4, Provision at the end of year 2, is found by equating

${}_2V \times 1.06 + 5 = {}_3V \times p_{52}$

which gives

$_2V = (14.15 \times 0.996848 - 5) / (1.06) = 8.59$

Provision at end of year 1, is found by equating

$_1V \times 1.06 - 20 = {}_2V \times p_{51}$

which gives

$_1V = (8.59 \times 0.997191 + 20) / (1.06) = 26.95$

Cashflow at the end of year 1 is now:

$-10 - {}_1V \times p_{50} = -10 - 26.95 \times 0.997492 = -36.882409$

the revised profit vector is

(-36.88, 0, 0, 0, 40)

[Total 6]

Que 5)

$\ddot{a}^{(4)}_{45} = \ddot{a}_{45} - 3/8 = 18.823 - 0.375 = 18.448$

$a^{(12)}_{55} = a_{55} + 11/24 = 14.873 + 0.458 = 15.331$

$$\ddot{a}^{(2)}_{50:\overline{20}|} = (\ddot{a}_{50} - ¼) - v^{20}\, {}_{20}p_{50}\, (\ddot{a}_{70} - ¼)$$
$$= 17.194 - 0.45639 * (8054.0544 / 9712.0728) * 10.125$$
$$= 17.194 - 3.832$$
$$= 13.362$$

[Total 3]

Que 6)

From first principles, we can write the expected present value of the term assurance as :

$2{,}50{,}000\ (v\, q^*_{35} + v^2\, p^*_{35}\, q^*_{36})$

where * denotes impaired mortality. Now

$p^*_{35} = \exp\left(-\int_0^1 \mu^*_{35+t}\, dt\right) = \exp\left(-\int_0^1 5\,\mu_{35+t}\, dt\right) = (p_{35})^5$

So the expected present value of the benefit payable to the impaired life is:

$250000\ (v[1-(p_{35})^5] + v^2\, (p_{35})^5[1-(p_{36})^5]$

$=250000\ (\ [1/1.04] \times [1- 0.999311^5] + (1/1.04)^2 \times (0.999311)^5 \times (1- 0.999276^5))$

$=1659.63$

[Total 6]

Que 7)

a) The difference between direct standardization and indirect standardization is that the direct method uses the age-specific mortality rates of the groups being compared whereas the indirect method adjusts the crude death of each group.

Direct standardization, therefore, uses the structure of the standard population, whereas, indirect standardization uses the structure of the local population.

Two disadvantages of direct standardization are:

- it requires the age-specific rates for each group to be known; and
- the figures are heavily influenced by the mortality rates at older ages.

(3)

b) The directly standardized rate (DSR) is given by

$$DSR = \left(\sum^{x} E_{x,t}^{\ cs}\, m_{x,t} \Big/ \sum^{x} E_{x,t}^{\ cs}\right)$$

The standardized mortality rate (SMR) is given by

$$SMR = \left(\sum^{x} E_{x,t}^{\ c}\, m_{x,t} \Big/ \sum^{x} E_{x,t}^{\ c}\, m_{x,t}^{\ s}\right)$$

For 10-year policies

DSR = (6991 x 0.86 + 6462 x 1.74 + 5815 x 11.55 + 3454 x 71.53)/
(6991 + 6462 + 5815 + 3454)

= (331484.01 / 22722)
= 14.58868

SMR = (6013 x 0.86 + 5438 x 1.74 + 4942 x 11.55 + 2754 x 71.53) /
(6013 x 1.08 + 5438 x 2.05 + 4942 x 13.26 + 2754 x 75.70)

= (268707.02 / 291650.66) = 0.921332

For 20-year policies

DSR = (6991 x 2.12 + 6462 x 3.68 + 5815 x 22.94 + 3454 x 97.7)/
(6991 + 6462 + 5815 + 3454)

= (509452.98 / 22722)

= 22.42113

SMR = (978 x 2.12 + 1024 x 3.68 + 756 x 22.94 + 481 x 97.70) /
(978 x 1.08 + 1024 x 2.05 + 756 x 13.26 + 481 x 75.70)

= (70178.02 / 49591.70) = 1.415116

(3)

[Total 6]

Que 8)

The independent rates of withdrawal are:

Age attained	Rate
20	0.05
21	0.0525
22	0.055125

probability of survival to age 22 =

$0.95 \times 0.9475 \times l_{22} / l_{20} = 0.95 \times 0.9475 \times (9970.6346 / 9982.2006)$

$= 0.899082$

$(aq)_{22}^{w} = q_{22}^{w} (1 - 0.5\, q_{22}^{d}) = 0.055125 \times (1 - 0.5 \times 0.000572)$
$= 0.0551092$

Required probability = $0.899082 \times 0.0551092$
$= 0.0495477$

[Total 5]

Que 9)

Dependent rates

	c1	c2	c3=c1*(1-0.5*c2)	c4=c2*(1-0.5*c1)	c5=1-c3-c4	
	qx	wx	(aq)x death	(aq)x surr	(ap)x	t-1 (ap)
45	0.001201	0.1	0.001141	0.09994	0.898919	1.000000
46	0.001557	0.05	0.001518	0.04996	0.948521	0.898919
47	0.001802	0.05	0.001757	0.04995	0.948288	0.85264351
48	0.002008	0.05	0.001958	0.04995	0.948092	0.80855169

Unit-Fund

	c1	c2=Annual Premium*Allocation rate	c3=c2*5%	c4=(c1+c2-c3)*6%	c5=(c1+c2-c3+c4)*0.5%	c6=c1+c2-c3+c4-c5
Year	Units at start	Allocation	Bid-Offer spread	Interest	AMC	value of units at end
1	0.00	2500.00	125.00	142.50	12.59	2504.91
2	2504.91	5125.00	256.25	442.42	39.08	7777.00
3	7777.00	5125.00	256.25	758.75	67.02	13337.47
4	13337.47	5125.00	256.25	1092.37	96.49	19202.10

Non-Unit Fund

Year	Year	Age	c1 Unallocated premium	c2 Bid-Offer spread	c3 expenses	c4=(c1+c2-c3)*4% interest	c5 AMC
1	1	45	2500.00	125.00	1650	39.000	12.59
2	2	46	-125.00	256.25	375	-9.7500	39.08
3	3	47	-125.00	256.25	375	-9.7500	67.02
4	4	48	-125.00	256.25	375	-9.7500	96.49

Year	c6=max(unit value, 50000)*qx+t Extra Death Benefit	c7=c1+c2-c3+c4+c5-c6 End of year cashflow	c8 Probablility in force	c9 Discount factor	c10=c7*c8*c9 Profit Signature	Premium signature
1	54.1895	972.398	1.000000	0.925926	900.3685	5000.000
2	64.0977	-278.517	0.898919	0.857339	-214.6472	4161.663
3	64.4142	-250.892	0.852644	0.793832	-169.8176	3655.022
4	60.2961	-217.303	0.808552	0.735030	-129.1453	3209.272

NPV of profit	386.758404	**PV of premium**	16025.956
		Profit Margin	2.41%

[Total 13]

Que 10)

Most of the surplus which arises comes from investment return, and the company will get this compound. Under the simple bonus, if it distributes the surplus as "simple" interest then there is a mismatch between incidence and outgo. Further, under this method it is likely that the surplus is distributed earlier than with the other two systems.

The compound bonus satisfies the objective of not distributing the surplus before it arises. However, under this method the surplus is distributed as it arises rather than deferring it.

The super-compound bonus system defers the distribution of surplus, thereby creating a cushion of undistributed capital that will permit the company to pursue a higher-risk higher-return investment strategy. However, it has the disadvantage of increased complexity, both in administering the increases and in communicating them to policyholders.

[Total 3]

Que 11)

(i) Premium Equation:

$P\,\ddot{a}_{[40]:20} \;=\; 50000\,\mathbf{A}_{[40]:20} + 50000\,(\mathbf{D}_{60}/\,\mathbf{D}_{[40]}) + 650 + 0.02\,P\,(\ddot{\mathbf{a}}_{[40]:20} - 1)$

$P * 13.93 = 50000 * 0.46423 + 50000 * (882.85/\,2052.54) + 650 + 0.02 * P * 12.93$

$13.93\,P = 45367.78 + 0.2586\,P$

$P = (45367.78/13.6714) = 3318.44$

(3)

(ii) Reserves:

Prospective Reserve:

${}_{5}\mathbf{V}^{pro} \;=\; 50000\,\mathbf{A}_{45:15} + 50000\,(\mathbf{D}_{60}/\,\mathbf{D}_{45}) + 0.02\,P\,\ddot{\mathbf{a}}_{45:15} - P\,\ddot{\mathbf{a}}_{45:15}$

$= 50000 * 0.56206 + 50000\,(882.85 / 1677.97) - 0.98 * 3318.44 * 11.386$

$= 54410.085 – 37028.083 = 17382.002$ **(3)**

${}_{5}\mathbf{V}^{retro} = (\mathbf{D}_{[40]} / \mathbf{D}_{45})\,[P\,\ddot{a}_{[40]:5} - 50000\,\mathbf{A}^{1}_{[40]:5} - 650 - 0.02\,P\,(\ddot{\mathbf{a}}_{[40]:5} - 1)]$

$= (2052.54 / 1677.97)\,[\,3318.44 * (20.009 – (1677.97/2052.54)*18.823) –$
$\quad 50000 * (0.23041 – (1677.97/2052.54)*0.27605) - 650 – 0.02 * 3318.44 * 3.6210274$

$= (2052.54 / 1677.97)\,[\,15334.602 – 50000 * 0.0047366 - 650 - 240.32]$

$= (2052.54 / 1677.97) * 14207.447$

$= 17378.948$ **(3)**

[Total 6]

(The difference of Rs 3.054 is due to the rounding errors).

(iii) Insurer's Profit:

Reserve per policy at the end of FY 2004-2005

${}_{4}\mathbf{V}^{pro} \;=\; 50000\,\mathbf{A}_{44:16} + 50000\,(\mathbf{D}_{60}/\,\mathbf{D}_{44}) + 0.02\,P\,\ddot{\mathbf{a}}_{44:16} - P\,\ddot{\mathbf{a}}_{44:16}$

$= 50000 * 0.541 + 50000 * (882.85/\,1747.41) – 0.98 * 3318.44 * 11.934$

$= 52311.67 - 38810.22 = 13501.45$

The reserves required on 1 April 2005 total
205* 13501.45 = 2767797.30

The premiums received on 1 April 2005 total
205 * 3318.44 = 680280.2

Expenses paid during the FY 2005-2006 was
2* 0.02 * 205 * 3318.44 = 27211.21

Interest earned during the FY 2005-2006 was
2* 0.04 * (2767797.30 + 680280.20 – 27211.21) = 273669.30

There were 15 deaths during the FY 2005-2006. So the reserves required as at 31st March 2006 total:
(205-15) * 17382.002 = 3302580.40

So the profit earned in the FY 2005-2006 was
2767797.30 + 680280.20 – 27211.21 + 273669.30 – 15 * 50000 – 3302580.40
= 358044.79
i.e a loss of approximately Rs 3,58,000 (6)

[Total 15]

Que 12)

i) The expected present value (EPV) of a benefit of 12,000 p.a. payable continuously to a life now aged ‘x’ and healthy whenever ‘x’ is sick, with the benefit ceasing at age 65.

(2)

ii) EPV of a benefit of 10,000 p.a. payable continuously to a life aged 35 and healthy throughout their first period of sickness, with the benefit ceasing at age 65 in any event.

(2)

[Total 4]

Que 13)

Let us give some credit to students who define the symbols they use and assumptions made.
The value of the pension benefit is

$= (2000 / 80\ {}^{s}D_{30})\ ({}^{z}\overline{R}_{30}^{ia} + {}^{z}\overline{R}_{30}^{ra})$

$= (25 / 41558)\ (1502811 + 4164521)$

$= 3409.29$

The value of the employer’s contribution which commences at 100 is

$= (100)\ ({}^{s}\overline{N}_{30} / {}^{s}D_{30})$

$= 100\ (680611 / 41558)$

$= 1637.74$

The value of the return of contributions on death and withdrawal is

$= (100 / {}^{s}D_{30}) ({}^{sj}\overline{R}_{30}^{d} + {}^{sj}\overline{R}_{30}^{w})$

$= (100 / 41558) (27531 + 95926)$

$= 297.07$

Then if the initial employer's contribution is C and it is assumed that the value of this contribution can be found from the value of the employer's contribution by proportion and by multiplying by 1.02 as on average it is paid half a year earlier, it will be

$$(C / 100) (1.02) (1637.74) = 16.70\, C$$

Then equating the values of contributions and benefits we have

Value of Employer's Contributions	+	Value of Employee's Contributions	=	Value of Pension benefits	+	Value of Return of Contributions on death or withdrawals

i.e. $16.70\, C + 1637.74 = 3409.29 + 297.07$

⇨ $C = 123.87$

That is the employer's contribution is a percentage of salary is (123.87 / 2000) = 6.19%

[Total 10]

Que 14) Let X denote the present value of the insurer's profit.

X= PV of premiums – PV of benefits
If the curtate future lifetime of a policyholder is K years, then the present value of the premiums received is $P\, \ddot{a}_{\overline{K+1}|}$ and the present value of the benefits paid is $S\, v^{K+1}$

Since $\ddot{a}_{\overline{K+1}|} = (1- v^{K+1})/d$, it follows that

$X= P \{ (1- v^{K+1})/d\} - S\, v^{K+1} = P/d – (P/d + S)\, v^{K+1}$

So the variance of the insurer's profit is :
$Var (X) = (P/d + S)^2\, Var (v^{k+1}) = (P/d + S)^2\, ({}^{2}A_x – (A_x)^2)$

Substituting the values for P, S, d, x etc we have,

$Var (X) = (P/d + S)^2\, ({}^{2}A_{40} – (A_{40})^2)$

$= (650 * 1.06/0.06 + 50000)^2\, (0.02707 – 0.12313^{\,2})$

$= 45018417 = (6709.57)^2$

[Total 4]

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