# Actuarial Society of India 

SA5: Finance

May 2006 Examination

Indicative Solution

Q1)a) - A short hedge involves taking a short position in a futures contract to hedge price fluctuations in respect of an underlying asset which the investor owns (is long of)

- A short hedge is appropriate when the hedger already owns the asset[ or will own it] and expects to sell it sometime in the future
- Likewise a long hedge involves taking a long position in a futures contract. A long hedge is appropriate when a company (hedger) knows that it will have to purchase a certain asset in the future and wants to lockin a price now.
Q1)b) - The hedge ration is the ratio of the size of the position taken in the futures contracts to the size of the underlying exposure
- Derivations of the Optimal [Minimum Variance] Hedge ratio:

Define
$\Delta S=$ Change in spot price $S$, during a period of time equal to the life of the hedge
$\Delta F=$ Change in futures price F , during a period of time equal to the life of the hedge
$\sigma S=$ Standard deviation of $\Delta S$
$\sigma F=$ Standard deviation of $\Delta F$
$\rho=$ correlation coefficient between $\Delta S$ and $\Delta F$
$h^{*}=$ hedge ratio that minimizes the variance of the hedger's position
$h=$ hedge ratio
If the hedge involves a long position in the asset and a short position in the futures contract, then the change in the value of the portfolio $=\Delta S-h \Delta F$ If the other hand, if the hedge involves a short position in the asset and a long position in the futures contract, then the change in the value of the portfolio $=h \Delta F-\Delta S$
In either case, the variance of the change in the value of the portfolio is given by $v=\sigma_{s}^{2}+h^{2} \sigma_{F}^{2}-2 h \rho \sigma_{S} \sigma_{F}$
» $\frac{\partial v}{\partial h}=2 h \delta_{F}^{2}-2 \rho \sigma_{S} \sigma_{F}$
Setting $\frac{\partial v}{\partial h}=0$ we get $h^{*}=\rho \frac{\sigma_{S}}{\sigma_{F}}$
Also $\frac{\partial^{2} v}{\partial h^{2}}=2 \sigma_{F}^{2}>0$
Therefore the minimum variance (optimal) hedge ratio is given by the expression $h^{*}=\rho \frac{\sigma_{S}}{\sigma_{F}}$
Q1)c) Mismatch \& Mitigation

- The fund value will change over a 6 month period depending on movements in the equity and bond markets. This would be expected to be broadly matched by any movements in the value of the liabilities
However the transfer value is fixed in nominal terms and will not change.

For example, if equity and bond markets fall, the proportion of the fund's assets to be transferred out of the fund will rise from the current proportion of $2 \mathrm{bln} / 4 \mathrm{bln}=50 \%$
However the proportion of the liabilities transferred will not change.
Hence the fund will lose and the solvency position of the remaining fund will be weakened.
The reverse will occur if the markets rise over the coming 6 months and the fund will gain.
However, the uncertainty does represent a risk that needs to be hedged The fund can resolve the problem by hedging its exposure to the extent of $50 \%$ of its exposure to the equity and bond market, 1 bln exposure to bonds and 1 bln exposure to equities
This can be achieved in the futures market by short hedging. The fund should sell appropriate volumes of futures in bonds and equities
Q1)d)i) - Exposure to 1bln rupees of equity stocks can be achieved through $\frac{1 \times 10^{9}}{4000 \times 100}=2500$ contracts of Nifty Index Futures

- The optimal hedge ratio $h^{*}=0.75 \times \frac{0.12}{0.18}=0.500$
- Therefore the fund should sell $0.5 x 2500=1250$ Nifty Index Futures contracts
- Effectiveness of this hedge is given by $\rho^{2}=0.75^{2}=0.5625$ which is considerably below the optimal level of 1 . Thus much of the risk in the equity portfolio has not been hedged effectively
- Equivalent exposure to the bond market is given by
$\frac{1 \times 10^{9}}{100000 \times 1.243}=8045$ long gilt futures contracts
Optimal hedge ratio $=0.9 \times \frac{0.03}{0.04}=0.675$
Therefore the fund should sell $0.675 \times 8045=5430$ long gilt futures contracts
Effectiveness of this hedge $\rho^{2}=0.9^{2}=0.81$ which is reasonably close to the optimal level of 1 . Thus the hedge is a relatively effective one.
Q1)d)ii) - Some basis risk arises from the fact that the underlying asset of the futures contract is not identical to the portfolio being hedged. This is the "Crosshedging" aspect of basis risk and is indicated by the level of hedge effectiveness - a lower hedge effectiveness corresponds to a higher degree of cross-hedging risk.
- Basis risk is also caused by timing mismatches
- The fund does not know exactly when the cash has to be transferred and thus cannot hedge the exposure precisely - the fund is only aware six months later that the transfer will occur sometime
- The transfer will take place six months later but the available
traded gilt futures contracts expire after 7 months thus the fund will need to sell the futures contracts prior to expiry, which could cause basis risk.


## Methods for Reducing Basis Risk

- The fund could reduce the cross-hedging risk by:
- Negotiating an OTC forward contract with another party based on a value share index portfolio
- Effecting a total return swap with a counter party based on the total return on a value share portfolio - the fund pays the total return on a value share index and receives fixed interest payment [or floating rate payments]
- Selling a basket of individual stock futures[if available] that more closely reflect the value portfolio
- Finding a combination of equity index futures contract[if available] that more closely matches the underlying portfolio of value shares
- The timing risk can be reduced if the fund seeks more detailed information on the date of transfer from the receiving fund manager and the receiving scheme trustees
- The timing risk caused by the longer term of traded gilt futures contract can be reduced by using OTC derivates as above; and specifying more precisely the expiry date of the contracts.
Q1)e) Option Positions
(i) Bull Spread: Involves buying a call option on the Nifty Index Futures with a certain strike price say, $\mathrm{K}(1)$; and selling a call option on the same future with a higher strike price say, $K(2)$ - both options having the same expiry date.
This position will give a positive payoff if the Nifty Index rises, but the payoff is limited in amount. It will give a negative payoff if the market falls, but the loss is also limited. The payoff diagram is shown below

Payoff at Expiry of options

(ii) Strap: A strap is created using a long position in two calls and one put
with the same strike price and exercise date
This gives a positive payoff if the market moves significantly in either direction, but a larger payoff if the movement is upwards The payoff diagram will be as follows

(iii) Calendar spread : Involves selling a call option with a near expiry date and buying a call option with a far-dated expiry date. Both option have the same strike price. Because the far-dated option will be more expensive, this position will incur an intial cost.
The payoff is positive if the Nifty Index future remains close to its current value. The payoff is usually considered at the expiry of the near-dated option, with the far-dated option being sold at this time The payoff diagram is as follows;


- Risk Weighted Assets - Calculation

| Asset | Book Value | Risk Weightage | Risk Weighted <br> Value |
| :--- | :--- | :--- | :--- |
| LTGoI Bonds | 450 | 0 | 0 |
| TBs | 900 | 0 | 0 |
| Corp Bonds \& 1350 | 1 | 1350 |  |
| Comm Loans |  |  |  |
| Housing loans <br> Total | 1350 | 0.5 | 675 |
|  | 4050 |  | 2025 |

- Tier I Capital $=$ Share Capital + Reserves

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=45+45=90
$$

- Tier II Capital = Min [Tier I Capital Subordinated loans]

$$
=\text { Min }[90,225]
$$

$$
=90
$$

- Basel I Ratio $=\frac{(90+90)}{2025} \times 100$

$$
=8.9 \%
$$

- Comments:
- This is very close to the minimum level of $8 \%$. This implies that the bank can breach the minimum requirement in the event of bad credit experience.
- The advantage of a low ratio is of course, that the return on capital is likely to be high [although part of the reason for the low ratio in this case is also the disallowance of the subordinated loan stock]
- Methods of improving the ratio:
- The ratio can be improved by either
- Reducing the risk-weighted assets
- Increasing the regulatory capital
- Some measures for reducing the risk-weighted assets are:
- Switch some of the corporate bonds into government bonds
- Reduce the size of the balance sheet by selling some corporate bonds and reducing the deposit balance
- Obtain the same credit exposure through off-balance sheet instruments. For example, rather than holding a corporate bond, the bank could hold a government bond and sell a credit default swap on the corporate bond counter party thereby reducing the regulatory capital
- Redirect its lending towards housing loans (mortgages) portfolio which are risk weighted $50 \%$ rather than corporate lending
- The regulatory capital can be increased by issuing shares

Q2)b) Calculation of VaR-Methodology
Historical simulation method of calculating VaR:

- The first step is to describe each asset and liability in terms of a number of chosen "market factors"
- These market factors can be:
- 3 - month interest rates
- 4 - year interest rates
- cash, etc
- The next step would be to collect historical data sets for the chosen market factors over a selected period in the past. For example, we might look at changes in the above market factors over various non-overlapping 99- day periods over the last 10 years. This would give us about 40 data sets. Alternatively, we might consider taking overlapping 99-day periods that start at monthly intervals to get 120 data sets or even weekly intervals to get 520 data sets
- We would then calculate the change in the market value of the bank's assets and liabilities when subjected to the changes experienced in the past i.e, 40 (or 120) observed changes
- Due to the small percentage tail chosen, we would then fit a distribution to the results - for example, the normal distribution might be selected. This would allow us to draw the graph showing (value of asset-value of liabilities) on the $x$ - axis and the percentage likelihood on the $y$-axis
- We would then select the point on the $x$-axis that represent the $0.5 \%$ tail which would be an appropriate number of standard deviations from the mean value of (assets-liabilites). In fact, in this case we would look for (minus) 2.576 standard deviations.
- The difference in value between the (assets-liabilites) at this point and the current value of (assets-liabilities) would be the desired VaR figure
- Drawback of Historical simulation method

Because the historical simulation method uses a specific set of market factor movements, if may be that this set is not representative of the real correlations and volatilities of the market factors. Other methods such as the analytical variance/covariance methods allow us to specify the behaviour of the market factors using a suitable economic model, which may be more accurate.

- Drawbacks of VaR in general:
- One potential problem is the choice of holding period-99 days in this case. VaR assumes that bank's position can be liquidated (both assets and liabilities) within 99 days if it was required. Although this might be the case for most of the liquid asset categories, it would probably not be the case for mortgages nor for most part of the corporate bonds and commercial loans portfolio
- Many assets do not exhibit a linear relationship to movements in the market factors- they exhibit a degree of curvature. This is particularly important with option-like assets and liabilities. Thus
the calculated value over 99 days cannot be scaled up or down to longer or shorter periods without an assumption of linearity. This approach implicitly assumes that the variance of the distribution grows linearly with time, which is often not the case.
- If the bank is exposed to other risks such as equity price movements, the VaR result becomes very dependent on the correlation data that links movements between the market factors. This is more important when analytical covariance or Monte Carlo methods are used to generate results. This correlation data is often the most unreliable, particularly between asset categories
- The choice of the observation period over which to collect the data[ inspection of what method is used] will have a large impact on the result. If the period is short, it has the advantage of data being more relavant, but the disadvantage of having too little data with possibly no extreme movements. If the period is long, then the amount of data is large but the relevance may be questionable.
- In general, the VaR collects data during the normal trading periods and uses them to predict the losses under more extreme events. Infact, during market turbulence, the variances and co-variances break down and the values collected are no longer valid at all. Thus the predicted loss under extreme circumstances might be completely different and possibly much larger.
Q2)c) Exposure to Interest Rate Risk
Maturity Gap Analysis
- A maturity gap analysis aims to measure the bank's exposure to changes in interest rates by sorting the bank's asset and liabilities into:
- Those that reprice within a certain time period (or gap) when interest rates change - referred to as rate- sensitive assets/liabilities
- Those that do not reprice within a certain period when interest rates change
An asset or liability is repriced either when it matures and must be reissued at a competitive contract rate or when its contract rate is reset periodically prior to maturity
The gap is chosen to suit the institution. Here we use 6 months as the gap.
- From the balance sheet we find the rate-sensitive assets to be
- Floating rate housing loans
- Treasury bills

Totaling about Rs. 2,250 million in accounting terms

- Other assets (LT GoI bonds plus corporate bonds and commercial loans) totaling Rs. 1,800 million are rate insensitive; and would not reprice
- Similarly the rate sensitive liabilities will be
- Current deposits
- 90 - day deposits [floating rate, even though > 3 months average term]
totaling about Rs. 3,735 million in accounting terms
- the sub-ordinated loan would be rate-insensitive share capital and
reserves will also deemed to be rate insensitive
- The maturity gap is defined as the difference between the amount of ratesensitive liabilities and rate-sensitive assets. If the maturity gap is positive, an increase in rates will increase the cost of servicing the liabilities more than it will increase the additional interest generated from the assets. This implies that profits will fall.
- In this case the gap is around Rs. 1485 million [=Rs. 3735 mln -Rs. 2250 mln ]. This is quite a high proportion of the bank's total asset and represents a fairly high level of interest rate risk
- The advantages of this approach are as follows
- It is very simple to complete and to understand
- It can be presented as table where the assets and liabilities are placed in repricing "buckets" of various terms
- It can be completed in each of the currencies in which the bank is active and can be used to analyse the currency mismatch of assets and liabilities if desired
- If used over short gaps it gives information about the liquidity risk of the bank i.e. its ability to pay near-term liability out of cashflow
- The disadvantages of this approach are as follows:
- It demonstrates the size of the gap but not the effect of the change in interest rates on the value of the bank's asset and liabilities
- It uses accounting data rather than market value data
- It is very difficult to incorporate off-balance sheet instruments
- It ignores default risk in the sense that it assumes that each loan has a value equal to its nominal value
Duration Analysis:
- Duration analysis measures the sensitivity of the value of asset and liabilities to changes in interest rates
- An outsized mismatch between the duration of assets and duration of liabilities places the value of the company's equity at great risk to changes in interest rates
- The change in the market value of equity $\Delta E$ in response to a change in market yields $\Delta r$ is given by the expression:
$\Delta E=-\left(A \times D_{A}-L \times D_{L}\right) \Delta r$
where $\mathrm{A}=$ market value of the financial assets
$\mathrm{L}=$ value of the financial liabilities
$D_{A}=$ duration of the financial assets
$D_{L}=$ duration of the financial liabilities
$\mathrm{E}=$ market value of equity $=\mathrm{A}-\mathrm{L}$
- Implicit duration of equity is defined as:
$D_{E}=\left(A \times D_{A}-L \times D_{L}\right) / E$
- Since the given information does not contain the exact coupons and cashflows of the assets and liabilities, it is difficult to estimate what the duration gap is. Hence we will make the following assumptions:
- The LT GoI bonds are zero coupon bonds. Hence duration of
these bonds= 4 years = term to maturity
- Duration of floating rate instruments [including housing loans, 90 - day term deposits and current deposits] $=0.25$ [i.e., the reset frequency]
- Duration of corporate bonds and commercial loans is approximately 4.5 years
- The subordinated loan is of the zero coupon type; hence duration of the sub-ordinated loan $=10$ years
- Duration of TBS $=1$ month (or) 0.08 years
- Therefore

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D_{A}=\frac{[(450 \times 4)+(1350 \times 4.5)+(1350 \times 0.25)+(900 \times 0.08)]}{[450+1350+1350+900]}
$$

$=2.05$ years
$D_{L}=\frac{[225 \times(10)+3735 \times(0.25)]}{[225+3735]}$
$=0.80$ years
Thus $\Delta E=-(4050 \times 2.05-3960 \times 0.8) \times(+0.01)$

$$
=-51.35
$$

$D_{E}=(4050 \times 2.05-3960 \times 0.8) / 90$
$=57.05$ years

- Thus the bank is very highly exposed to changes in interest rates in that a $1 \%$ increase in interest rates would reduce the value of the equity of the bank by 51 mln - that is a large amount given that the total value of the equity is only Rs. 90 mln in accounting terms
- Clearly the equity of the bank is very sensitive to changes in interest rates and will behave in a similar fashion to a bond with a duration of 55 years
- The advantages of duration analysis are as follows:
- It gives information about the sensitivity of the bank's asset and liabilities to changes in interest rates
- It is easlier [as compared to the maturity gap analysis] to incorporate off- balance sheet instruments
- It can be adapted to use market values, if necessary
- It can be used to formulate appropriate hedging strategies
- The disadvantages of this approach are as follows:
- It only looks at value changes and gives no information on the timing on the expected cashflows
- It ignores default risk in the sense that each loan is valued at its nominal value [ this problem can be mitigated by using market values that reflect credit risk]

Q3)a) Possible Motives for the merger

- The merger is a horizontal merger as it involves companies that operate at the same stage of the production process
- In general such mergers have one or more of the following reasons:
- To benefit from economics of scale - using one set of corporate office staff in the UK or a single branch network
- To benefit from complementary resources - fund manager's expertise or utilizing the tied selling agents of both companies or complementary client bases
- To eliminate inefficiencies - under performing management and branches
- There are other reasons that apply to almost any type of merger or acquisition
- Utilization of unused tax benefits - if two proprietary life companies have different tax positions, then one may takeover the other in order to ensure that any valuable unused tax shields are utilized
- It is also possible that LL has a cash pile which it would like to utilize and one of the ways of utilizing the cash pile is to make the acquisition
- Protection against the threat of takeover - by increasing the size [albeit modestly] of the business
- Enhancement of earnings per share [EPS] - LL may be able to increase its EPS by taking over SL which may have a lower P/E [price-earnings] ratio. The EPS of the merged company will increase if the merger reduces the total number of shares in existence, without affecting total earnings
- Exploitation of lower financing costs - often large companies can raise debt or equity at higher prices [an hence lower costs]. Again this is likely to be of minor significance in this case
Q30b) Approach for valuing SL
- The business activities of SL will be dividend into those that will continue to operate after the merger and those that will not
- Those that will not continue [ the investment management function?] will either be written off - although there may be costs in closing down certain parts of the operation or sold on to a third party after the merger. If the operation is saleable, then an estimate of its realizable value should be made
- Those operations that are to remain will either be run down over a period of time and used to generate cash and earnings, or they will require investment in order to continue to sell the products [in which case costs are involved]
- Estimates of future income will involve detailed analysis of estimated not cash flows, starting with past performance in the existing line of activity, net of tax. Factors affecting future performance will also need to be considered.
- Where accounts have been prepared under different accounting standards, historical figures will need to be recast on the acquirer's own accounting basis
- It is important to ascertain any "hidden" issues exist, such as hidden
liabilities or overvalued assets. Similarly the extent to which reported profits are enhanced by freehold property occupation or low rental leased needs to be investigated, as well as any cuts in advertising and sales promotion expenditure
- In assessing likely future profits, consideration must be given to all possible ways in which the firm could develop - run down or built up, continue to sell its own products or switch to LL products
- Opportunities for cost saving must be evaluated and allowances need to be made for capital expenditure
- Evaluation must be based on the differential net cash flow which can be attributed to the takeover alone. It needs to be noted that it is the future cashflows, not accounting profit, that needs to be evaluated.
- Impact of the contingent liabilities (such as capital commitments) on the future cash flows must be considered
- Tax aspects such as tax losses which can be offset against purchaser's profits, capital allowances and other tax "carry forward" need to be taken into account
- It needs to be decided whether to keep the SL brand name or whether that should be written off
Q3)c) Project Appraisal Methods
- Profit-Based Analysis
- This is useful for investigating the effect on the P\&L
- However this analysis ignores an important aspect of any project cash flow. Many projects with positive profit effect can blunder because the product does not bring in sufficient cash flow in the early years. Therefore the appraisal should use certain cash flow methods like pay back analysis to look at this aspect
- This method also misses certain qualitative aspects of projects which may make the project important even if it’s not profitable or cash generative
- Hurdle Rate
- A high hurdle rate as $20 \%$ will cause problems because
- Must projects will not meet this criterion and will be rejected even though such projects provide a perfectly reasonable return
- Very risky projects might pass because of the high returns offered
- Short term projects will be favoured over long term
- It may lead managers to intentionally inflate the profitability of their project to get them passed
- The hurdle rate criterion can result in rejecting projects with higher NPV but a lower IRR - for example using this criterion is isolation can result in accepting a project with a $25 \%$ IRR \& NPV of say Rs. 1 mln and rejecting a project with an IRR of $15 \%$ and NPV of Rs. 10 mln
- Not Reviewing the method of Appraisal:

One of the key ideas of project appraisal is that project managers should learn from their projects. This can only be done if the project appraisal process itself is reviewed before a new project is launched.

- Passed on Management Judgments
- This is normal in many companies. Often these are projects that cannot meet financial criteria but are nevertheless important to business on qualitative or other bases
- However it is important that financial criteria be applied fairly and consistently wherever they can be applied. Otherwise there can be inter-departmental conflicts where certain projects are approved by the management whilst others are rejected on financial criteria

