Institute of Actuaries of India

Subject ST8 — General Insurance: Pricing

May 2012 EXAMINATION

INDICATIVE SOLUTION

Solution 1 :

Rainfall index-based insurance for agriculture.

a. Possible exclusions

- Indemnity for expenses incurred
- access to other natural or man-made resources of water
- crop not sown as per predetermined schedule or lower quantity sown in the area
- volatility in rainfall from unnatural causes like nuclear risk

b. risk factors

- uncertainty of experience
- Location
- Season of coverage
- Period
- Lack of rainfall
- Exposure

c. rating factors

- extent of past data available
- location
- cover start and end dates
- trigger and exit levels
- sum insured
- area of field
- crop planned to be covered

[8]

Solution 2 :

Approaches to rating and circumstances when they will be used

- Frequency severity – useful approach where excesses, deductibles or limits apply
- Burning cost – where little individual claims data are available
- Original loss curves – where historical data for a risk are sparse and/or lacking in relevance, so experience-based rates are not credible
- Generalised linear modelling – when there are multiple identified factors influencing claims and their relative relationships can be used to predict claims
- Multivariate models – when there a large number of levels within some factors in a GLM
- Credibility theory – when most relevant past data for an insured or insurer is to be used along with a more reliable market-wide data

[6]

Solution 3 :

a. Issues likely to be faced by the company when there is inadequate data capturing on information systems:

- inappropriate pricing
- insufficient reserving
- wrong claims settlements
- inadequate reinsurance arrangements
- insufficient reinsurance recoveries
- lost opportunities for identifying more profitable segments
- lost opportunities for cross-selling
- wrong management decisions
- insufficient capital to deal with additional expenses as consequence of inadequate data

b. Benefits of capturing information on systems:

Policy stage

- Will aid more appropriate rating of risks
- Premiums can be more competitive
- Can cross-sell other products depending on the characteristics of policyholders captured at issuance
- can lead to more competitive terms from reinsurers
- can help in identifying and retaining profitable customers
- can help in better customer service
- can aid predictive modelling which will be useful in capital or cashflow modelling exercises

Claims stage

- Information can be matched with that provided at issuance stage
- useful in determining profitable and loss-making segments by area / line / cause of claim etc
- Can enable detailed claim analysis so different features of claims can be accurately determined
- Can aid more accurate reserving leading to more accurate pricing
- Can help in filtering out invalid claims
- Can help in earlier identification of possible reinsurance recoveries to be made

[10]

Solution 4 :

Items in which controls can be imposed by regulator or statutory bodies:

- Product design
- Amount of cover
- Extent of reinsurance cover
- Maximum commissions / fees
- Pricing methodology

- Expenses of management
- Profit margin or any other performance measurement criteria
- Review of existing similar or same product
- Maximum / minimum premium
- Credit taken for investment income
- Credit taken for favourable reinsurance arrangement
- Extent of credit taken for mid-year cancelled policies with little or no refund of premium

[6]

Solution 5 :

Competitive motor insurance industry

a.Issues the company currently faces while pricing:

- Competition driving lower margins in premiums
- Higher loss ratios
- lower profits
- maybe capital support required
- insufficient capital can lead to insolvency
- there may be little or no reinsurance support available at such rates
- higher than normally expected losses on catastrophes

b.Issues the company is likely to face when the industry moves to more profitable rates due to exit of loss-making companies:

- Higher premiums might lead to loss of customers who cannot afford the cover from any insurer
- More uninsured risks
- Higher per policy expenses
- Systems not functioning to full capacity, ie. Higher costs

[5]

Solution 6 :

a.Factors to consider before finalising a reinsurer:

- Credit rating
- Expertise of reinsurers in the new product
- Training that will be provided
- Any system support
- Other technical support such as experience analysis for product review
- And regulatory restrictions
- Value for money

b.Stages of a product life where a reinsurer is likely to be involved:

- Product design
- Product pricing
- Proposal underwriting
- Claims underwriting
- Claims settlement

[5]

Solution 7 :

Factors to consider while determining the extent of loading for catastrophic risk while pricing:

- Volume of business
- Level of retention
- Extent of concentration of exposures to single area
- Extent of concentration of exposure to single risk
- Catastrophe XL premium
- the extent to which rating takes account of exposure to catastrophes
- the size of insurer
- the loading used by other companies
- any cap on cover
- type of cover
- any regulatory requirements

[6]

Solution 8 :

Upward sloping yield curve. A portfolio of personal accident policies

- if the insurer is small to medium sized, has a slowing growth pattern and just satisfies the statutory solvency requirement
    - liabilities are short tailed
    - will want assets to be matched as much as possible by term to liabilities
    - lower free assets will allow lesser investment freedom for mismatching
    - slowing growth pattern might not provide the required cash inflows to meet liquidity requirements
    - so less likely to invest in long term bonds
    - so less likely to take higher rate into account while pricing

- if the insurer is large sized with a high solvency ratio
    - insurer has higher investment freedom for mismatching
    - so more likely to invest in long term bonds to take advantage of the high returns
    - so more likely to take higher rate into account while pricing

[4]

Solution 9 :

a.The one way relativity of a factor explains the effects of that factor alone on the dependant variable. A one way relativity gives no consideration to interdependencies between variables in the model.

1. A) FALSE: IF this had been true then one way relativity would have been lower than GLM relativity. That is to say that implication of scanty population and fewer would mean reduced overall exposure to risk. Therefore one ways would be more favourable

(b) FALSE: This is not true as building height was not a rating factor. Therefore the effect of height would get ignored in both one way relativities and GLM model thus not being a contributing factor for the difference observed in relativities

(c) TRUE: GLM distinguishes the effects of one factor with that of the other factors whereas one-way analysis only takes the effect of that factor (ignoring the possible contributions of other factors). Overall experience could have adverse in location X. Oneways attributes the cause to location alone. Whereas GLM will try to separate out the effects into those attributable to location and those attributable to old buildings (both being rating factors considered in the model).

[4]

Solution 10 :

a.

A translated gamma distribution is a gamma variable with a shift parameter. If a random variable $X \sim Gamma(\alpha, \beta)$ then a translated gamma variable Y with shift parameter k is nothing but gamma variable X whose values are shifted (or translated) by a constant amount k (where k may be positive or negative). Therefore where X takes values $[0,\infty)$ the new variable $Y = k + X$ will take values in the range $[k,\infty)$

The translated gamma distribution generally gives a better fit since a gamma distribution has positive skewness which is also expected of the aggregate claims distribution variable S in many practical situations.

b.

Random variable X follows a translated gamma variable with

- Shift parameter λ,
- Shape parameter α and
- Scale parameter β

Let Y be the Gamma variable and Z be the Chi-square variable

$Y = X - \lambda \sim Gamma(\alpha, \beta)$

$Z = 2\beta(X - \lambda) \sim \chi^2_{2\alpha}$

c.

Let S be compound Binomial variable

$E[S] = E[N]E[X]$ where N and X are claim number and claim amount distributions

$N \sim B(2000,0.1)$, $X \sim LogN(10,1)$

$E[N] = 200$, $E[X] = 36315 = m_1$

$Var[N] = 180$, $Var[X] = 2266097112 = (47603)^2$

$E[S] = E[N]E[X] = 7263101$

$Var[S] = E[N]Var[X] + Var[N]E[X]^2 = (831027)^2$

$E[X^3] = \exp(10*3 + (9 * ½)) = (98715.77)^3 = m_3$

$E[X^2] = Var[X] + E[X]^2 = (59874.14)^2 = m_2$

Skew (S) =

$$nqm_3 - 3nq^2m_1m_2 + 2nq^3m_1^3$$

Substituting above, Skew(S) = $(569569)^3$

Coefficient of skewness = $(569569)^3 / (831027)^3 = .3219539$

We now equate means, variances and coefficient of skewness of S with that of the translated gamma variable where

$E[S] = k + \alpha/\beta$

$Var[S] = \alpha/\beta^2$

Coefficient of Skewness (S) = $2/\text{sqrt}(\alpha)$

Therefore:

$\alpha = 38.59$

$\beta = 7.475*10^{-6}$

$k = 2100703$

Prob(S>80%*11Mn)
= Prob(2β(S-k)>100.16)
=Prob($\chi^2_{78} > 100.16$)
χ^2_{78} @ 5% is 99.61 and @ 4% is 101.17

That means probability that S > 80% of 11 Mn is slightly below 5%. Therefore the actuary would still want to review the prices.

[16]

Solution 11 :

a.Excess of Loss reinsurance

A form of reinsurance whereby the reinsurer indemnifies the cedant for the amount of a loss above a stated excess point, usually up to an upper limit. The excess point and upper limit may be fixed, or indexed as specified in a stability clause. Usually this type of reinsurance relates to individual losses, but it can be a form of aggregate excess of loss reinsurance covering the total of all losses in a period and subject to a total aggregate claim limit.

b.Stop Loss

An aggregate excess of loss reinsurance that provides protection based on the total claims, from all perils, arising in a class or classes over a period. The excess point and the upper limit are often expressed as a percentage of the cedant's premium income rather than in monetary terms.

c.Probable Maximum Loss

The term "probable maximum loss" represents an attempt to quantify exposure, used in rating or to judge requirements for outwards reinsurance. It may be used as another term for estimated maximum loss, depending on the class of business.

ii

Assumption: Claims as mentioned are the only incurred claims for the year

Loss No.	Limit/PML	Loss	Threshold	Maximum	Limit over and above Max	Retained %	Ceded %	Retained Amt	Ceded Amt	Total
	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
			as given	*(c) * 10 (no. of lines)*	*max((a)-(d),0)*	*[(c) + (e)]/(a)*	*1 - (f)*	*(f) * (b)*	*(g) * (b)*	*(h) + (i)*
1	40,000,000	19,000,000	20,000,000	200,000,000	-	50%	50%	9,500,000	9,500,000	19,000,000
2	40,000,000	38,500,000	20,000,000	200,000,000	-	50%	50%	19,250,000	19,250,000	38,500,000
3	60,000,000	57,500,000	20,000,000	200,000,000	-	33%	67%	19,166,667	38,333,333	57,500,000
4	120,000,000	23,000,000	20,000,000	200,000,000	-	17%	83%	3,833,333	19,166,667	23,000,000
5	20,000,000	8,000,000	20,000,000	200,000,000	-	100%	0%	8,000,000	-	8,000,000
6	80,000,000	46,500,000	20,000,000	200,000,000	-	25%	75%	11,625,000	34,875,000	46,500,000
7	100,000,000	1,500,000	20,000,000	200,000,000	-	20%	80%	300,000	1,200,000	1,500,000
8	220,000,000	4,500,000	20,000,000	200,000,000	20,000,000	18%	82%	818,182	3,681,818	4,500,000
9	30,000,000	26,500,000	20,000,000	200,000,000	-	67%	33%	17,666,667	8,833,333	26,500,000
10	50,000,000	41,000,000	20,000,000	200,000,000	-	40%	60%	16,400,000	24,600,000	41,000,000
TOTAL	760,000,000	266,000,000						106,559,848	159,440,152	266,000,000

Amount Ceded under Surplus Treaty: = INR 159.44 MN ---------- (A)

Amount ceded under Stop Loss:

Gross Loss = INR 266 MN (Gross Incurred Claims)

GEP (given) = INR 239.02 MN

LR = 111.29%

Lower Limit = 105%

Reinsurer Share of Loss = 85%

Amount ceded = (111.29%-105%)*85%*GEP = INR 12.77 MN -------- (B)

Amount ceded under Cat XL Cover:

Total amount (retained after Surplus Treaty) of Cat Losses = INR 72.483 MN

XS amount INR 50 MN

Therefore amount ceded = INR 22.483 MN --------- (C)

Amount ceded under Portfolio Aggregate XL Cover:

Total amount (retained after Surplus Treaty) of Losses = INR 106.56 MN

Less Amount ceded in Cat XL = INR 22.483 MN

Amount of losses before Aggregate XL = INR 84.076 MN

Aggregate XS = INR 20 MN

Aggregate XL layer = INR 30 MN

Amount ceded = minimum of (30, 84.076 – 20) = INR 30 MN -------- (D)

Total amount ceded = INR 224.698 MN [(A) + (B) + (C) + (D)]

Total amount retained = Gross Loss – Amount ceded = INR 41.302 MN

Gross Incurred Claims Ratios:

Earned basis = GIC/GEP = 266/239.02 = 111.29%

Written Basis = GIC/GWP = 94.59%

Net Incurred Claims ratios:

Earned Basis = NIC/NEP = 41.302/119.51 = 47.88%

Written Basis = NIC / NWP = 41.302/125.8 = 32.83%

[16]

Solution 12 :

a.Credibility is a statistical measure of the weight to be given to a statistic.

To estimate standard of full credibility for severity we need the claims size distribution.

Let F be the standard of full credibility for frequency and let the coefficient of variation of the claims size distribution be denoted by CV_s.

Then standard for full credibility for severity S can be estimated by:

$S = F * CV_s^2$

b. The square root rule of partial credibility states that if n_F is the size statistic for full credibility (i.e. to assign 100% credibility) and the observed sample size is n (where $n < n_F$) then the weight to be assigned to the estimate based on this smaller sample size is $\sqrt{n}/\sqrt{n_F}$

Derivation of square root rule:

Let X_i be a random variable denoting the number of claims arising from i^{th} policy.

$E[X_i] = \mu$ and $Var[X_i] = \sigma^2$

So if the number of policies is N then total number of observed claims from this exposure is given by: $\sum_{i=1}^{i=N} X_i$

Let n be expected number of claims, then n is given by $E\left(\sum_{i=1}^{i=N} X_i\right) = N\mu$

Let the statistic that needs to be estimated be frequency denoted by $\overline{X}$. Then $\overline{X}$ is estimated as:

$$\overline{X} = Z * \left|\frac{\sum_{i=1}^{i=N} X_i}{N}\right| + (1\text{-}Z) * \text{other}$$

Now say no. of claims $\sum_{i=1}^{i=N} X_i$ is the minimum no. of claims to be assigned full credibility (under an assumed a claim number distribution and using normal approximation). Then the estimate of the statistic $\overline{X}$ given above would be assigned 100% credibility i.e. Z = 1.

Therefore $\overline{X} = \dfrac{\sum_{i=1}^{i=N} X_i}{N}$

$$\text{Var}(\overline{X}) = \text{Var}\left(\frac{\sum_{i=1}^{i=N} X_i}{N}\right)$$

$$\frac{\sum_{i=1}^{i=N} Var(X_i)}{N^2}$$ *Assuming that the numbers of claims per policy are independent of each other*

$$= \sigma^2/N \quad \text{---(1)}$$

Now let us say we have a smaller sample i.e. no. of policies is M (M< N)

Based on this sample

No. of observed claims = $\sum_{i=1}^{i=M} X_i$

Expected no. of claims, m = Mμ

The estimate of the statistic frequency $\overline{X}$ ' is

$$Z * \left|\frac{\sum_{i=1}^{i=M} X_i}{M}\right| + (1 - Z) * \textit{other (constant)}$$

Since M < N, 0<Z<1

$$\frac{1}{Z^2} \frac{\sum_{i=1}^{i=M} Var(X_i)}{M^2} \quad \text{Var}(\overline{X}') =$$

$$= Z^2\sigma^2/M \quad \text{---(2)}$$

Using the concept of limiting the fluctuations in classical credibility theory, the credibility factor Z is calculated so that the expected variation in $\overline{X}$ ' is limited to the variation allowed in a full credibility estimate $\overline{X}$. Therefore, equating (1) and (2)

Z^2 = M/N

$$= \frac{m/\mu}{n/\mu}$$

= m / n

Therefore Z = $\sqrt{m/n}$ which is the square root rule for partial credibility

---(2)

c.

For X ~ Bin(n, p), E(X) =np and var(X) = np(1− p) . Using the normal approximation we have:

μ = np and

σ2 = np(1− p)

Given p = 25%

The probability that the value lies ±7.5%of the expected value is approximately 85% is given by:

P(0.925np<=X<=1.075np) = 85%

$$Prob\left(\frac{-0.075*n*0.25}{\sqrt{n*0.25*(1-0.25)}} \le Z \le \frac{0.075*n*0.25p}{\sqrt{n*0.25*(1-0.25)}}\right) = 85\%$$

$$Prob\left(-0.0433*\sqrt{n} < Z < 0.0433*\sqrt{n}\right) = 85\%$$

---(2)

By trial and error

Where n= 500, LHS = 66.708%

Where n = 1000, LHS = 82.91%

Where n = 1200, LHS =86.64%

Using linear interpolation between n=1000 and n=1200 we get n = 1112 (giving a probability of 85.12% approximately)

Therefore:
Exposure standard for full credibility is 1112 and
Standard for no. of claims is 25% * 1112 = 278 claims

--(2)

Pure Premium = Z * *estimate* + (1-Z) * *other*
Z = $\sqrt{n}/\sqrt{n_F}$ = $(173/278)^{0.5}$ = 0.7886
Therefore, pure premium = 0.7886 * 5315 + (1-0.7886)*6281
= Rs. 5518.96

-- (2)

[14]

[Total Marks – 100]

****