Institute of Actuaries of India

Subject CT8 – Financial Economics

MAY 2012 EXAMINATION

INDICATIVE SOLUTIONS

Solution 1 :

a. The multifactor model attempts to explain returns on assets by relating them to a series of *n* factors known as indices:

$$R_{i=}\ a_i + b_{i,1} I_i + \cdots . + b_{i,n} I_n + c_i$$

- a_i, c_i are the constant and random parts of the return, specific to asset *i*
- I_1, I_2,..., I_n are the *n* indices explaining the returns on all the stocks
- $b_{i,k}$ is the sensitivity of the return on stock *i* to factor/index *k*
- $E[c_i] = 0$ and
- $Cov[c_i, c_j] = 0$ for all $i \pi j$ and $cov[c_i, I_k] = 0$ for all stocks and indices. }

b. Using

$$R_i = \lambda_0 + \lambda_1 b_{i,1+} + \lambda_2 b_{i,2}$$

Solve 3 simultaneous equations

$\lambda_0 = 7.75$

$\lambda_1 = 5$

$\lambda_2 = 3.75$

$\check{R}_i = 7.75 + 5\, b_{i,1} + 3.75\, b_{i,2}$ is the equation that describes equilibirium return

c. *Portfolio D is constructed by investing equally investing ⅓ in X; Y; and Z*

$E(D) = ⅓*15 + ⅓*14 + ⅓*10$

$= 13$

$b_{p,1} = ⅓*1 + ⅓*0.5 + ⅓*0.3 = 0.6$

$b_{p,2} = ⅓*.6 + ⅓*1.0 + ⅓*0.2 = 0.6$

By the law of one price, 2 portfolios that have the same risk cannot sell on different expected return. Arbitrageurs will buy Portfolio E and sell equal amount of Portfolio D and make a riskless profit

[10]

Solution 2 :

a.

u(w) = -1/w^0.5

Differentiating

u'(w) = w^(-3/2)/2

u''(w) = (-3/4)w^(-5/2)

Co-efficients

A(w) = -U''(w) / U'(w)

R(w) = wA(w)

Therefore

A(w) = (3/2w)

R(w) = 3/2

b.

To study the effect calculate

A'(w) = (-3/2w^2) <0

R'(w) = 0 =0

This shows that investor exhibits decreasing ARA i.e

So as wealth increases he will hold more dollars in risky assets.

However constant RRA implies as a % invested in risky assets is unchanged as wealth increases.

c.

A			B		
Outcome	Probability	Utility	Outcome	Probability	Utility
5	0.33	-0.45	4	0.25	-0.50
6	0.33	-0.41	7	0.50	-0.38
7	0.33	-0.38	10	0.25	-0.32

C			D		
Outcome	Probability	Utility	Outcome	Probability	Utility
1	0.20	-1.00	1	0.40	-1.00
9	0.60	-0.33	6.5	0.50	-0.39
18	0.20	-0.24	31.8	0.10	-0.18

EU (A)	-0.41114
EU (B)	-0.39304
EU (C)	-0.44714
EU(D)	-0.61385

Where Expected U (i) = $\Sigma\ p_i * u_i$

The investor is risk averse. If, however, the investor does not invest in any of the portfolios, then his or her expected (and certain) utility is

$$-\frac{1}{\sqrt{6.5}} = -0.3922$$

Thus, as investing in any of the portfolio gives the investor a lower expected utility, he or she will not invest in any of the portfolios.

[9]

Solution 3:

a.

Variance main advantage is ease of use for optimal portfolio within MVPT theory and also it is mathematically tractable

However the main disadvantage is that most investors do not dislike uncertainty of returns; rather they dislike downside risk of low investment returns which variance doesn't capture.

b.

I. Mean is given by =-10*0.1+5.5*0.9 = 3.95

II. Variance = $(3.95-(-10))^2 * 0.1 + (3.95 - 5.5)^2 * 0.9 = 21.62$

III. 95% Value at Risk at

VaR(X) = -t where t = max { x : P(X<x) ≤ 0.05}

P(X<-10) = 0 and P(X< 5.5)= 0.1

t = -10

Since t is a percentage investment return per annum, the 95% value at risk over one year on a Rs. 20 crores portfolio is 20x0.10 = Rs. 2 crores. This means that we are 95% certain that we will not make profit of less than Rs. -2 crores over the next year.

IV. The expected shortfall in returns below -10% is given by

$$E(\min(-10-X,0)) = \sum_{x<-10} (-10 - x)P(X=x)$$

$$= 0$$

On a portfolio of Rs. 20 crores, the 95% TailVaR = 0. This means expected reduction in profit below Rs. -2 crores is zero. That is, profit can not fall below Rs. -2 crores.

[9]

Solution 4 :

I. Ornstein Uhlenbeck Process

II. Consider

$Y_t = X_t e^{rt}$

Thus $d(Y_t) = d(X_t e^{rt})$

$= e^{rt} dx_t + X_t r e^{rt} dt$

$=-r x_t e^{rt} dt + e^{rt} dB_t + r x_t e^{rt} dt$

$= e^{rt} dB_t$

$Y_t = Y_0 + \int_0^t e^{rs} dB_s$

$= 0 + \int_0^t e^{rs} dB_s$

$$X_t = e^{-rt} Y_t = \int_0^t e^{-r(t-s)} dB_s$$

III. f(x,t) to be a martingale , it has to satisfy

$$Af = \frac{\partial f}{\partial t} - rx\frac{\partial f}{\partial x} + \frac{1}{2}\frac{\partial f^2}{\partial x^2}$$

Now $A(xe^{rt}) = re^{rt}x - rxe^{rt} + 0$ since $\frac{\partial f^2}{\partial x^2} = 0$

$= 0$

Therefore $X_t e^{rt}$ is a martingale

IV. $A(x^2 - a)\, e^{2rt} = 2r\,(x^2 - a)\, e^{2rt} - rx2xe^{2rt} + \frac{1}{2}\, 2\, e^{2rt}$

$= (2ra - 1)\, e^{2rt}$

$=0$

$A = \frac{1}{2r}$

$A(x^4 - bx^2 + c)\, e^{4rt} = 4r\,(x^4 - bx^2 + c)\, e^{4rt} - rx(4x^3 - 2bx)e^{4rt} + \frac{1}{2}\,(12x^2 - 2b)\, e^{4rt}$

$= x^2 e^{4rt}(-4rb + 2rb + 6) + (4rc - b)e4rt$

Thus $2rb = 6$ or $b = \frac{3}{r}$

And $4rc = b$; $c = \frac{b}{4r} = \frac{3}{4r^2}$

V. We have

$$X_t = \int_0^t e^{-r(t-s)}\, dBs$$

$$\text{Var}(X_{t1}) = \int_0^{t1} e^{-2r(t1-s)}\, dBs = \int_0^{t1} e^{-2rs}\, dBs$$

$$= \frac{1 - e^{-2rt_1}}{2r}$$ [1.5]

$$\text{Var}(X_{t2}) = \int_0^{t1} e^{-2r(t2-s)}\, dBs = \int_0^{t1} e^{-2rs}\, dBs$$

$$= \frac{1 - e^{-2rt_2}}{2r}$$

$0<t1<t2$

$$\text{Cov}(X_{t1};X_{t2}) = \int_0^{t1} e^{-r(t1-s)}\; e^{-r(t2-s)}\, dB_s$$

$$= e^{-r(t1+t_2)} \int_0^t e^{-2rs}\; ds$$

$$= \frac{e^{-r(t2-t_1)} - e^{-r(t2+t_1)}}{2r}$$

[12]

Solution 5 :

a. $E(r_M) = 18\%$, $r_f = 6\%$ and $\beta = 1.2$

Therefore, the expected rate of return is:

$$6\% + 1.2(18\% - 6\%) = 20.4\%$$

If the stock is fairly priced, then E(r) = 20.4%.

b. If r_M falls short of your expectation by 2% (that is, 16% – 18%) then you would expect the return for Infosys to fall short of your original expectation by: $\beta \times 2\% = 2.4\%$
Therefore, you would forecast a "revised" expectation for Infosys of: 20.4% – 2.4% = 18%

c. Given a market return of 16%, you would forecast a return for Infosys of 18%. The actual return is 21%. Therefore, the surprise due to firm-specific factors is 21% – 18% = 3% which we attribute to the settlement. Because the firm is initially worth Rs. 900 million, the surprise amount of the settlement is 3% of Rs. 900 million, or Rs. 27 million, implying that the prior expectation for the settlement was only Rs.18 million.

[5]

Solution 6 :

a. Consistent. Based on pure luck, half of all managers should beat the market in any year.

b. Inconsistent. This would be the basis of an "easy money" rule: simply invest with last year's best managers.

c. Consistent. In contrast to predictable returns, predictable *volatility* does not convey a means to earn abnormal returns.

d. Inconsistent. The abnormal performance ought to occur in January when earnings are announced.

e. Inconsistent. Reversals offer a means to earn easy money: just buy last week's losers.

[5]

Solution 7 :

The desirable characteristics of a model for the term structure of interest rates are

- The model should be arbitrage-free.
- Interest rates should be positive.
- Interest rates should be mean-reverting over the long term.
- Bonds and derivative contracts should be easy to price.
- The model should produce realistic interest rate dynamics.
- It should fit historical interest rate data adequately.
- It should be easy to calibrate to current market data.
- It should be flexible enough to cope with a range of derivatives.

[4]

Solution 8 :(i)

Let R(t) denote the rainfall in year t.

Then $R(t) \sim N(1150, \ 110^2)$

$Probability(s \text{ is 'Good' year}) = P(1150 - 110 \leq R(s-1) \leq 1150 + 110)$

$= P(1040 \leq N(1150, 110^2) \leq 1260) = 68.27\%$

(ii) Let I(t) denote the force of inflation in year t.

Wilkie's updating for the force of inflation I(t) is

$I(t) = QMU + QA[I(t-1) - QMU] + QSD.QZ(t)$

where:

- QMU represents the long-run mean value of I(t)
- QA is the autoregressive parameter – the "speed" at which I(t) reverts to the long-run mean value
- QSD is the standard deviation of error terms – the magnitude of the influence of random fluctuations
- QZ(t) is a series of independent and identically distributed standard normal variables

(iii) Since rainfall in 2011 was within one standard deviation of mean, 2012 is a good year.

In a good year the force of inflation follows Wilkie's updating equation.

$I(2012) = 5\% + 0.9[I(2011) - 5\%] + 2\%.N(0,1) = 0.9I(2011) + N(0.5\%, 0.04\%)$

$I(2012) \sim N(7.7\%, 0.04\%)$

$\therefore P[I(2012) < 10\%] = P[N(7.7\%, 0.4\%) < 0.1] = 87.49\%$

(iv) We know that 2012 was a good year. Following are the possible combinations for the years running up to 2013

	Year			
Scenario	***2012***	***2013***	***2014***	***Probability***
1	Good	Good	Good	0.6827*0.6827 = 46.61%
2	Good	Bad	Good	(1-0.6827)* 0.6827 = 21.66%
3	Good	Good	Bad	0.6827*(1-0.6827) = 21.66%
4	Good	Bad	Bad	(1-0.6827)* (1-0.6827) = 10.07%

Under Scenario 1 (S1) we have

$I(2014)|S1 = 0.9I(2013) + N(0.5\%, 0.04\%)$

$= 0.9\left[0.9I(2012) + N(0.5\%, 0.04\%)\right] + N(0.5\%, 0.04\%)$

$= 0.81N(7.7\%, 0.04\%) + 0.9\ N(0.5\%, 0.04\%) + N(0.5\%, 0.04\%)$

$= N\left(0.81 * 7.7\% + 0.9 * 0.5\% + 0.5\%, 0.81^2 * 0.04\% + 0.9^2 * 0.04\% + 0.04\%\right)$

$I(2014)|S1 = N(7.187\%, 0.098644\%)$

$\therefore P[I(2014)|S1 < 0] = P[N(7.187\%, 0.98644\%) < 0] = 1.11\%$

Under Scenario 2 (S2)

$I(2014)|S2 = 0.9I(2013) + N(0.5\%, 0.04\%)$

$= 0.9N(0.1, 0.01) + N(0.5\%, 0.04\%)$

$I(2014)|S2 = N(9.5\%, 0.85\%)$

$\therefore P[I(2014)|S2 < 0] = P[N(9.5\%, 0.85\%) < 0] = 15.14\%$

If 2014 is a "Bad" year (scenario 3 and 4) then I(2014) is independent of past inflation and

$I(2014)|(S3\ or\ S4) \sim N(10\%, 1\%)$

$\therefore P\left[I(2014)|(S3\ or\ S4) < 0\right] = P\left[N(10\%, 1\%)\right] = 15.87\%$

By multiplying the respective probabilities we get

$P[I(2014)|(I(2011) = 8\%\ and\ 2012\ is\ a\ good\ year)] < 0] = 8.8\%$

[17]

Solution 9 :

(i) The Intrinsic Value of an option is the value assuming expiry of the option immediately rather than at some time in the future. For a put option the intrinsic value at time t is:

$\max(K - S_t, 0)$

The *Time Value* of an option is defined as the excess of an option's value over its intrinsic value. It primarily represents the value of the *choice* that the option provides to its holder. For a put option the intrinsic value at time t is Total Value – Intrinsic Value.

Theta measures the sensitivity of the price of an option to changes in time.

$$\Theta = \frac{\partial f}{\partial t}$$

(ii) To estimate the profit or loss on the portfolio we need to calculate the value of the portfolio at time 0, V(0), and at time 3/12, V(3/12)
At time 0 we know the following

$$S_0 = 100;\ K = 100;\ r = 5\%;\ \sigma = 15\%$$

Using Black-Scholes formula we get:

$$V(0) = 2.3928 - 3.0581 = -0.6653$$

We are given that the value of the parameters remain unchanged in three months time.

Since 3 months have passed and the price of the security is still 100, the 3 month put option expires with no liability.
However, the other put option still has 3 months to expiry and has some Time Value left. The value of the portfolio at time 3 is equal to the value of the yet to expire put option.
(Candidates can simply use the value of 3 months to expiry option from the calculation above)

$$V\left(\frac{3}{12}\right) = 2.3298$$

Overall profit from the strategy = 2.3298 – 0.6653 = 1.7276
(Ignoring any loss of interest. Award full marks if candidates allow for interest on initial outlay for three months at the risk free rate.)

(iii) The intrinsic value for at the money options (both options in this case) is zero. Hence the entire value of the option is due to Time Value. As the option approaches maturity the time value approaches to zero, all else being equal.

The near to maturity (3months) put option's (time) value decreases at a rate (Time decay) faster than the far from maturity (6 months) put option. This difference in rate of decrease could result in profit as demonstrated in calculation above.

(The above explanation is one of many different ways in which the emergence of profit can be explained. Award marks for other reasonable explanations)

[13]

Solution 10 : (i) Let A(t) be the value of the company's assets at time t.

Let X(5) be the payment to Senior Debt holders at time 5. It can be represented as

(working in units of millions hereafter, unless otherwise specified)

$$X(5) = \begin{Bmatrix} 5 \text{ if } A(5) \geq 5 \\ A(5) \text{ if } A(5) < 5 \end{Bmatrix}$$

Equivalently,

$$X(5) = \min\{A(5), 5\} \text{ or}$$
$$X(5) = 5 - \max\{0, 5 - A(5)\}$$

The second term in the above equation is the payoff for a European put option on A(t) maturing at time 5 with strike price 5, say, Put(5,5).

Hence, the value of the Senior Debt is
$$Price_{X(5)}(0) = Present\ value\ of\ 5 - Price_{Put(5,5)}(0)$$

(ii) Similarly, let Y(5) be the payment to Subordinated debt holders at time 5. It can be represented as

$$X(5) = \begin{Bmatrix} 8 \text{ if } A(5) \geq 13 \\ A(5) - 5 \text{ if } 13 < A(5) \leq 5 \\ 0 \text{ if } A(5) < 5 \end{Bmatrix}$$

Equivalently,

$$X(5) = \min\{13, A(5)\} - \min\{5, A(5)\} \text{ or}$$

$$X(5) = 13 - \max\{0, 13 - A(5)\} - 5 + \max\{0, 5 - A(5)\}$$
$$X(5) = 8 - \max\{0, 13 - A(5)\} + \max\{0, 5 - A(5)\}$$

The last two terms in the above equation correspond to the payoff for European put options on A(t) maturing at time 5 with strike price of 13 and 5 respectively.

Hence, the value of the Subordinated debt is
$$Price_{Y(5)}(0) = Present\ value\ of\ 8 - Price_{Put(5,13)}(0) + Price_{Put(5,5)}(0)$$

(iii) The price of call options can be calculated using the Black Scholes formula:

We are given the following parameters

$$A(0) = 17, r = 5\%; \ \sigma = 30\%$$

Applying the Black-Scholes formula for option price we get
$$Price_{Put(5,5)}(0) = 0.0257; \ Price_{Put(5,13)}(0) = 1.0743$$
The fair price of the Senior debt is

$Price_{X(5)}(0) = Present\ value\ of\ 5 - Price_{Put(5,5)}(0)$

$Price_{X(5)}(0) = 5e^{-5\times5\%} - 0.0257 = 3.8683$ or 77.37 per 100 nominal

The fair price of the subordinated debt is

$Price_{Y(5)}(0) = Present\ value\ of\ 8 - Price_{Put(5,13)}(0) + Price_{Put(5,5)}(0)$

$8e^{-0.05\times5} - 1.0743 + 0.0257 = 5.1819$ or 64.77 per 100 nominal

Fair price of equity is the total value of assets less the value of senior and subordinated debt

$i.e.\ 17 - 5.1819 - 3.8683 = 7.9499$ or 39.75 per 100 nominal

(iv) The implied yield on a non interest paying debt can be calculated by solving the following equation

$Price = Maturity\ Proceed\ e^{-Time\ to\ Maturity \times Implied\ Yield}$

Hence the implied yield on Senior debt is 5.13% and the credit spread is 13 basis points.

Similarly, the implied yield on subordinated debt is 8.69% and the credit spread is 369 basis points.

[16]

[TOTAL MARKS – 100]

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