Institute of Actuaries of India

Subject CT5 – General Insurance, Life & Health Contingencies

May 2012 Examinations

INDICATIVE SOLUTIONS

Solution 1 :

(a) $a_{[50]}^{(2)} = a_{[50]} + ¼$

$= (ä_{[50]} - 1) + ¼$
$= (17.454 - 1) + ¼$
$= 16.704$

(b) $_{10|10}q_{[60]+1} = (l_{71} - l_{81}) / (l_{[60]+1})$
$= (7854.4508 - 4901.4789) / 9209.6568$
$= 0.32064$

c) $_{5|}a_{60} = a_{60} - a_{60:\overline{5|}}$

$= (ä_{60} - 1) - (ä_{60:\overline{5|}} - 1 + v^5 \; _5p_{\overline{60}})$

$= (14.134 - 1) - (4.550 - 1 + (1/1.04)^5 * (8821.2612 / 9287.2164)$

$= 13.134 - 4.331 = 8.803$

Alternative approach:

$_{5|}a_{60} = v^5 \; _5p_{60} \; a_{65}$

$= 1.04^{-5} * (8821.2612 / 9287.2164) * (12.276 - 1)$ where $a_{65} = ä_{65} - 1$

$= 8.803$

[3]

Solution 2 :

$$48 = 1000\int_0^1 e^{-(\mu+0.04)t}(\mu+0.04)dt = 1000\left(1 - e^{-(\mu+0.04)}\right)$$

$e^{-(\mu+0.04)} = 0.952$

$\mu = 0.009$

We require ,$1000\int_3^4 e^{-0.049t}(0.009)dt$

$$= 1000\frac{0.009}{0.049}\left(e^{-(0.049)(3)} - e^{-(0.049)(4)}\right) = 7.6$$

[5]

Solution 3 :

$$\overline{\text{K}}\bar{a}_{xy} + K(\bar{a}_x - \bar{a}_{xy}) + 0.5K(\bar{a}_y - \bar{a}_{xy}) = 10{,}000\bar{A}_{\overline{xy}}$$

But, $\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy}$

Now, $\bar{A}_x = 1 - \delta\bar{a}_x = 0.4$, $\bar{A}_y = 1 - \delta\bar{a}_y = 0.25$, $\bar{A}_{xy} = 1 - \delta\bar{a}_{xy} = 0.5$

Hence, $K = \frac{0.4+0.25-0.5}{10+(12-10)+0.5(15-10)}$ * 10,000

= 103.45

[4]

Solution 4 :

(i) Class Selection

Each group of lives is specified by a category or class of a particular characteristic of the population, eg sex with categories of male and female. The stochastic models are different for each class. There are no common features to the models, they are different for all ages.

Examples:

- Different races have different susceptibilities to disease
- Individuals who have lived abroad away may have been exposed to tropical diseases

Comment: Only one example was required. Any other valid example was given full marks

(ii) Adverse Selection

Adverse selection is characterized by the way in which the select groups are formed rather than by the characteristics of those groups. Adverse selection usually involves an element of self-selection, which acts to disrupt (act against) a controlled selection process which is being imposed on the lives. This adverse selection tends to reduce the effectiveness of the controlled selection.

Example:

Individuals who purchase an annuity at retirement are more likely to be in good health than the general population. If these individuals thought that they were likely to die in the near future they would not convert a capital lump sum into a lifetime annuity as this would represent a poor investment.

[5]

Solution 5 :

(a) *Accumulated value* = $100{,}000 * \ddot{a}_{30:\overline{30|}} * (1.06)^{30} \,/\, (l_{60}/\, l_{30})$

= $100{,}000 * 14.435 * (1.06)^{30} / (9287.2164 / 9925.2094)$

= Rs 8,860,268

(b) *Let P be the monthly annuity. Then:*

Expected Present Value = $12 * P * \left(\ddot{a}_{\overline{5|}}^{(12)} + v^5 \;{}_{5|}\ddot{a}_{65}^{(12)}\right)$

= $12 * P * \left[\ddot{a}_{\overline{5|}}^{(12)} + v^5 \;{}_5p_{60} \; * \left(\ddot{a}_{65} - \frac{11}{24}\right)\right]$

= 12P [(1 - 0.74726)/0.058128 + 0.74726 * (8821.2612/9287.2164) * (10.569 – 11/24)]

= 12 P (4.34799 + 7.17623)

= 138.29 P

Using equivalence principle,

⇨ P = 5,000,000 / 138.29 = Rs 36,155

[7]

Solution 6 :

(a) (2/3) (30,000) (s_{40}/s_{39}) $({}^{z}M_{40}^{ia} / {}^{s}D_{40})$

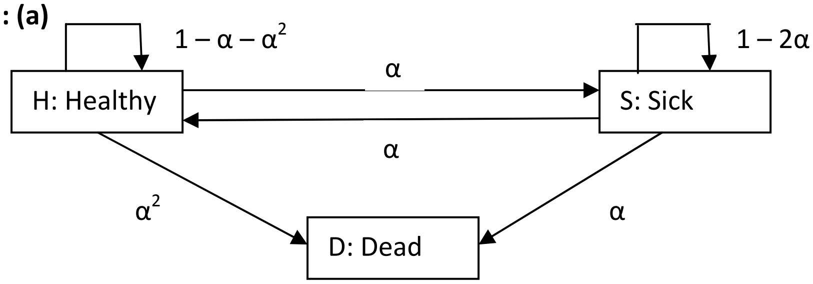
= (2/3) (30,000) (7.814/7.623) (58,094/25,059) = Rs. 47,527.5

(b) (30,000/80) (s_{40}/s_{39}) $(18\ {}^{z}M_{40}^{ra} + {}^{z}\overline{R}_{40}^{ra} - {}^{z}\overline{R}_{62}^{ra}) / {}^{s}D_{40}$

= (2/3) (30,000) (7.814/7.623) (18 * 128,026 + 2,884,260 – 159,030) / 25,059 = Rs. 77,153.7

[5]

Solution 7 : (a)



(b) Transition probabilities must lie in [0, 1]. Thus we need $\alpha \geq 0$, $1 - 2\alpha \geq 0$ and $1 - \alpha - \alpha^2 \geq 0$.

Solution of the quadratic is the interval $[-1/2 - \sqrt{5}/2, -1/2 + \sqrt{5}/2]$, thus for all the conditions to be satisfied simultaneously, α must lie in the range [0, 1/2].

(c) Required Probability = $P_{HH}P_{HS} + P_{HS}P_{SS}$

= 0.76 * 0.2 + 0.2 * 0.6 = 0.272

(d) Benefit reserve at the end of Year 2 if the policyholder is in Healthy (H) state is

$= (100000 * \alpha^2 + 75000 * \alpha) / (1 + i)$

= Rs. 17,924.5

[7]

Solution 8 :

(a)

The "death strain at risk" for a policy for year $t + 1$ (t = 0, 1, 2) is the excess of the sum assured (i.e. the present value at time $t + 1$ of all benefits payable on death during the year $t + 1$) over the end of year provision.
i.e. DSAR for year $t + 1 = S - {}_{t+1}V$

The "expected death strain" for year $t + 1$ (t = 0, 1, 2) is the amount that the life insurance company expects to pay extra to the end of year provision for the policy.
i.e. EDS for year $t + 1 = q(S - {}_{t+1}V)$

The "actual death strain" for year $t + 1$ (t = 0, 1, 2) is the observed value at $t+1$ of the death strain random variable
i.e. ADS for year $t + 1$ $= (S - {}_{t+1}V)$ if the life died in the year t to $t + 1$
$= 0$ if the life survived to $t + 1$

(b) (i) Let P be the net premium for the policy payable annually in advance. Then:

$$P\,\ddot{a}_{30} = 200{,}000 * A_{30}$$

$$\Rightarrow \quad P = 200{,}000 * 0.16023 / 21.834 = \text{Rs } 1467.7$$

Net premium reserve at the end of 10^{th} policy year is

$$\Rightarrow \quad {}_{10}V = 200{,}000\, A_{40} - 1467.7\, \ddot{a}_{40}$$
$$= 200{,}000 * 0.23096 - 1467.7 * 20.005$$
$$= \text{Rs } 16{,}750.7$$

(ii) Death Strain at Risk = 200,000 – 16750.7

= 183,249.3

EDS = 689 * q_{39} * 183,249.3

= 689 * 0.000870 * 183,249.3

= 109,845.1

ADS = 1 * 183,249.3 = 183,249.3

Mortality Profit = EDS – ADS

= 109,845.1 – 183,249.3

= Rs -73,404.1 (ie a loss)

[14]

Solution 9 :

(a) Let P be the annual premium. Then:

EPV of premiums = $P * \ddot{a}_{35:\overline{25|}}$

= 13.390 P

EPV of benefits = 100,000 * [$q_{35} * v + {}_{1|}q_{35} * (1+b) * v^2 + ... + {}_{24|}q_{35} (1+b)^{24} * v^{25}$]

+ 100,000 * $(1+b)^{25} * v^{25} * {}_{25}p_{35}$

= $\frac{100,000}{(1+b)}$ * [$q_{35} * (1+b) * v + {}_{1|}q_{35} * (1+b)^2 * v^2 + ... + {}_{24|}q_{35} (1+b)^{25} * v^{25}$]

+ 100,000 * $(1+b)^{25} * v^{25} * {}_{25}p_{35}$

= $\frac{100,000}{(1+b)} * A^1_{35:\overline{25|}}$ @ i' + + 100,000 * $v^{25} * {}_{25}p_{35}$ @ i'

Where (1+i') = (1+i)/(1+b) =1.04 = > I' = 4%

= $\frac{100,000}{1.0192308}$ * [$A_{35:\overline{25|}} - v^{25} * {}_{25}p_{35}$] + 100,000 * $v^{25} * {}_{25}p_{35}$ @ 4%

= $\frac{100,000}{1.0192308}$ * (0.38359 – 0.37512 * $\frac{9287.2164}{9894.4299}$) + 100,000 * 0.37512 * $\frac{9287.2164}{9894.4299}$

= 3,089.66 + 35,209.92

= Rs 38,299.58

EPV of expenses (@6%)

$= 0.09\,P + 800 + 0.05P * (\ddot{a}_{35:\overline{25|}} - 1) + 400 * A^{1}_{35:\overline{25|}} + 200 * v^{25} *{}_{25}p_{35}$

$= 0.09\,P + 800 + 0.05P * (13.390-1) + 400 * [A_{35:\overline{25|}} - v^{25} *{}_{25}p_{35}] + 200 * v^{25} *{}_{25}p_{35}$

$= 0.09\,P + 800 + 0.05\,P * 12.390 + 400 * (0.24208 - 0.23300 * \frac{9287.2164}{9894.4299}) + 200 * 0.23300 * \frac{9287.2164}{9894.4299}$

= 0.09 P + 800 + 0.6195 P + 9.3516 + 43.7402

= 0.7095 P + 853.0918

Equation of value gives:

EPV of premium = EPV of benefits + EPV of expenses

= > 13.390 P = 38,299.58 + 0.7095 P + 853.0918

= > 12.6805 P = 39,152.67

= > P = Rs 3087.6

(b) Gross prospective policy value

$V^{prospective} = \frac{130{,}000}{(1+b)} * A^{1}_{45:\overline{15|}}$ @ I''

$+ 130{,}000 * v^{15} *{}_{15}p_{45}$ @ I''

$+ 0.05\,P * \ddot{a}_{45:\overline{15|}} - P * \ddot{a}_{45:\overline{15|}} + 400 * A^{1}_{45:\overline{15|}} + 200 * v^{15} *{}_{15}p_{45}$

$= \frac{130{,}000}{(1+b)} * [A_{45:\overline{15|}} - v^{15} *{}_{15}p_{45}]$ @ 4%

$+ 130{,}000 * v^{15} *{}_{15}p_{45}$ @ 4%

$- 0.95P * \ddot{a}_{45:\overline{15|}}$ @6%

$+ 400 * [A_{45:\overline{15|}} - v^{15} *{}_{15}p_{45}]$ @6%

$+ 200 * * v^{15} *{}_{15}p_{45}$ @ 6%

$= \frac{130{,}000}{1.0192308} (0.56206 - 0.55526 * \frac{9287.2164}{9801.3123})$

$+ 130,000 * 0.55526 * \frac{9287.2164}{9801.3123}$

$- 0.95 * 3087.6 * 10.149$

$+ 400 * (0.42556 – 0.41726 * \frac{9287.2164}{9801.3123})$

$+ 200 * 0.41726 * \frac{9287.2164}{9801.3123}$

= 4582.05 + 68,396.9 + 29,769.25 + 12.07 + 79.07

= Rs 43,300.84

[11]

Solution 10 :

(a) Since Surrender Value (SV) is to be determined prospectively, the recursive formula which can be used to determine the SV at time t is:

$(SV_t + P_{t+1}) * (1 + i) = Exp_{t+1} * (1 + i) + q_{t+1} * SA_{t+1} + (1 – q_{t+1}) * SV_{t+1}$

We have boundary condition, SV_5 = 100,000 since at the end of exact 5 years i.e. at maturity, no future expenses or future premiums or future death benefit is to be incurred except the maturity benefit payable on the next day.

Substituting the required values in the above equation, we get

$(SV_4 + 25000) * (1 + 8.5\%) = 200 * (1 + 8.5\%) + 0.0018 * 100000 + (1 – 0.0018) * SV_5$

SV_4 = 67,365.9

Similarly, $(SV_3 + 25000) * (1 + 8.5\%) = 200 * (1 + 8.5\%) + 0.0016 * 100000 + (1 – 0.0018) * SV_4$

SV_3 = 37,336.5

SV_1 and SV_2 are not required as there is no surrender value before the end of third policy year.

Paid-up value at the end of 3rd year = 3/5 * 100000 = 60,000

Paid-up value at the end of 4th year = 4/5 * 100000 = 80,000

Paid-up value at the end of 5th year = 5/5 * 100000 = 100,000

End of Year	Surrender Value (1)	Paid-up Value (2)	Surrender value scale = (1)/(2)
3	37,336.5	60,000	62.2%

4	67,365.9	80,000	84.2%
5	100,000	100,000	100.0%

(b) (i) There will be no impact on surrender value as 7% p.a. for the first two years is irrelevant since there is no surrender value before the end of the third policy year.

(ii) Increase in expense will tend to increase the surrender value while increase in discount rate will tend to reduce the surrender value. Surrender value may hence increase or decrease.

[10]

Solution 11 :

(a) Let P be the single premium. Then:

P = Present value of the benefits

$= (100{,}000 + 5{,}000) * A^{1}_{[40]:\overline{5}|} - 5{,}000 * (IA)^{1}_{[40]:\overline{5}|}$

$= 105{,}000 * [\ A_{[40]} - v^5 * \frac{l_{45}}{l_{[40]}} * A_{45}\ ]$

$- 500{,}000 * [\ (IA)_{[40]} - v^5 * \frac{l_{45}}{l_{[40]}} * (5 * A_{45} + (IA)_{45}\ )\ ]$ where $v^5 = 0.82193$

=> $P = 105{,}000 * (0.23041 - 0.82193 * \frac{9801.3123}{9854.3036} * 0.27605)$

$- 5{,}000 * [7.95835 - 0.82193 * \frac{9801.3123}{9854.3036} * (5 * 0.27605 + 8.33628)]$

$= 105{,}000 * 0.004736 - 5{,}000 * 0.014989$

= Rs 422.3

(b) $P = 100{,}000 * q_{[x]} * v + 95{,}000 * {}_{1|}q_{[x]} * v^2 + 90{,}000 * {}_{2|}q_{[x]} * v^3$

$+ 85{,}000 * {}_{3|}q_{[x]} * v^4 + 80{,}000 * {}_{4|}q_{[x]} * v^5$ where x = 40

Now, $q_{[40]} = 0.000788$, $p_{[40]} = 0.999212$

$q_{[40]+1} = 0.000962$, $p_{[40]+1} = 0.999038$, ${}_2p_{[40]} = p_{[40]}.p_{[40]+1} = 0.998251$

$q_{42} = 0.001104$, $p_{42} = 0.998896$, ${}_3p_{[40]} = {}_2p_{[40]} * p_{42} = 0.997149$

$q_{43} = 0.001208$, $p_{43} = 0.998792$, ${}_4p_{[40]} = {}_3p_{[40]} * p_{43} = 0.995944$

q_{44}=0.001327   and v = 1/1.04 = 0.96154

= >   P   = 100,000 * (0.000788 * 0.96154 + 0.95 * 0.999212 * 0.000962 * 0.924555

+ 0.90 * 0.998251 * 0.001104 * 0.888995

+ 0.85 * 0.997149 * 0.001208 * 0.854803

+ 0.80 * 0.995944 * 0.001327 * 0.821925)

= 100,000 * (0.000758 + 0.000844 + 0.000882 + 0.000875 + 0.000869)

= Rs 422.8 which is equal to value calculated by formulas (the slight difference in answers can be due to rounding)

c) Let y be the initial premium. Then:

= >   422.3   = (y + 10) $* \ \ddot{a}_{[40]:\overline{5}|}$ - 10 $* \ (I\ddot{a})_{[40]:\overline{5}|}$

= (y+10) * ($\ddot{a}_{[40]- v5*\frac{l45}{l[40]}} * \ddot{a}_{45})$ - 10 * [$(I\ddot{a})_{[40]}$ - $v^5 * \frac{l_{45}}{l_{[40]}} *$ (5 * $\ddot{a}_{45}$ + $(Ia)_{45}$)]

= (y+10) (20.009 – 0.82193 * 0.99462 * 18.823)

- 10 [313.323 – 0.82193 * 0.99462 * (5 * 18.823 + 272.647)]

= (y+10) * 4.62105 – 10 * 13.492

= 4.62105 y - 88.7095

= > 4.62105 y = 511.01

= > y = Rs 110.58

[13]

Solution 12 :

(a) Weighted unit growth rate = equity proportion * equity return + debt proportion * debt return

Before the stress

Weighted unit growth rate = 70% * 10% + 30% * 6% = 8.8% per annum

After the stress

Debt value = 30% * 100,000 = 30,000

Equity value before the stress = 70% * 100,000 = 70,000

Equity value after the stress = (1 – 20%) * 70,000 = 56,000

Value of unit fund after the stress = 30,000 + 56,000 = 86,000

Proportion of equity = 56,000 / 86,000 = 65.1%

Proportion of debt = 100% – 65.1% = 34.9%

Weighted unit growth rate = 65.1% * 8% + 34.9% * 6% = 7.3% per annum

(b)

Year	InForce at start of year	Mortality rate	Deaths	InForce at end of year
1	1.0000	0.0015	0.0015	0.9985
2	0.9985	0.0020	0.0020	0.9965
3	0.9965	0.0025	0.0025	0.9940

Before the stress

Year	UF at start of year	Premium	Allocation Charge	UF after charge	UF after growth	FMC	UF at end of year
	(1)	(2)	(3) = (2)*3%	(4) = (1)+(2)–(3)	(5) = (4)*(1+8.8%)	(6) = (5)*1%	(7) = (5)-(6)
1	100,000	25,000	750	124,250	135,184	1,352	133,832
2	133,832	25,000	750	158,082	171,993	1,720	170,273
3	170,273	25,000	750	194,523	211,642	2,116	209,525

Year	Allocation Charge	FMC	Expense	Commission	Interest	Profit	Profit InForce
	(8) = (3)	(9) = (6)	(10)	(11) = (2)*2%	(12) = [(8)-(10)-(11)]*6%	(13) = (8)+(9)-(10)-(11)+(12)	(14) = (13)*IFstart
1	750	1,352	100	500	9	1,511	1,511
2	750	1,720	100	500	9	1,879	1,876
3	750	2,116	100	500	9	2,275	2,267

NPV @12.5% pa = $1511v + 1876v^2 + 2267v^3 = 4,418$

After the stress

Year	UF at start of year	Premium	Allocation Charge	UF after charge	UF after growth	FMC	UF at end of year
	(1)	(2)	(3) = (2)*3%	(4) = (1)+(2)–(3)	(5) = (4)*(1+7.3%)	(6) = (5)*1%	(7) = (5)-(6)
1	86,000	25,000	750	110,250	118,301	1,183	117,118
2	117,118	25,000	750	141,368	151,691	1,517	150,174
3	150,174	25,000	750	174,424	187,161	1,872	185,289

Year	Allocation Charge	FMC	Expense	Commission	Interest	Profit	Profit InForce
	(8) = (3)	(9) = (6)	(10)	(11) = (2)*2%	(12) = [(8)-(10)-(11)]*6%	(13) = (8)+(9)-(10)-(11)+(12)	(14) = (13)*IFstart
1	750	1,183	100	500	9	1,342	1,342
2	750	1,517	100	500	9	1,676	1,673
3	750	1,872	100	500	9	2,031	2,024

NPV @12.5% pa = $1342v + 1673v^2 + 2024v^3 = 3,936$

(c) The weighted unit-growth rate will decrease as money market return is less than the equity return. The projected unit-fund will hence decrease and so will the fund management charge. The net present value will hence reduce.

(d) Unit fund will fluctuate if it is invested in equities. Unit fund with zero equity investment will thus have no impact to movements in equity market and hence no impact on profit.

Even if the unit fund is invested in equities, having a charging structure which is not linked to unit fund will have no impact on profit. Example, having no fund management charge.

[16]

[TOTAL MARKS – 100]

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