INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

31st May 2012

Subject ST6 — Finance and Investment B

Time allowed: Three hours (9.45* – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. **** You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.***

3. ***You must not start writing your answers in the answer sheet until instructed to do so by the supervisor***

4. ***The answers are not expected to be any country or jurisdiction specific. However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***

5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

6. ***Mark allocations are shown in brackets.***

7. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) Prove that the cost of setting up a bull spread using call options is higher than the cost of setting up the bull spread using put options. How does it relate to the profit?

[5]

Q. 2) Rakesh, portfolio manager of HDFC Mutual Funds Limited, plans to use Treasury-bond futures contracts to hedge a bond portfolio against the fluctuation in market interest rates over the next six months. The current market value of the portfolio is Rs. 500 million. The portfolio will have a duration of 5 years at end of six months. The current quoted futures price is 120 (the quoted price is for a bond with a face value of Rs. 100), and each future is on Rs. 500,000 of bonds. The cheapest to deliver bond for the Treasury-bond futures contract is expected tobe a 10-year 10% per annum coupon bond which will have a duration of 8 years at the maturity of the futures contract.

a) What position in futures contracts is required to hedge the portfolio against the fluctuation in market interest rates? (2)

b) What adjustments to the hedge are required if after one month the bond that is expected to be the cheapest to deliver changes to one with a duration of 6 years? (2)

c) Assume that all interest rates increase over the next 6 months, but short-term and medium-term rates increase more than the long-term rates. What is the effect of this increase in market interest rates on the performance of the hedge? (2)

[6]

Q. 3) Assume that the price of USD (US dollar) in terms of the price of INR (Indian rupee) follows the process:

$$dP_\$ = (r_I - r_{US})P_\$ dt + \sigma P_\$ dz$$

Where dz is the wiener process, σ is the volatility of the price of USD (in terms of INR), r_{US} is the risk-free interest rate in the US and r_I is the risk-free interest rate in India. Please derive the process followed by the prices of INR expressed in terms of price of USD? Please comment on your result.

[6]

Q. 4) Consider two future dates T0, T1 with T0 < T1. A forward start call option is a contract in which the holder receives at time T0, at no extra cost, a call option with expiry date T1 and strike price equal to ST0 (the asset price at time T0). Assume that the stock price evolves according to a standard two-period binomial model, assume that $T0 = \frac{T1}{2}$ and the asset price at time T0 is either S_0u or S_0d, and at time T1 is one of S_0u^2, S_0ud and S_0d^2 with $d \le 1 < \min\{e^{rT0}, e^{r(T1-T0)}\} \le \max\{e^{rT0}, e^{r(T1-T0)}\} < u$, where r denotes the risk-free interest rate.

Find the fair price of such a call option at time zero. Comment on your result.

[7]

Q. 5) **a)** What is the swap rate for a yearly paying swap with 5 year term and Rs 10,000 as notional. The zero coupon spot rate for the first year is given as 8% and decreasing by 0.5% for each following year. (2)

b) An insurance company has a liability of duration 30 years and the longest available bond in the market is of duration 20 years. How can a company match the payouts for the above durations using a combination of swaps and available long duration bonds? (2)

c) An insurance company calculates its future liability using gilt yield at YE 11. This liability is matched by holding receiver swaps and cash as assets for pay offs. What will happen to the balance sheet if gilt yield remain the same and the spread between the gilt yield and the swap rate increases (swap spread -> swap rate – gilt yield). (2)

d) Design a hedging strategy using forward swaps and forward gilts contracts to reduce the variability of the balance sheet due to swap spreads in the next valuation (1 year from now). Show the impact on the following instruments due to changes in the swap rates in the table given below:

Impact on…	**increase in swap spread**	**decrease in swap spread**
Hedge		
Liability		
Receiver swap		

(5)

[11]

Q. 6) **a)** Define synthetic CDOs and explain what are attachment points. (2)

b) We assume that the reference portfolio consists of N names, each of equal notional amount A and recovery rate δ. Assuming there are j defaults at time t,write the equation for the loss in the tranche at time t with attachment points $Lower_i$ and $Upper_i$.

Write down the equation for the risk neutral expected loss at time t in terms of the joint default probability($P_{default}(j,T_i)$). Use this to derive an expression for the present value of the contingent payout. Now assuming a premium for default risk in a tranche is paid only for the non-defaulted amount left for that tranche, and the CDO is priced as zero at t=0. Calculate the value of this premium in terms of $P_{default}(j,T_i)$. (7)

c) Describe a way of estimating this joint distribution of probability. What is the key factor to consider. (2)

[11]

Q. 7) Suppose that yield Y on a zero-coupon bond follows the process:

$$dY = mdt + \sigma dz$$

where m and σ are functions of Y and t, and dz is a wiener process.
Show that the volatility of the zero-coupon bond price declines to zero as it approaches to maturity.

[3]

Q. 8) **a)** (W_t stands for Brownian motion) Show that given t_0, t_1, t_2,…. t_n and W_{t_0}, W_{t_1},….. W_{t_n} we can always write

$$\sum_{j=1}^{n}\left[t_j W_{t_j} - t_{j-1}W_{t_{j-1}}\right] = \sum_{j=1}^{n}\left[t_j(W_{t_j} - W_{t_{j-1}})\right] + \sum_{j=1}^{n}\left[(t_j - t_{j-1})W_{t_{j-1}}\right]$$

How is this different from the standard formula for the differentiation of the products: d(uv) = u dv + v du (4)

b) Use the above to show that

$$\int_0^t s dW_s = tW_t - \int_0^t W_s ds$$

(3)

c) In the above equation there are two integrals. Which one is defined in the sense of Ito only? (1)

[8]

Q. 9) **a)** Define 'chooser option'. What would be the value of chooser option at the time (t) when the chooser option can be exercised? (2)

b) Let the T be the expiration date of the underlying call/put options, S_t be stock price at time t and K the strike price. Let the value of the call option be as given below:

$C(S_t,t)=e^{-r(T-t)}E\left[\left(\max\left(S_T\text{-}K,0\right)|\mathcal{F}_t\right)\right]$ where C is the value of the call, r is the risk free interest rate and $\mathcal{F}_t$ represents the filtration (information until t).

Similarly let $P(S_t,t)$ be the value of the put option and $H(S_t,t)$ be the value of the chooser option.

Using the above show that

$$C(S_t,t)\text{-}P(S_t,t)=S_t - e^{-r(T-t)}K$$

Now, show that the value of the chooser option is given by (3)

$$H(S_t,t)=\max\langle C(S_t,t),\ C(S_t,t) + e^{-r(T-t)}K\text{-}S_t\rangle$$

c) Therefore, show that the option price at time = 0 would be given by $H(S,0)=C(S,0) + e^{-rT}E\left[\max\langle K - S\, e^{rT}\, e^{\sigma W_t - \frac{1}{2}\sigma^2 t}\rangle\right]$ where S is the underlying price observed at time 0. (4)

d) Now use the Girsanov theorem to evaluate the expectation in the above formula. (3)

e) Write down the final formula for valuing the chooser option. (3)

[15]

Q. 10) **a)** Define 'numeraire' associated with a probability measure. What is the numeraire associated with the risk neutral measure? (2)

b) Write equation of the forward rate at time t between time T and S in terms of a bond price, P(t,T/S) (1)

c) Obtain a stochastic differential equation for F(t:T,S) with P(t,S) as numeraire in a form where F(t;T,S) follows a geometric Brownian motion. How does this equation change under the risk neutral measure? (6)

d) Which interest rate model does the above equation represent and what are the advantages and disadvantages of the above model? (3)

[12]

Q. 11) A bank has a portfolio of options on an asset. The delta of the options is -30 and the gamma is -5. Explain how these numbers can be interpreted. The asset price is 20 and its volatility is 1% per day. Using the quadratic model, calculate the first three moments of the change in the portfolio value. Calculate a 1-day 99% VAR using the first two moments and using the first three moments.

[6]

Q. 12) **a)** Suppose you are allowed to short-sell and there are no transaction costs and no margins though bid-offer spread exists. Assume you have the following information

i) USD 1 year interest rate 3%/3.1%

ii) INR 1 year interest rate 9%/9.25%

iii) Spot rate USD/INR 50.00/50.10 and 1 year Forward Rate 52.25/52.3

What transactions should you be making in the market? What changes would take place in the market, due to your actions? (5)

b) Suppose the forward rate moves to 52.77/52.87 as a consequence of your actions and there are no other changes in the market. Now you receive a request to invest 1 mn USD in the money markets for a duration of one year. Explain two approaches that you could use to make this investment and explain the return you would expect under each approach. (3)

c) Comment on your results obtained from the above and give reasons why such arbitrage opportunities may persist in the market. (2)

[10]

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