

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

28th May 2012

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*
5. *Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) a) If the nominal rate of interest convertible quarterly is 8% per annum calculate the value of

i) i (1)

ii) $\ddot{s}_{\overline{5}|}^{(12)}$ (2)

b) Prove from first principles (i.e. using integration method)_that

$$(\overline{Ia})_{\overline{n}|} = (Ia)_{\overline{n}|} * \left(\frac{\overline{a}_{\overline{n}|}}{a_{\overline{n}|}} \right) \quad (5)$$

[8]

Q. 2) An investor wants to purchase a financial instrument which will provide her with a continuous payment at time t of Rs $(500 * \exp(0.1t+0.001t^2))$ p.a. from time $t = 2$ to $t = 4$ and a payment of Rs 15,000/- at time $t = 6$.

The underlying force of interest, $\delta(t)$, is a function of time t and is given by

$$\delta(t) = \begin{cases} 0.07 + 0.002t & \text{for } 0 < t \leq 5 \\ 0.085 - 0.001t & \text{for } 5 < t < 10 \end{cases}$$

Calculate the price the investor should pay for the instrument. [7]

Q. 3) a) State the main differences between preference shares and ordinary shares issued by a company (4)

b) Define a currency swap (2)

[6]

Q. 4) A bond with a coupon rate of 6% per annum, payable half-yearly and having a term of 10 years has a current price of Rs 105/- per Rs 100/- nominal. The next coupon is due in six months' time.

An investor holds a short position in a forward contract on Rs 10 lacs nominal of this bond, with a delivery price of Rs 112/- per Rs 100/- nominal and maturity in one year, immediately following the coupon payment then due.

The effective rate of interest per annum for terms of six months and one year from now are 5% per annum and 5.5% per annum, respectively.

Calculate the value of this forward contract to the investor assuming no arbitrage.

[5]

- Q. 5)** a) List the reasons, why, in general, interest rates vary over time. (2)
- b) What would happen to yields of fixed interest securities if
- i) The government issues high volumes of fixed interest bonds.
- ii) Investors decide to invest more in fixed interest securities. (2)
- [4]**
- Q. 6)** A Life Insurance Company sells a 25 year annuity certain plan to its customers. In return for a single premium, the insurance company would pay the customer an inflation linked annuity payable annually in advance. Discuss the cash flows to the insurance company. (3)
- [3]**
- Q. 7)** a) A fund has a liability to pay Rs 20,000/- at the end of five years. To meet its liability, the fund manager invests in five-year zero coupon bonds which provides any one of the annual effective return of 5% or 7% or 9% with probability 0.2, 0.45 and 0.35 respectively over the next five years. The fund manager invests the present value of the liability at the expected rate of interest, it would earn on the zero-coupon bond.
- i) Calculate the amount invested. (2)
- ii) Calculate the expected profit of the fund due to this investment at the end of five years. (2)
- b) The monthly return i on a fund has a mean of 1.70% and standard deviation of 2%. Assume that $(1 + i)$ is log normally distributed.
- Calculate the value of k such that the probability of i being greater than k is 80%. (5)
- [9]**
- Q. 8)** On 1st April 2008, an Oil company launched a 4-year financial instrument where, for each unit of financial instrument, the purchaser had to pay a premium of Rs 1,500/- per annum at the beginning of each of the 4 years. In return, the purchaser would get a lump sum at the end of the 4 year term. The effective rate of return to the purchaser was fixed at 9.5% per annum from 1st April, 2008 to 1st April, 2010 and 10.5% per annum thereafter.
- The Oil company invested the premiums as and when they were received in an overseas asset which paid a return to the company as per the \$ price of one barrel crude oil. The crude oil price (\$ per barrel) and exchange rate (Rs per \$) over the past few years are given below.

Date	Crude oil price in \$ (per barrel)	Exchange rate (Rs per \$)
1 st April, 2008	55	45.5
1 st April, 2009	53.5	48.1
1 st April, 2010	70	50
1 st April, 2011	90	48
1 st April, 2012	111	49.5

Calculate the amount of accumulated profit (in Rs) the Oil company makes on 1st April, 2012 for each unit of the financial instrument. Assume that the Oil company takes out all the amounts invested in the overseas asset on 1st April, 2012 to meet its liability.

[7]

Q. 9) PQR Health Insurance Company wants to setup a small subsidiary. All the particulars of the project are given below:

The estimated capital expenditure at the start is Rs 50 Lacs.

Each policy holder will get health cover for a one year period and they need to pay a single premium in advance.

The company will incur a yearly cost of Rs 500/- per policy at the start of each year.

Commission per policy sold will be 20% of the premium and will be paid to the agents at the time of sale.

The premium per policy will be Rs 5,000/-.

The claim amount is 5 times premium and the probability of a claim is 5%. The company incurs the claim cost at the end of the year.

The company plans to sell x , $2x$, $3x$, $4x$ and $5x$ number of policies over a five year period starting from the third year. Thereafter the sales will remain constant at $5x$ policies per year. The policies are sold at the beginning of each year.

a) Assuming an effective rate of interest 7.5% per annum, show that the minimum value of x for which the business will be breakeven at the end of the seventh year is 200.

(5)

b) PQR Health Insurance Company requires at least 18% per annum effective rate of return on the project. Assuming that the business will run in perpetuity with the value of x the same as in (a), determine whether the project is viable.

(5)

[10]

- Q. 10)** An insurance company has a liability of Rs 20,00,000/- per annum payable continuously for the next 10 years. The company values these payments using an effective interest rate of 7% per annum. This is also the interest rate at which the current prices of all bonds are calculated. In order to meet its liabilities, the company invests an amount equal to the present value of these liabilities in the following two assets:
- a zero coupon bond redeemable in 3 years, and
 - a fixed interest bond redeemable at par in 7 years' time which pays a coupon of 8% per annum annually in arrears.
- a) Calculate the present value and the duration of the liabilities. (5)
- b) Calculate the amount of money that should be invested now in each asset if the duration of the assets is to be equal to that of the liabilities. (9)
- [14]**
- Q. 11)** a) A fixed interest security pays coupons of 6% per annum payable half yearly on every 1 July and 1 January. The security is redeemable at par at the option of the lender on any 1 July between 1 July 2014 and 1 July 2018 both inclusive, after payment of the coupon then due.
- An investor who is subject to income tax of 20% on the coupon payments purchased Rs 10,000/- nominal of this security on 1 September 2006 at a price to give her a minimum net yield of 4.80% per annum effective.
- Calculate the price paid for the bond (4)
- b) On 1 July 2012, immediately after the payment of the coupon then due, the investor sold the whole stock of this security to another investor who pays income tax at a rate of 25% and capital gains tax at a rate of 30%. The bond is purchased by the second investor to provide her with a minimum net return of 5% per annum effective.
- i) Calculate the price paid by the second investor. (4)
- ii) Calculate, to one decimal place, the annual effective rate of return earned by the first investor during the period for which the bond was held. (5)
- [13]**

Q.12) Akash wants to take a loan of Rs 10 Lacs from ABC bank for his higher education. The bank charges an interest of 7.5% per annum convertible quarterly on the loan. The bank offers the following two options for the loan repayment:

Option (i) – No repayment will be required for next 2 years. After two years, the accumulated loan amount will need to be repaid over 20 equal half-yearly installments payable in advance.

Option (ii) – Only interest will need to be paid over next 2 years annually in arrears. The loan outstanding will need to be paid over 10 increasing annual installments, increasing at the rate of Rs 10,000/- per annum payable in advance.

- a) Calculate the first installment amount to be paid after first two years of the loan for each of the above options. (8)
- b) Calculate the total amount Akash needs to pay in the case of option (ii). (2)
- c) Calculate the amount of capital repaid in the 9th installment, in the case of option (ii). (4)

[14]
