

Institute of Actuaries of India

May 2011 EXAMINATION

**Subject ST7 — General Insurance: Reserving &
Capital Modelling**

Indicative Solution

Q1.]

The assets are made up of both insurance funds and shareholders' funds and the investment policy should reflect this and not be directed entirely towards the shareholders' funds.

Match assets and liabilities by term, amount, nature, currency. Since the company is small, need to have extra consideration of the level of uncertainty in reserves, so more secure, liquid assets required.

Motor property damage claims are mainly short tailed, so need liquid assets, need to hold cash on deposit, very short dated assets such as short dated government securities to match liability outgo.

Motor third party claims are longer tailed and costs are influenced by inflation, need to hold some longer dated real assets (index linked securities if available or low risk equities).

Consider regulatory requirements :

- restrictions on assets that can be held
- prescription to hold assets
- custodianship of assets mismatching allowed

Availability of additional capital (e.g. parent company, shareholders)

Size of the free reserves (in excess of solvency requirements)

– As the company is small, the company is less likely to be able to accept the risk of investing in higher risk/reward investments (e.g. property)

Expected growth plans and resultant needs to invest in the business

Shareholders and management's attitude to risk

If company growing less of a problem as cash-flow normally positive except for large claims when may need to sell equities when price low: volatile and illiquid.

Also, that difference between price for buying and sale of equities large relative to other investments so need to keep for longish period to recover this cost.

[Total Marks – 5]

Q2.]

It is true that the BF method relies heavily, even critically, on the a priori estimate which is a strength if the a priori is chosen well and a weakness if chosen badly.

- The company should be trying to book a best estimate result which means using the best information available and should take precedence over an

established practice which in this instance looks like may not give good results.

- CL is very unreliable when development factor is high (early part of development period) so credibility formula (BF) better between CL and a – priori e.g. <20% credibility for CL for 2010. Also, very essential to get as accurate estimates for a-priori. Budgeted LR may not be representative of given the wide differences between BLR and estimated ULR for prior periods
- BF not that useful for earlier years as CL should be good enough and using BF does not add anything.

[Total Marks – 3]

Q3.]

i) A copula is a mathematical relationship between the individual distributions of random variables and the joint distribution of those variables. Copulas allow us greater flexibility when we model multiple dependencies than single correlation factors allow us.

ii) Suppose we have two variables X and Y.

Let $F_X(x)$ and $F_Y(y)$ are the respective cumulative distribution functions.

However, X and Y are not independent then we need to consider the joint distribution function.

$$F_{XY}(x,y) = P(X \leq x, Y \leq y)$$

A copula function (C) is a function that allows us to calculate the joint distribution function from the values of the marginal distributions so that

$$F_{XY}(x,y) = C[F_X(x), F_Y(y)]$$

$$\text{if we define } C(u,v) = F(F^{-1}(u)), F^{-1}(v))$$

we get

$$C(F_X(x), F_Y(y)) = F(F^{-1}(F_X(x)), F^{-1}(F_Y(y))) = F(x,y)$$

The function $C(u,v)$ is known as a copula.

iii) Gumbel copula function is commonly used in insurance applications.

$$C(u,v) = \exp[-((- \ln u)^a + (- \ln v)^a)^{1/a}]; a > 1$$

The Gumbel copula gives a strong tail correlation and is also non symmetric, giving more weight to the right of the distribution.

[Total Marks – 7]

Q4.]

i) Market Risk : This risk could be modeled using an economic model that would enable a stochastic asset valuation at the end of the period by simulating returns for each asset type. Alternatively could use asset movement stress and scenario testing. Such risk could be correlated with insurance risk as dependent upon inflation. Also could be correlated to liquidity risk for example in the case of reinsurance failure.

Insurance Risk: This risk is likely to separate catastrophe claims experience from attritional experience. Likely to model risk in respect of claims, reserves etc. using ECM (economic capital model). Correlations with most other risks, e.g. credit risk following effect of cat claim, liquidity risk for reinsurance failure.

Credit Risk: Simulate counterparty risk using an ECM stress and scenario test. Credit risk arising from a reinsurer failure could have an impact on other insurers leading to a market risk, and also as mentioned above could lead to a liquidity risk.

Liquidity Risk: In this case would likely to produce a cashflow model. As stated above this is closely linked to insurance risk if large claims occurs.

Group Risk: In this case each part of the group would be modeled separately, i.e. parent or subsidiaries. There is a link here with market risk if an event takes place that affects all areas of the group.

Operational Risk: This risk is likely to occur as a result of poor management leading to monetary loss. As such stress and scenario testing against the risk register is likely to be used there is a correlation with Insurance risk as bad management could lead to high loss ratios.

Mention of the following: Strategic Risk, Political Risk, Enterprise Risk.

[Total Marks – 7]

Q5.]

- i)** The items that may need to be included in a reinsurance treaty document.
 - a** Period of cover
 - b** Name / address of Cedant
 - c** Name / address of Reinsurer(s)
 - d** ...and percentages placed with each if more than one
 - e** Proportional or non-proportional
 - f** Quota share or Surplus for proportional

- g** Alternatively risk XL, aggregate XL, Cat XL, stop loss for non-proportional
- h** Name/address of Broker
- i** Territorial scope
- j** Classes of business covered
- k** Exclusion to cover
- l** Definitions of loss occurrence
- m** Ceding company retentions
- n** Cover granted / limits
- o** Reinstatement provision
- p** Stability clauses/ indexations
- q** Premium rate
- r** Premium payment terms
- s** Ceding commissions payable
- t** Profit commission amount
- u** Profit commission calculation method
- v** Brokerage
- w** Claim notification arrangement
- x** Claims payment arrangement
- y** Bases of cover – risk inception or accident year
- z** Cash loss clause
- aa** Rendering / settlement of accounts
- bb** Currency clause
- cc** Access to records clause
- dd** Termination terms
- ee** Sunset clause
- ff** Arbitration clause

ii) Exposure Measure:

Basic unit used by insurer to measure amount of risk. Usually over a given period e.g. vehicle year.

Risk Factor:

A factor which is expected to influence the intensity of risk (i.e either frequency or severity of claims). Usually, though not necessarily, statistically backed.

Rating Factor:

A factor used to determine the premium rate charged.

Measurable, verifiable, objective

A risk factor or a proxy for a risk factor

[Total Marks - 8]

Q6.]

i) Reserves with reasons which would be required for this class of business.

- Generally a multiyear contract paid for by a single premium, therefore most significant reserve will be UPR
- Possible URR (premium deficiency reserve) if the business was in-adequately priced
- Reported and IBNR outstanding claims will be very low as claims are reported and repairs authorized quickly.
- May be a need for a catastrophe equalization reserve in case of catastrophes.

(ii)

- a** For UPR need a pattern of risk over the period of the contract. This may be over e.g. 3 years where in the first year there is no risk due to the manufacturer's guarantee period.
- b** So no uniform earnings pattern.
- c** There may also be significant risk near the end of the contract when customers instigate repairs with a view to recovering from the insurance.
- d** If past claims data are available then the risk profile can be derived from the pattern of claims emerging over the life of the contract.

- e If no data then can estimate risk profile by examining the product and assessing when various components may fail.

(iii) Earned Premium is given below

Calendar Year	Underwriting Year				
	2008	2009	2010	Plan 2011	Total
2008	-	-	-	-	-
2009	6.25	-	-	-	6.25
2010	12.50	12.50	-	-	25.00
Plan 2011	12.50	25.00	12.50	-	50.00

$$\text{UPR} = \text{Written Premium (cumulative)} - \text{Earned Premium (cumulative)}$$

$$\text{Rs } 350 \text{ cr} - 81.25 \text{ cr} = 268.75$$

Assumptions:

- No premium earned in first year, any other assumption is wrong
- Premium earned evenly (or some reasonable assumptions) over next 4 years
- Business is written evenly over the year
- No business written prior to 2008 (or if so then answer will be different)

Comment: given the info that 50 written in 2008, 100 for each of 2009 to 2011 I would have made the assumption that all the 2008 premium was written in 2nd half of 2008 which would change earned premium in 2008 year to 3.125.

Comment: An appropriate assumption to make and suitable marks would be given to the student working on the assumption

[Total Marks - 11]

Q7.]

(a) MSM proportional to technical reserves

One of the purposes of the solvency margin is to provide a cushion for fluctuations in the value of assets or liabilities (technical reserves). As fluctuations are generally proportional to the amounts, it might be appropriate to have the margin proportional to the amount of the technical reserves.

(b) MSM proportional to square root of premium income

Consistent with statistical theory – the variation in the total claim amount, allowing for variation in the number of claims, increases roughly in line with the square root of premium income.

Basis does give a reasonable MSM for new and growing companies.

(c) Statutory basis for calculating technical reserves

Necessary/desirable as free reserves are defined as assets less technical reserves. This basis prevents possibility of a specified MSM being satisfied through weakening the reserving basis.

Aims to provide consistency between companies.

(d) Technical reserves + MSM invested in government securities

Ensures that the assets are secure (assuming government doesn't default), so there is no danger of policyholder claims not being met through investment failure.

Provides the government with funding !

Suitability

The suitability or otherwise of each suggestion can be considered against the following criteria but should be considered in conjunction with existing solvency requirements which are not given. Note that the final outcome might be a combination of the suggestions.

(1) MSM might be met by the company using a weak reserving basis: applies to all suggestions except (c), and especially for (a).

(2) Some judgment might be appropriate within the reserving basis to allow for peculiar circumstances and different classes of business (so (c) not ideal).

(3) A new or fast growing company needs a strong MSM; (b) is OK, but (a) is not.

(4) A contracting company (or one writing no new business) needs a margin in case outstanding claims are greater than expected. So (a) is OK, but (b) is not.

(5) Credibility theory is based on a pure approach, and does not make full allowance for the vagaries of the real world – so (b) may not be fully appropriate.

(6) Different classes of business may produce different levels of ruin probability – this is not incorporated in any of the suggestions. For example, a company writing 10 industrial fire policies may satisfy all of the criteria (a) to (d), but could go bust on a single claim from any one of the policies.

(7) Solvency margin should make allowance for (i) the valuation basis of the assets, and (ii) the quality of the assets. Only (d) makes any attempt to do this.

(8) The actual investments should reflect the type and term of the liabilities; (d) is restrictive in this respect, and probably results in lower investment returns and hence higher premiums.

(9) None of the bases enforce adequacy of premium rates so a company may have written unprofitable business for some time before it is found to be insolvent.

(10) The legislation may need to be tailored to suit the class(es) written – there is no mention of this in (a) to (d).

(11) Under (c) it may be very hard to establish appropriate statutory reserving bases, especially for unusual risks.

(12) Under (d): if these were only fixed interest investments, monetary security would not protect against rampant inflation.

[Total Marks - 12]

Q8.]

i) The following issues which needs to be addressed in the note

- Discounting liabilities means that the treatment of income on assets side of the balance sheet needs to be altered.
- Even if claims run off is as planned, every year an amount will need to be transferred from the asset income in order to compensate the claims reserves . The amount will be determined by the rate of discount.
- The discount rate needs to be determined. If the rate is set too high then there is a risk that the future years income will not be enough to cover the unwinding of the discount.
- Even if the discount rate is set to a realistic level, there is still a risk that under reserving could occur because the interest rates vary unpredictably.
- To avoid any future shocks, the discount rate should be set to a prudent level where by we can be reasonably sure to be able to unwind the discount as the claims run off.
- The risk of setting the discount rate too high may be offset by careful matching of assets to liability by term, although the degree of variability of general insurance reserves makes it more difficult to achieve than for life insurance companies.
- The derivative market may also be of assistance in this.
- The amount of discount also depends on the assumed future payment pattern. This can be assessed by studying the historical run offs and the technics used to reserve the business.
- Any changes observed in the rate of settlement will need to be carefully assessed. If we assume too slow a pattern of run off then the reserves would be understated.
- Practical solutions will depend on local considerations, regulators approach and Tax authorities.

ii)

- Expected payment pattern may be difficult to derive for some classes
- Companies with significant amounts that are not invested (broker balances, reinsurance balances) would be under reserved if the balances are taken as an asset at face value
- Unless the discount rate can be adjusted
- Proposal not clear on currency effects
- Should local currency bond be used – what about liability denominated in foreign currency
- No government bonds of suitable term and hence not possible to determine an appropriate redemption yield
- What if bond yield is higher than actual asset yield – then there is problem of reserves plus investment return not meeting liabilities

- Additional administrative burden for companies and government
- To what extent should classes be combined (best estimates for sum of classes may not be equivalent to sum of best estimates)
- As at what date should gross redemption yield be taken
- Problems of allowing for re-insurance recoveries
- Subjective nature of what is “best estimates”

[Total Marks - 11]

Q9.]

i) Balance sheet Assumptions

- All yearly business
- No reinsurance
- Risks written uniformly across year
- Risk is uniform across policy year

Assets	Company 1	Company 2	Company 3
Total investments	188	7000	2500
Current Assets	8	160	75
Deferred Acquisition Costs	14	360	75
Total Assets	209	7520	2650
 Liabilities			
O/S claims reserves	45	1700	1750
Additional URR	23	200	0
UPR	38	2000	313
Current Liabilities	17	200	100
Free Reserves	87	3420	488
Total Liabilities	209	7520	2650

ii) Assumptions

- assume GWP = GEP (i.e. UPR b/f = UPR c/f)
- assume AURR as at 31/03/2011 = AURR as at 31/03/2010
 - assume outstanding claims reserves include IBNR

	Company 1	Company 2	Company 3
Loss Ratio			
Incurring Claim / GEP	90%	38%	40%
Expense Ratio			
Acquisition ratio + non acquisition cost / GWP	46%	31%	36%
Underwriting Ratio			
Loss ratio + expense ratio	136%	68%	76%
Solvency Ratio			
Free reserves / GWP	116%	86%	78%
Return on capital employed			
(Earned premium - claims incurred - expenses + investment income) / free reserves	-26%	43%	39%



Accounts Ques

iii)

Comments

- Company 1 may have suffered from adverse claims experience as shown by its higher loss ratio compared to the other companies.
- Each company may be writing different classes or mix of business, each at a different point in their respective market cycle
- Company 1 expense ratio is higher due to higher acquisition expense ratio.
- ...The company is smaller than 2 and 3 and it may be spending money to expand rapidly.
- Company 1 solvency ratio is higher than the other companies.
- ...This may be the result of a recent capital injection to expand the business.
- Company 3 has the lowest solvency ratio, suggesting that the company is less financed than the other companies.
- ...Or it may have stronger valuation basis for its assets and liabilities
- Company 3 return on capital employed is the highest, supported by a larger relative investment return compared to the other companies
- Company 2 and 3 both have high returns on capital employed, supported by good underwriting results

Any other relevant point to be considered

iv) There are estimates for unknowns which make up the stated profits. The estimates may turn out to be incorrect.

The estimates of reserves for outstanding claims are subject to uncertainty and to differences in the strength of the reserving basis used.

The basis of valuation of assets will also affect the level of apparent profitability. Anything to effect that published accounts are result of more than one year of business so difficult to see what result of current business is.

v) For internal monitoring of true profitability the general insurer can follow the following steps

- Replace the actual large and weather related claims with the budgeted level of large and weather related claims
- Remove any prior year adverse / favorable claims development
- Remove one off expense / commission items
- Add back any movement of reserve margin for the current year which has an impact on the profitability

[Total Marks - 17]

Q10.]

i)

- The decision on what to publish in the accounts rests with the Board of Directors. The actuary's role is to provide the necessary advice to the Board in order to make an informed decision.
- In framing the advice the actuary will make a number of assumptions regarding the model and parameters used. Both of these sets of assumptions are subject to varying degree of uncertainty.
- The quality of data has to be relied upon.
- Also each of the assumptions used may contain margins for prudence. Such margins include the discount rate, reinsurance recoveries, inflation, tail factor and treatment of future premiums.
- Due to the degree of uncertainty regarding the "true" model and parameters there is a range of values which can be considered reasonable and prudent.
- Provided the assumptions made remain above the lower end of their respective reasonable and prudent level then their use is adequately justified.
- Where the assumptions fall below a level considered reasonable and prudent by the actuary then they should not be used in framing advice.
- General accounting and actuarial guidelines also indicate that arbitrary changes should not be made to assumptions.

- Any proposed changes should be for reasons indicated by past experience or known future changes.
- In practice, good reasons will usually exist to justify small changes to any assumptions made.
- With a reserve of Rs 100 crores a change in profit of Rs 1 crore is relatively immaterial. There will undoubtedly be small margins in some of the assumptions which can be trimmed to produce the desired result.
- However, in the unlikely event that all margins have already been set at the lowest level deemed acceptable to the actuary, then any review of the calculation should make that conclusion clear.

ii) Bootstrapping – Characteristics:

1. A method of estimating the parameter uncertainty surrounding an estimate of the reserves.
2. To estimate the process uncertainty need to use in conjunction with e.g. Mack method or over-dispersed Poisson model
3. Calculate the expected values and the residuals for each point in the claims triangle
4. Re-sample (with replacement) from the residuals to obtain a new triangle
5. Refit the chain ladder model to the new triangle to obtain a revised reserve estimate
6. By repeating this process thousands of times we can generate standard deviations, confidence intervals.
7. Can be applied to paid or incurred data, and accident year or underwriting year cohorts.

ii. Bootstrapping – Pros / cons

1. + Easy to apply for most datasets
2. + Customizable
3. - Basic method, very restrictive in terms of how development factors are selected
4. + However method can be applied to subjectively derived development factors

5. - Assumes incremental claims are positive for all development periods which restricts use for incurred claims, necessitating adjustment for the method to work

iii. Mack's Method – Characteristics:

1. An analytical method based on the chain ladder for estimating the uncertainty inherent in the reserve estimate for a given accident or underwriting year.
2. A standard chain ladder method is applied to the cumulative triangle to determine the incremental development factors.
3. Variability between the actual and expected development at each point in the triangle is calculated
4. Then the variability across the rows is aggregated to produce a standard error for each accident / underwriting year.
5. Can extend to derive a standard error of the overall reserve estimates
6. However, if percentiles are required, in order to produce a range, a distribution needs to be assumed via a deterministic calculation or bootstrap approach based on chain ladder method.
7. So it assumes underlying chain ladder assumptions are appropriate
8. The method can be applied to paid or incurred data, and accident year or underwriting year cohorts

iv. Mack's Method pros/cons:

1. + No assumption of prior distribution
2. - Although assumption that variance of cumulative claims to time t being proportional to cumulative claims amount to time $t-1$ might not be appropriate
3. + A tail factor can be incorporated as a deterministic multiple
4. - Limited judgment possible
5. - Full predictive distribution not derived, may need to approximate e.g. using log-normal distribution

v. Both Methods – pros / cons:

1. + Require few assumptions
2. Easy to use – can be run in MS Excel or other proprietary software may contain a version of the method
3. + Increasing usage of methods in insurance industry

4. - Dependent on the quality of data used
5. - Output may reflect variability of data which is a feature of data errors/inconsistency rather than the underlying claim features
6. - Any variability not included in the data will not be reflected in the derived range
7. - This is a particular problem where limited data are available
8. - Difficult to explain to non-technical audience
9. + Objective
10. + Can audit and peer review

[Total Marks - 19]

[Total Marks – 100]
