Institute of Actuaries of India

May 2011 Examinations

Subject ST6 — Finance and Investment B Specialist Technical

Indicative Solution

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

SOLUTION ST-6-MAY 2011

Q.1)

(a) The instrument provides a six month return equal to
Max(0, 0.3R)
Where R is the return on NSE Nifty index. Suppose that S_0 is the current value of NSE Nifty and S_T is the value of Nifty in one year.

(b) When an amount of Rs. 5,000,000 is invested, the return (in Rs.) received at the end of one year is:

$$5{,}000{,}000 \times Max(0, 0.3 \times \frac{S_T - S_0}{S_0}) = \frac{1{,}500{,}000}{S_0} \times Max(0, S_T - S_0)$$

This is 1,500,000/S_0 of at-the-money European call options on NSE Nifty. They have value:

$$\frac{1{,}500{,}000}{S_0}[S_0 e^{-0.02}\Phi(d_1) - S_0 e^{-0.06}\Phi(d_2)] = 1{,}500{,}000[e^{-0.02}\Phi(d_1) - e^{-0.06}\Phi(d_2)$$

Where

$$d_1 = \frac{(0.06 - 0.02 + 0.5 \times 0.20^2)}{0.20} = 0.30$$

$$d_2 = 0.30 - 0.20 = 0.10$$

$\Phi(0.30) = 0.6179$; $\Phi(0.10) = 0.5398$

Thus value of the European call options being offered is:
1,500,000(0.6057 – 0.5084) = Rs. 145,927.90
This is the present value of the payoff from the instrument. If an investor buys the instrument he or she avoids having to pay Rs. 145,927.90 at time zero for the underlying option. The minimum cash flows to the investors are therefore
Time 0: -5,000,000 + 145,927.90 = 4,854,072.10
After one year: 5,000,000

(c) The minimum return with continuous compounding is ln(5000000/4854072.10) = 2.96% per annum. The instrument is therefore less attractive than a risk-free investment.

[Total Marks – 8]

Q.2)

(a)

Payoff of Portfolio A:

	$S_T \le 100$	$100 < S_T \le 150$	$150 < S_T \le 200$	$S_T > 200$
Long Call (100)	0	$S_T - 100$	$S_T - 100$	$S_T - 100$
Short Call (150)	0	0	$-(S_T - 150)$	$-(S_T - 150)$
Long Call (200)	0	0	0	$S_T - 200$
Long Put (100)	$100 - S_T$	0	0	0
Short Put (150)	$-(150 - S_T)$	$-(150 - S_T)$	0	0
Long Put (200)	$200 - S_T$	$200 - S_T$	$200 - S_T$	0
Total	$150 - S_T$	$-50 + S_T$	$250 - S_T$	$-150 + S_T$

Payoff of Portfolio B:

	$S_T \le 100$	$100 < S_T \le 150$	$150 < S_T \le 200$	$S_T > 200$
Long Call (150)	0	0	$S_T - 150$	$S_T - 150$
Long Put (150)	$150 - S_T$	$150 - S_T$	0	0
Debt	-50	-50	-50	-50
Total	$100 - S_T$	$100 - S_T$	$S_T - 200$	$S_T - 200$

(b)

	$S_T = 0$	$S_T = 100$	$S_T = 150$	$S_T = 200$
Portfolio A	150	50	100	50
Portfolio B	100	0	-50	0

(c)

At all the Nifty values at time T, the payoff of portfolio A is more than payoff of portfolio B. Thus portfolio A requires greater initial outlay to establish.

(d)

Cost of establishing portfolio A = $c_1 - c_2 + c_3 + p_1 - p_2 + p_3$

Cost of establishing portfolio B = $50e^{-rT} + c_2 + p_2$

From 2(b), we have

$c_1 - c_2 + c_3 + p_1 - p_2 + p_3 > 50e^{-rT} + c_2 + p_2$

$c_1 - 2c_2 + c_3 + p_1 - 2p_2 + p_3 > 50e^{-rT}$

From put-call parity theorem, we have

$c_1 - 2c_2 + c_3 = p_1 - 2p_2 + p_3$

$2(c_1 - 2c_2 + c_3) > 50e^{-rT}$

$c_1 - 2c_2 + c_3 > 25e^{-rT}$

[Total Marks – 11]

Q.3)

(a)

A self financing portfolio (h^B, h^X, h^Y) has value process

$$dV^h = \left(h^B rB + h^X(\mu^X + \delta^X)S^X + h^Y(\mu^Y + \delta^Y)S^Y\right)dt + \left(h^X\sigma^X S^X + h^Y\sigma^Y S^Y\right)dW$$

From Girsanov's theorem, we have

$$dW_t = \theta_t dt + d\tilde{W}_t$$

Where $\tilde{W}$ is Q-Wiener process.

$$dV^h = \left(h^B rB + h^X(\mu^X + \delta^X + \theta\sigma^X)S^X + h^Y(\mu^Y + \delta^Y + \theta\sigma^Y)S^Y\right)dt + \left(h^X\sigma^X S^X + h^Y\sigma^Y S^Y\right)d\tilde{W}$$

By no-arbitrage argument, V^h must have drift r under Q, which implies

$$\mu^X + \delta^X + \theta\sigma^X = r$$

$$\mu^Y + \delta^Y + \theta\sigma^Y = r$$

The Q-dynamics of S^X is given as

$$dS_t^X = (\mu^X + \sigma^X\theta_t)S_t^X dt + \sigma^X S_t^X d\tilde{W}_t$$

$$dS_t^X = (r - \delta^X)S_t^X dt + \sigma^X S_t^X d\tilde{W}_t$$

The Q-dynamics of S^Y is given as

$$dS_t^Y = (\mu^Y + \sigma^Y\theta_t)S_t^Y dt + \sigma^Y S_t^Y d\tilde{W}_t$$

$$dS_t^Y = (r - \delta^Y)S_t^Y dt + \sigma^Y S_t^Y d\tilde{W}_t$$

(b)

We need to compute γ such that

$$e^{-rT}E^Q[D_T^X - \gamma D_T^Y] = 0$$

$$\gamma = \frac{E^Q[D_T^X]}{E^Q[D_T^Y]}$$

$$E^Q[D_T^X] = E^Q\left[\int_0^T \delta^X S_t^X dt\right]$$

$$= \delta^X \int_0^T E^Q[S_t^X]dt$$

$$= \delta^X s_X \int_0^T \exp\{(r - \delta^X)t\}dt$$

$$= \frac{\delta^X s_X}{r - \delta^X}\left(\exp\{(r - \delta^X)T\} - 1\right)$$

Similarly

$$E^Q[D_T^Y] = \frac{\delta^Y s_Y}{r - \delta^Y}\left(\exp\{(r - \delta^Y)T\} - 1\right)$$

$$\gamma = \frac{E^Q[D_T^X]}{E^Q[D_T^Y]} = \frac{\delta^X s_X (r-\delta^Y)(\exp\{(r-\delta^X)T\}-1)}{\delta^Y s_Y (r-\delta^X)(\exp\{(r-\delta^Y)T\}-1)}$$

[Total Marks – 10]

Q.4)

(a)
For Q to be a martingale measure (with B as numeraire), the parameters must satisfy the following relation.

$$m(t,T) = s(t,T)\int_t^T s(t,u)du$$

This is the HJM condition
This condition follows since all discounted price processes must be martingales under Q. That is, all zero coupon bond prices $P(t,T) = \exp\{-\int_t^T F(t,u)du\}$ must have drift $r(t) = F(t,t)$.
To derive the condition, the dynamics of P(t,T) is derived from the dynamics of F(t,T). Putting the drift equal to r(t) leads to a relation (in integrated form) for the parameters m and s. The HJM condition is obtained by differentiating that relation with respect to T.
z

(b)
On integrated form

$$F(t,T) = F(0,t) + \int_0^t m(u,T)du + \int_0^t s(u,T)dz(u)$$

$$= F(0,0) + \int_0^t F_T(0,u)du + \int_0^t m(\tau,\tau)d\tau + \int_\tau^T m_T(\tau,u)du$$

$$+ \int_0^t s(\tau,\tau)dz(\tau) + \int_\tau^T s_T(\tau,u)dudz(\tau)$$

Putting T = t, we get

$$F(t,t) = r(t) = F(0,0) + \int_0^t F_T(0,u)du + \int_0^t m(\tau,\tau)d\tau + \int_0^t \int_\tau^t m_T(\tau,u)dud\tau$$

$$+ \int_0^t s(\tau,\tau)dz(\tau) + \int_0^t \int_\tau^t s_T(\tau,u)dudz(\tau)$$

$$r(t) = r(0) + \int_0^t F_T(0,u)du + \int_0^t m(u,u)du + \int_0^t \int_0^u m_T(\tau,u)d\tau du$$

$$+ \int_0^t s(u,u)dz(u) + \int_0^t \int_0^\tau s_T(\tau,u)dz(\tau)du$$

$$r(t) = r(0) + \int_0^t [F_T(0,u) + \int_0^u m_T(\tau,u)d\tau + \int_0^\tau s_T(\tau,u)dz(\tau)]du$$

$$+ \int_0^t m(u,u)du + \int_0^t s(u,u)dz(u)$$

$$r(t) = r(0) + \int_0^t F_T(u,u)du + \int_0^t m(u,u)du + \int_0^t s(u,u)dz(u)$$

$$dr(t) = \alpha(t)dt + \sigma(t)dz(t)$$

Where $\alpha(t) = F_T(t,t) + m(t,t)$ and $\sigma(t) = s(t,t)$

The drift condition, with T = t implies $m(t,t) = 0$ but does not imply a particular condition relationship between $\alpha(t)$ and $\sigma(t)$.

[Total Marks – 11]

Q.5)

a. $V(t)^h = h(t)^B B(t) + h(t)^S S(t) = B(t) + S(t)$

$$dV(t)^h = dB(t) + dS(t) \neq \frac{S(t)}{B(t)} dB(t) + \frac{B(t)}{S(t)} dS(t)$$

The portfolio defined by $h(t) = \left(\frac{S(t)}{B(t)}, \frac{B(t)}{S(t)}\right)$ is therefore not self financing.

b. $dS(t) = rS(t)dt + \sigma S(t)dz(t)$

under the martingale measure Q (z denotes a Q-Wiener process)

$$dY(t) = -\alpha S(t)^{-\alpha-1} dS(t) + \frac{1}{2}(-\alpha)(-\alpha-1)S(t)^{-\alpha-2}(dS(t))^2$$

$$= -\alpha r S(t)^{-\alpha} dt - \alpha\sigma S(t)^{-\alpha} dz(t) + \frac{1}{2}(\alpha + \alpha^2)\sigma^2 S(t)^{-\alpha} dt$$

Putting $\alpha = \frac{2r}{\sigma^2}$, we get

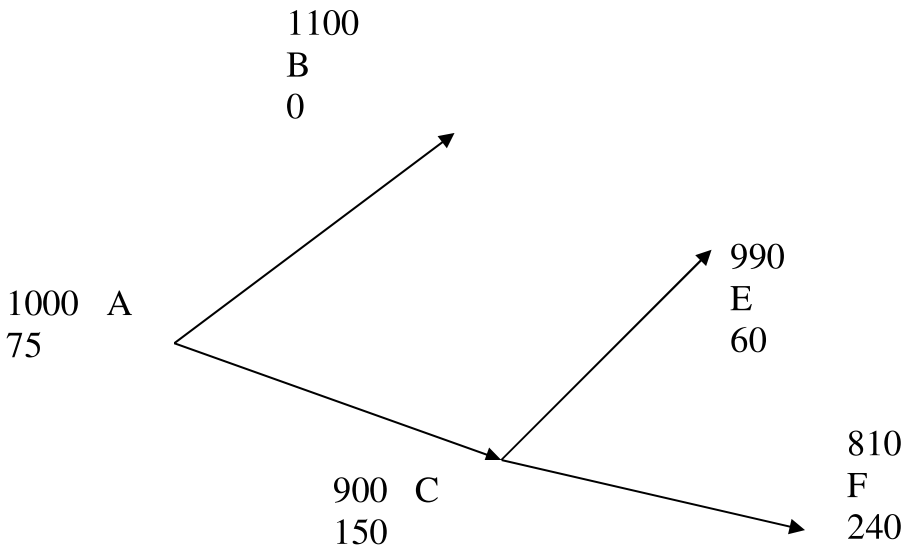
$$dY(t) = rY(t) - \alpha\sigma Y(t)dz(t)$$

Since the process has a local rate of return of r under the martingale measure Q, it represents a tradable asset

[Total Marks – 8]

Q.6)

(a)



$$p = \frac{1-0.90}{1.10-0.90} = 0.5$$

Value of the option = Rs. 75

(The risk neutral probability is 0.5 and the real world probability of 0.55 is not used)

(b)

To obtain the replicating portfolio at a one has to solve the following set of equations:

$1100\phi + \psi = 0$

$900\phi + \psi = 150$

$\phi = -\frac{150}{200} = -0.75$; $\psi = 825$

It is not required to maintain the replicating portfolio at B, since at B the option ceases to exist as the stock price reaches the barrier level (H = 1050). Thus, the value of the replicating portfolio at B must be zero.

To obtain the replicating portfolio at C one has to solve the following set of equations:

$990\phi + \psi = 60$

$810\phi + \psi = 240$

$\phi = -1$; $\psi = 1050$

That the portfolio is self financing is seen from the following equation

$825 - 0.75 \times 1100 = 0$

$825 - 0.75 \times 900 = 1050 - 1 \times 900 = 150$

[Total Marks – 6]

Q.7)

Maturity (years)	Price	YTM	Forward rate
1	934.58	7.00%	
2	869.37	7.25%	7.50%
3	804.96	7.50%	8.00%
4	741.88	7.75%	8.50%
5	680.58	8.00%	9.00%

(a) For each three-year zero the investor buys today, issue:

804.9606/680.5831 = 1.182751 five-year zeros

For 10,000 (10,000,000/1000) three-year zero coupon bonds the investor buys today, issue

1.182751 X 10,000 = 11,827.51 five-year zero coupon bonds

The time-0 cash flow equals zero.

(b) Your cash flows are thus as follows:

Time	Cash Flow (Rs.)	
0	0	
3	10,000,000	The 3-year zero purchased at time 0 matures; receive $1,000 face value
5	-11,827,510	The 5-year zeros issued at time 0 mature; issuer pays face value

(c) This is a synthetic two-year loan originating at time 3.

The effective two-year interest rate on the forward loan is:

11,827,510/10,000,000 – 1 = 0.1827 = 18.27%

(d) The one-year forward rates for years 4 and 5 are 8.5% and 9%, respectively. Notice that:

$(1+f_4)(1+f_5) = 1.085 \times 1.09 = 1.1827 = (1+f_{3,2})$

1 + (two-year forward rate on the 3-year ahead forward loan)

[Total Marks – 9]

Q.8)

(a) While it is true that short-term rates are more volatile than long-term rates, the longer duration of the longer-term bonds makes their prices and their rates of return more volatile. The higher duration magnifies the sensitivity to interest-rate savings.

(b) The minimum terminal value that the manager is willing to accept is determined by the requirement for a 5% annual return on the initial investment. Therefore, the floor is:

Rs. 10 million $\times (1.05)^6$ = Rs. 13.40 million

Four years after the initial investment, only two years remain until the horizon date, and the interest rate has risen to 7%. Therefore, at this time, in order to be assured that the target value can be attained, the manager needs a portfolio worth:

Rs. 13.40 million/$(1.07)^2$ = Rs. 11.70 million

(c) The bondholders have, in effect, made a loan which requires repayment of B dollars, where B is the face value of bonds. If, however, the value of the firm (V) is less than B, the loan is satisfied by the bondholders taking over the firm. In this way, the bondholders are forced to "pay" B (in the sense that the loan is cancelled) in return for an asset worth only V. It is as though the bondholders wrote a put on an asset worth V with exercise price B. Alternatively, one might view the bondholders as giving the right to the equity holders to reclaim the firm by paying off the B dollar debt. The bondholders have issued a call to the equity holders.

(d) You want to protect your cash outlay when the bond is purchased. If bond prices increase, you will need extra cash to purchase the bond. Thus, you should take a long futures position that will generate a profit if prices increase.

(e) The Indian rupee is depreciating relative to the US dollar. To induce investors to invest in India, the India interest rate must be higher.

(f) The bond callable at 104 should sell at a lower price because the call provision is more valuable to the firm. Therefore, its yield to maturity should be higher.

[Total Marks – 11]

Q.9)

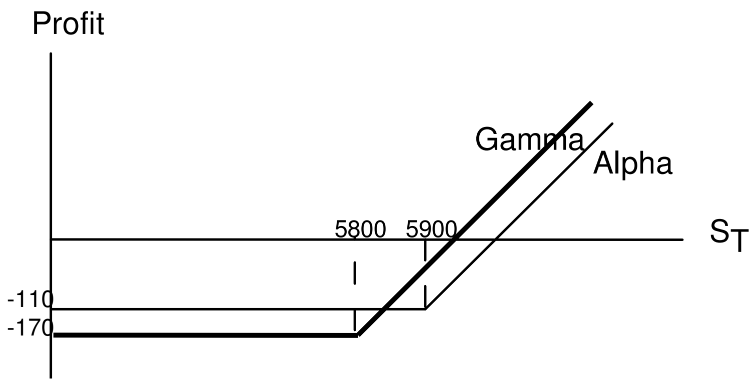
(a) Alpha's strategy

Position	Cost	Payoff	
		$S_T \le 5900$	$S_T > 5900$
NSE Nifty Fund	5900	S_T	S_T
Put option, K = 5900	110	$5900 - S_T$	0
Total	6010	5900	S_T
Profit = payoff – 6010		–110	$S_T - 6010$

Gamma's strategy

Position	Cost	Payoff	
		$S_T \le 5800$	$S_T > 5800$
NSE Nifty Fund	5900	S_T	S_T
Put option, K = 5800	70	$5800 - S_T$	0
Total	5970	5800	S_T

Profit = payoff – 5970 –170 S_T – 5970



b. Gamma does better when the stock price is high, but worse when the stock price is low. The break-even point occurs at S_T = 5860, when both positions provide losses of 110.

c. Gamma's strategy has greater systematic risk. Profits are more sensitive to the value of the stock index.

[Total Marks – 6]

Q.10)

(a) Real-world probabilities of default should be used for calculating credit value at risk. Risk-neutral probabilities of default should be used for adjusting the price of a derivative for default.

(b) In a default-free world the forward contract is the combination of a long European call option and a short European put option where the strike price of the options equals the delivery price and the maturity of the options equals the maturity of the forward contract. If the no-default value of the contract is positive at maturity, the call has a positive value and the put is worth zero. The impact of defaults on the forward contract is the same as that on the call. If the no-default value of the contract is negative at maturity, the call has a zero value, the call has a zero value and the put has a positive value. In this case defaults have no effect. Again the impact of defaults on the forward contract is the same as that on the call. It follows that the contract has a value equal to a long position in a call that is subject to default risk and short position in a default-free put.

(c) A credit default swap insures a corporate bond issues by the reference entity against default. Its approximate effect is to convert the corporate bond into a risk-free bond. The buyer of a credit default swap has therefore chosen to exchange a corporate bond for a risk-free bond. This means that the buyer is long a risk-free bond and short a similar corporate bond.

(d) A CDO is created from a bond portfolio. Different tranches are created for the returns from the bonds and the different tranches have different credit risk exposure. Investors can invest in any one of the tranches. The first tranche might have an investment in 5% of the bond portfolio and be responsible for the first 5% of the losses.The next tranche might have an investment of 10% of the bond portfolio and be responsible for the next 10% of the losses and so on.

[Total Marks – 11]

Q.11)

(a) Using the true volatility (30%) and time to maturity T = 0.25 years, the hedge ratio for Infosys is $\Phi(d_1) = 0.5695$. Because you believe the calls are under-priced (selling at an implied volatility that is too low), you will buy calls and short 0.5695 shares for each call you buy.

(b) The calls are cheap (implied $\sigma = 0.28$) and the puts are expensive (implied $\sigma = 0.32$). Therefore, buy calls and sell puts. Using the "true" volatility of $\sigma = 0.30$, the call delta is 0.5695 and the put delta is: 0.5567 – 1.0 = –0.4305
Therefore, for each call purchased, buy: 0.5695/0.4305 = 1.323 puts

(c) According to the parity relation, the proper price for 6-month future is:

$$F_{0.5} = F_{0.25}(1 + r_f)^{1/4} = 3045 \times 1.06^{1/4} = 3089.68$$

The actual 6-month futures price is too high relative to the 3-month future price.You should short the 6-month contract and take a long position in the 3-month contract.

[Total Marks – 9]

[Total Marks – 100]

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