INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

12th May 2011

Subject CT4 – Models

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) (i) Define the following stochastic processes:

(a) a Compound Poisson Process

(b) a General Random Walk (2)

(ii) For each of the processes in (i), state whether it operates in continuous or discrete time, and whether it has a continuous or discrete state space. (2)

(iii) For each of the processes in (i), describe one practical situation in which an actuary could use such process to model a real world phenomenon. (2)

[6]

Q. 2) Due to recent regulatory changes you need to re-price all your products quickly and hence it requires extensive modelling work. The company's actuarial staff may not be able to model all the products in such a short timeframe. The Chief Executive Officer (CEO) of your company has suggested that the company should outsource the modelling work for some of the products to a leading actuarial consultancy.

(i) Evaluate the benefits and disadvantages of the CEO's suggestion. (4)

(ii) Your company has decided to outsource the modelling work for some products to the actuarial consultancy. The Chief Actuary of your company has asked you to ensure that the consultancy provides complete and accurate documentation of the models built by them. Briefly describe what items would you require in the documentation. (3)

[7]

Q. 3) Hindustan Motors is investigating causes of break-down of their legendary Ambassador cars. Despite various improvements over the past few years, negative reports in the recent press has highlighted that Ambassadors have the highest break-down incidences of any model in the market.

Most common industry-wide causes of break-downs are:

Cause of break-down	**Proportion of cars**
Flat / Faulty Battery	32%
Flat tyres	27%
Alternator	17%
Starter Motor	11%
Fuel	9%
Others	4%

It has been suggested that the industry-wide causes of break-down should be used as a model for investigating break-down of Ambassador Cars as well. The management has obtained the following statistics regarding the number of break-downs of Ambassador Cars in the last year:

Cause of break-down	Number of break-downs
Flat / Faulty Battery	3269
Flat tyres	2627
Alternator	1680
Starter Motor	1037
Fuel	839
Others	415

(i) Clearly state the null hypothesis and carry out a chi-squared test to verify the suggestion. (5)

(ii) List two possible differences that the chi-square test will not detect. (2)

[7]

Q. 4) A large life insurance company has been selling term assurance plan for number of years and has carried out a mortality investigation for the first time recently. Mr. Bimankak, the company's Appointed Actuary is generally satisfied with the results and has recommended that the mortality rates obtained be used for setting mortality assumptions in the future. However, Mr. Bimankak has also suggested that the crude rates from the investigation are 'smoothed' before using them in actuarial calculations. He has sought your opinion on the appropriate method for graduation. Briefly describe three methods of graduation that can be used, stating clearly the advantages and disadvantages of each as relevant to the insurance company.

[9]

Q. 5) Sachin Tendulkar has played 440 one-day international matches in his career. A one-day match has a maximum 50 overs per innings and you can assume that Tendulkar himself can play maximum 25 overs. You are given the following statistics regarding the number of overs he has played himself.

Overs played	**No. of matches (x)**	**∑x**	**Overs played**	**No. of matches (x)**	**∑x**
0	35	35	13	5	351
1	56	91	14	11	362
2	38	129	15	9	371
3	30	159	16	8	379
4	42	201	17	12	391
5	24	225	18	9	400
6	22	247	19	3	403
7	23	270	20	4	407
8	13	283	21	12	419
9	19	302	22	8	427
10	12	314	23	8	435
11	19	333	24	2	437
12	13	346	25	3	440

(i) Derive an expression, in terms of probability of batting through an over and probability of getting out in a given over, for the curtate expectation of number of overs Sachin will play, given that he has already played "x" overs. Define all symbols you use. (3)

(ii) Given that Sachin has already played 18 overs in a given match. Calculate:

a. The expected number of overs Sachin will play before he gets out

b. Probability that Sachin will bat till the end of the innings (i.e. he will play 25 overs himself) (3)

(iii) Sachin's strike rate in one day internationals is 86.36. If he has already scored 89 runs in the 18 overs he has played so far, calculate the expected final score and the variance of Sachin's final score in the match.

(Strike rate is the average number of runs per 100 balls faced. There are 6 balls in one over.) (4)

[10]

Q. 6) The Life Insurance Council of India is concerned that a systemic increase in lapse rates may adversely affect profitability of the life insurance industry as a whole. It has been suggested that younger policyholders are more likely to lapse their policies than their older counterparts, therefore the Council has commissioned a study into policy lapse rates by age.

(i) Explain what is meant by Central Exposed to Risk and specify what data is needed if this is to be calculated exactly. (3)

The Council collects the following data from a survey of all life insurance companies:

- Number of policies in-force for lives aged x next birthday on 31 December 2009, 31 December 2010 and 31 March 2011.
- Number of lapses during the period 1 January 2010 to 31 March 2011 given by age last birthday.

(ii) Derive a formula for the central exposed to risk that corresponds to the lapse data stating any assumptions that you make. (5)

(iii) Comment on the reasonableness or otherwise of the assumptions you made in your answer to part (ii). (3)

[11]

Q. 7) The local management of Serengeti Actuarial Consulting ("SAC") is interested in knowing how many student actuaries employed with the firm will remain in the firm when they qualify. It defines "survival probability" as the probability that a new recruit will still be employed at SAC after passing all the 15 exams. You are given the following employment history at SAC:

- SAC recruits six new graduates: A, B, C, D, E and F.
- A, E and F have never taken actuarial exams before. A leaves the firm when he only has one paper left for qualification while E and F continue to be employed with SAC when they qualify.
- B and C have two papers each when they join while D has already cleared 7 papers at the time of joining.
- C leaves SAC after having passed three more papers and D takes a transfer to another office when he still has two more papers remaining. The local management is unable to track D's exam progress after the transfer.
- At the end of the investigation, B continues to be employed with the firm and has passed 13 papers in total.

The following year, SAC recruits four more students: W, X, Y and Z. Of these,

- Z only has 3 more papers remaining when he joins SAC and continues to be with the firm when he qualifies.
- W has passed 4 papers before joining SAC. He takes a transfer within the firm to another office after passing 7 more papers and the local office is unable to track his exam progress thereafter. A couple of years later, W re-joins the local office at which time he has already qualified.
- X and Y both started with no papers and left the firm when each had passed 7 and 12 papers respectively.

(i) Calculate the Kaplan-Meier estimate of the survival function $STgrad\ (t) = P(T\text{grad} > t)$, i.e. probability that a new graduate recruit with no papers will still be employed at SAC after passing "t" papers. (8)

(ii) Calculate the Poisson estimate of q_{Grad} and construct a 95% confidence interval for qGrad based on this estimate. (5)

[13]

Q. 8) Roger Federer and Rafael Nadal are playing out what has become an epic US Open finale. The two have won two sets each, and the fifth set is currently going right down the wire. Fifth set ended up in a tie at 6-6, and a tie-breaker is being played out to decide the fifth set. The winner of this fifth set will be the 2011 US Open champion.

Tie-breaker score stands at 10-10 and the tie-breaker would continue until one of the players takes out a clear lead of two points over the opponent. If Roger Federer wins the next point, he would lead and the score would become 11-10 in his favour. If he wins the following point as well, the score would become 12-10 and the match would end as he gains a two-point lead. However, if Rafael Nadal were to wrest back a point when Roger Federer is leading at 11-10, the score would become 11-11 and tie-breaker would again be back tied.

The probability of Roger Federer winning a point is 55%. The probability of Rafael Nadal winning a point, therefore, is 45%.

(i) Describe how you can model the tie-breaker in this case as a Markov chain. Clearly specify the states of this Markov chain. (5)

(ii) Write down the transition matrix. (2)

(iii) State, with reasons, whether the chain is:

(a) Irreducible; and

(b) Aperiodic (2)

(iv) Calculate the number of points that must be played before there is more than 95% chance of the game being completed. (3)

(v) Calculate the probability that:

(a) Federer eventually wins the match.

(b) Nadal eventually wins the match.

Comment on your answers. (4)

[16]

Q. 9) A solar system has 8 planets revolving around its star. The scientist observes that first two inner planets are very close to the star, and are too hot for any form of life to develop. What's more, the two outer planets are so far from the star that they are in a state of eternal deep freeze. Life will never develop on the two inner and two outer planets.

The four central planets are rocky and have an atmosphere on their own. This allows for a hope that someday life can develop in these planets, subject of course to the availability of water and some other elements. Scientist discovers that some of the required elements are missing from these four central planets and none of these planets, therefore, are habitable as of now. Fortunately, such elements can often be supplied into such planets by comets from the outer space when they strike the planets. Thus, what is perceived to be an event of destruction is indeed the only event that could potentially lead to creation.

Comet collisions with the planets in this solar system occur according to a Poisson process with rate of 6 every million years. What happens upon a comet collision is not influenced by previous comet collisions if any. A comet is equally likely to strike any one of the eight planets. Half of the comets that strike the planets have the required elements which are missing from the central planets.

Central planets are massive and rocky and can absorb a comet strike without much damage. Should a planet become habitable following a comet strike supplying the required elements to it, there is no chance that it would cease to be habitable should another comet strike such planet again.

(i) Show how number of habitable planets at any time, *X(t)*, can be formulated as a Markov jump process by specifying the state space. (2)

(ii) Briefly describe, with reasons, the absorbing state for this process if any. (2)

(iii) Explain how the transition rate at which the process moves from k (number of habitable planets) to k+1 is $3 \times \frac{4-k}{8}$ every million years. (3)

(iv) Write down the generator matrix for this process. (4)

(v) State the distribution of the holding times of the Markov Jump process. (2)

(vi) Write down a generic formula to calculate the expected time until n planets become habitable. Calculate the expected time until

(a) 1 central planet becomes habitable

(b) 2 central planets become habitable

(c) 3 central planets become habitable

(d) All the four central planets become habitable (5)

(vii) Comment on your answers in (vi) above. (3)

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