INSTITUTE OF ACTUARIES OF INDIA

CT8 – Financial Economics

May 2010 EXAMINATION

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

1.

(a) The fund is 50,00,000 times the value of SENSEX. When the value of portfolio falls by 10% (7200 crores), the value of SENSEX will fall by 10% to 14,400. The manager requires European put options on 50,00,000 times the SENSEX with exercise price of 14,400.
So = 16000, K= 14400, r = 6%, σ = 25%, T - t = 6/12 = 0.5

Therefore
$d1 = [\ln(14400/16000) + (0.06 + 0.25^2 / 2) \times 0.5] / [0.25 \times \sqrt{0.5}] = 0.8541$

$d2 = d1 – (0.25 \times \sqrt{0.5}) = 0.6773$

$N(-d1) = 0.1965$ and $N(-d2) = 0.2491$

The value of one put option is
$14400\, e^{-r(T-t)}\ N(-d2) – 16000\, N(-d1)$
$= 14400\, e^{-0.06x(0.5)}\ 0.2491 – 16000 \times 0.1965$
$= 336.64$

Total cost of hedging is
50,00,000 x 336.64 = 168.32 crores.

(b) From put call parity

$ct + K\, e^{–r(T-t)} = pt + St$

$pt = ct + K\, e^{–r(T-t)} – St$

This shows that the single put option can be created by selling the stock equal to index value, buying a call option and investing remainder at the risk free rate of interest. Therefore, the fund manager should

1. Sell all 8000 crore of stock.
2. Buy call options on 50,00,000 times the SENSEX with exercise price of 14400 and maturity in six months
3. Invest the remaining cash at the risk free rate of 6% per annum.

This strategy gives same result as buying put options.

(c)
The Delta of put option should be invested in risk free securities to hedge the portfolio.

Delta of put option is $–N(-d1) = N(d1) -1$
$= N(.8541) -1 = 0.8035 -1 = -0.1965$

This indicates that 19.65% of the portfolio or 1572.19 crores should be initially sold and invested in risk free securities.

[13]

2.

(a) The process of holding suitable derivatives and the underlying assets in appropriate combinations to eliminate the market risk facing the portfolio is known as hedging.
In speculation, the investor has no market risk exposure to offset. The investor is betting on future movement in price of asset to create a profit.
Arbitrage is generally described as a risk-free trading profit. More accurately, an arbitrage opportunity exists if either an investor can make a deal that would give her or him an immediate profit, with no risk of future loss or an investor can make a deal that has zero initial cost, no risk of future loss, and a non-zero probability of a future profit.

(b) Consider the following two portfolios
Portfolio A: a European call futures option plus an amount of cash equal to Ke^{-rT}
Portfolio B: a European put futures option plus a long futures contract plus an amount of cash equal to $F_0 e^{-rT}$
In portfolio A, the cash can be invested at the risk-free rate, r, and grows to K at time T. Let F_T be the futures price at maturity of the option. If $F_T > K$, the call option in portfolio A is exercised and portfolio is worth F_T. If $F_T < K$, the call is not exercised and portfolio is worth K. The value of portfolio A at time T is therefore

$$Max(F_T, K)$$

In portfolio B, the cash can be invested at the risk-free rate to grow to F_0 at time T. The put option provides a payoff of $F_T - F_0$. The value of portfolio B at time T is therefore

$$F_0 + (F_T - F_0) + Max(K - F_T, 0) = Max(F_T, K)$$

Because the two portfolios have the same value at time T and European options can not be exercised early, it follows that they are worth the same today. The value of portfolio A today is

$$c + Ke^{-rT}$$

The value of portfolio B today is

$$p + F_0 e^{-rT}$$

Hence $c + Ke^{-rT} = p + F_0 e^{-rT}$

(c)
ct + K e^{-rT} = 8 + 95 $e^{-0.06x1}$ = 97.47 [0.5]
pt + F e^{-rT} = 8 + 100 $e^{-0.06x1}$ = 102.18 [0.5]

Put call parity shows that we should but one call, sell one put and sell a futures contract. This will not cost anything at time zero.
In one year, either we will exercise the call or the put will be exercised against us. In either case, we buy the share at Rs.95 and close out the futures position. The gain made in this scenario is 100 – 95 = 5 without any downside risk.

[11]

3

(a)
The risk neutral probability, p, is

p = ($e^{0.06 \text{ x } 3/12}$ – 0.97) / (1.04 – 0.97) = 0.6445

The tree of the movements in stock price and payoff is:

			Payoff
		108.16	6.16
	104		
100		100.88	0
	97		
		94.09	0

[1 for tree and 1 for payoff]

Therefore, the value of call option $Ct = 6.16 \times 0.6445^2 \times e^{-0.06x6/12} = 2.48$.
The value of option is Rs.2.48.

(b) The payoff in this case is

			Payoff
		108.16	0
	104		
100		100.88	1.12
	97		
		94.09	7.91

Therefore, the value of put option
$Pt = [1.12 \times 2 \times 0.6445 \times (1 - 0.6445) + 7.91 \times (1 - 0.6445)^2]\ e^{-0.06x6/12} = 1.47$.
The value of option is Rs.1.47.

(c)
The value of put option plus the price of share = 1.47 + 100 = 101.47

The value of call option plus the present value of strike price = $2.48 + 102\ e^{-0.06x6/12} = 101.47$.

Therefore the put call parity holds.

(d)
We know that the value at the first node at time zero for the European put option is Rs.1.47

The value at the up node (B) at the end of first three months period will be
$1.12 \times (1 - 0.6445)\ e^{-0.06x3/12} = 0.39$. [0.5]

The value at the down node (C) at the end of first three months period will be
$[1.12 \times 0.6445 + 7.91 \times (1 - 0.6445)]\ e^{-0.06x3/12} = 3.48$ [0.5]

Therefore the value of European option at various nodes at the tree is

		0
	0.39	
1.47		1.12
	3.48	
		7.91

The payoff from the American option at various nodes would be

		0
	0	
2		1.12
	5	
		7.91

Therefore. American option can be exercised at nodes A or C.

[10]

4

(a)

A one-factor model is one in which there is assumed to be only one source of randomness affecting the short rate. This randomness is usually modeled using Brownian motion.

(b)

If we look at historical interest rate data we can see that changes in the prices of bonds with different terms to maturity are not perfectly correlated, as one would expect to see if a one-factor model was correct.

Sometimes we even see, for example, that short-dated bonds fall in price while long-dated bonds go up.

If we look at the long run of historical data we find that there have been sustained periods of both high and low interest rates with periods of both high and low volatility. Again these are features that are difficult to capture without introducing more random factors into a model.

This issue is especially important for two types of problem in insurance:

1. the pricing and hedging of long-dated insurance contracts with interest rate guarantees
2. asset-liability modeling and long-term risk-management.

We need more complex models to deal effectively with derivative contracts that are more complex than, say, standard European call options. For example, any contract that makes reference to more than one interest rate should allow these rates to be less than perfectly correlated.

(c) The 6 month rate is

$e^{(r/2)} = 100 / 94$

$r = 2 \times \ln(100/94) = 12.38\%$

The 12 month rate is

$e^{(r)} = 100 / 89$

$r = \ln(100/89) = 11.65\%$

For 1.5 year rate,

$4\, e^{(-0.1238 \times 0.5)} + 4\, e^{(-0.1165 \times 1)} + 104\, e^{(-1.5\, r)} = 94.84$

$3.76 + 3.56 + 104\, e^{(-1.5\, r)} = 94.84$

$e^{(-1.5\, r)} = 0.8415$

$r = 11.5\%$

For 2 year rate,

$5\, e^{(-0.1238 \times 0.5)} + 5\, e^{(-0.1165 \times 1)} + 5\, e^{(-0.115 \times 1.5)} + 105\, e^{(-2\, r)} = 97.12$

$e^{(-25\, r)} = 0.7977$

$r = 11.3\%$

[12]

5.

(a) The outcome of a default may be that the contracted payment stream is:

- Rescheduled
- Cancelled by the payment of an amount which is less than the default-free value of the original contract
- Continued but at a reduced rate
- Totally wiped out.

(b) A credit event is an event that will trigger the default of a bond and includes the following:

- Failure to pay either capital or a coupon
- Loss event (ie where the company says that it is not going to make a payment)
- Bankruptcy
- Rating downgrade of the bond by a rating agency such as Standard and Poor's or Moody's.

[4]

6.

(i) It is usually assumed that people prefer more wealth to less. This is known as the principle of non-satiation and can be expressed as:

$U'(w)>0$

(ii) Ram's utility function is given by

$$U(w) = \begin{cases} 4\left(\frac{w}{10000}\right)^2, & w < 5{,}000 \\ \frac{3w}{5000} - 2, & 5{,}000 < w \leq 10{,}000 \\ \log_{10} w, & w > 10{,}000 \end{cases}$$

This implies

$$U'(w) = \begin{cases} \frac{8w}{10000^2}, & w \leq 5{,}000 \\ \frac{3}{5000}, & 5{,}000 < w \leq 10{,}000 \\ \frac{1}{w \log_e 10}, & w > 10{,}000 \end{cases}$$

Ram's marginal utility is strictly positive at all levels of wealth. We can therefore say that Ram is a non satiated investor.

(iii) Risk Averse Investor

A *risk-averse* investor values an incremental increase in wealth less highly than an incremental decrease and will reject a fair gamble. The utility function condition is:

$U''(w)<0$

Risk Seeking Investor

A *risk-seeking* investor values an incremental increase in wealth more highly than an incremental decrease and will seek a fair gamble. The utility function condition is:

$U''(w)>0$

Risk Neutral Investor

A *risk-neutral* investor is indifferent between a fair gamble and the *status quo*. In this case:

$U''(w)=0$

(iv) From (ii) above we have

$$U'(w) = \begin{cases} \dfrac{8w}{10000^2}, & w \le 5{,}000 \\ \dfrac{8}{5000}, & 5{,}000 < w \le 10{,}000 \\ \dfrac{1}{w \log_e 10}, & w > 10{,}000 \end{cases}$$

This implies

$$U''(w) = \begin{cases} \dfrac{8}{10000^2}, & w \le 5{,}000 \\ 0, & 5{,}000 < w \le 10{,}000 \\ \dfrac{-1}{w^2 \log_e 10}, & w > 10{,}000 \end{cases}$$

or

$$U''(w) \text{ is } \begin{cases} > 0, & w \le 5{,}000 \\ = 0, & 5{,}000 < w \le 10{,}000 \\ < 0, & w > 10{,}000 \end{cases}$$

As per the definition Ram is

$$Risk\ seeking\ for\ w \le 5000$$
$$Risk\ neutral\ for\ 5{,}000 < w \le 10{,}000$$
$$Risk\ Averse\ for\ w > 10{,}000$$

[10]

7.

(i) Two new, uncorrelated factors, F_1^* and F_2^* can be constructed as follows

First, let $F_1^* = F_1$

Then carry out a linear regression analysis to determine the parameters γ_1 and γ_2 in the equation:

$$F_2 = \gamma_1 + \gamma_2 F_1^* + d_2$$

Then set:

$$F_2^* = d_2 = F_2 - (\gamma_1 + \gamma_2 F_1^*)$$

By construction F_2^* is uncorrelated with F_1^*

(ii) If we define

V_{F1} as the variance of F_1

V_{ei} as the variance of e_i

and assume that

e_i is uncorrelated with F_1 and

e_i is independent of e_j for all $i \neq j$ then

a. $V_i = \mathrm{Var}(a_i + b_i F_1 + e_i) = b_i^2 \mathrm{Var}(F_1) + \mathrm{Var}(e_i) = b_i^2 V_{F1} + V_{ei}$

b. $C_{ij} = \mathrm{Cov}[R_i, R_j] = \mathrm{Cov}[a_i + b_i F_1 + e_i, a_j + b_j F_1 + e_j]$
$= b_i b_j \mathrm{Cov}[F_1, F_1] = b_i b_j V_{F1}$

c.
$$V_p = \mathrm{Var}\left(\frac{1}{n}R_1, \frac{1}{n}R_2, \frac{1}{n}R_3 \ldots n\ variables\right)$$
$$= \frac{1}{n^2}\left(\sum_{i=1}^{n} V_i + \sum_{i=1}^{n} \sum_{j \neq i}^{n} C_{ij}\right)$$
$$= \frac{1}{n^2}\left(\sum_{i=1}^{n} (b_i^2 V_{F1} + V_{ei}) + \sum_{i=1}^{n} \sum_{j \neq i}^{n} b_i b_j V_{F1}\right)$$
$$= \frac{1}{n^2}\left(\sum_{i=1}^{n} V_{ei} + \sum_{i=1}^{n} \sum_{j=1}^{n} b_i b_j V_{F1}\right)$$

(iii) From (ii) c above we have

$$V_p = \frac{1}{n^2}\left(\sum_{i=1}^{n} V_{ei} + \sum_{i=1}^{n} \sum_{j=1}^{n} b_i b_j V_{F1}\right)$$

$$= \frac{1}{n}\left\{\frac{1}{n}\sum_{i=1}^{n} \mathrm{Var}(e_i)\right\} + \left(\frac{1}{n}\sum_{i=1}^{n} b_i\right)\left(\frac{1}{n}\sum_{j=1}^{n} b_j\right) V_{F1}$$

$$= \frac{1}{n}DR + \beta_p^2 * \text{Variance of BSE Sensex}$$

Hence proved.

(iv) In the limiting case where n→ ∞ the DR term above disappears hence

$$V_p \to \beta_p^2 * \text{Variance of BSE Sensex}$$

As the number of securities 'n' gets large, the contribution to the portfolio variance of the variances of the individual securities diminishes and goes to zero.

However, the contribution of the covariance terms approaches the average covariance as n gets large.

The individual risk of securities can be diversified away, but the contribution to the total risk caused by the covariance terms cannot be diversified away.

[14]

8.

(i) The market price of risk is $\frac{E_M - r}{\sigma_M}$

where:

E_M is the expected return on the market portfolio

r is the risk-free rate of return

σ_M is the standard deviation of the return on the market portfolio

(ii) Using data from the table we have

$$E_M = [\ 100 * (0.8 * 40\% + 0.15 * 20\% + 0.05 * 10\%$$
$$+ 50 * (0.8 * 10\% + 0.15 * 15\% + 0.05 * 20\%$$
$$+ 25 * (0.8 * 0\% + 0.15 * 60\% + 0.05 * 20\%\ ] / 175$$

$$E_M = 24.92857\%$$

$$\sigma_M^2 = [\ 0.8 * (100 * 40\% + 50 * 20\% + 25 * 0\%)^2$$
$$+ 0.15 * (100 * 20\% + 50 * 15\% + 25 * 60\%)^2$$
$$+ 0.05 * (100 * 10\% + 50 * 20\% + 25 * 20\%)^2\ ] - E_M^2$$

$\sigma_M^2 = 0.062194\%$ or

$\sigma_M = 2.49387\%$

$r = 20\%$

Market price of risk is $\frac{E_M - r}{\sigma_M} = 1.97627$

[7]

9.

Examples of over-reaction to events (any two)

1. Past performance – past winners tend to be future losers and vice versa – the market appears to over-react to past performance.

2. Certain accounting ratios appear to have predictive powers, eg companies with high earnings to price, cashflow to price and book value to market value – generally poor past performers – tend to have high future returns. Again an example of the market apparently over-reacting to past growth.

3. Firms coming to the market; in the US evidence appears to support the idea that stocks coming to the market by Initial Public Offerings and Seasoned Equity Offerings have poor subsequent performance.

Examples of under-reaction to events (any two)

1. Stock prices continue to respond to earnings announcements up to a year after their announcement. This is an example of under-reaction to information which is slowly corrected.

2. Abnormal excess returns for both the parent and subsidiary firms following a de-merger. Another example of the market being slow to recognise the benefits of an event.

3. Abnormal negative returns following mergers (agreed takeovers leading to the poorest subsequent returns). The market appears to over-estimate the benefits from mergers, the stock price slowly reacts as its optimistic view is proved to be wrong.

[4]

10. (a) If we define $f(x,t) = xe^{(R-r)(T-t)}$ then $A(t) = f(S(t),t)$

We can also derive the following equations

$$\frac{\partial}{\partial x} f(x,t) = e^{(R-r)(T-t)} \text{ , } \frac{\partial^2}{\partial x^2} f(x,t) = 0 \text{ and } \frac{\partial}{\partial t} f(x,t) = -f(x,t)(R-r)$$

By ito's lemma we have

$$\begin{aligned} dA(t) &= e^{(R-r)(T-t)} dS(t) + 0 - f(S(t),t)[R-r]dt \\ &= e^{(R-r)(T-t)} S(t)[a\,dt + b\,dZ(t)] + 0 - f(S(t),t)[R-r]dt \\ &= A(t)[\,b\,dZ(t) + (a+r-R)\,dt] \end{aligned}$$

(i) In simple terms, a martingale is a stochastic process for which its current value is the optimal estimator of its future value

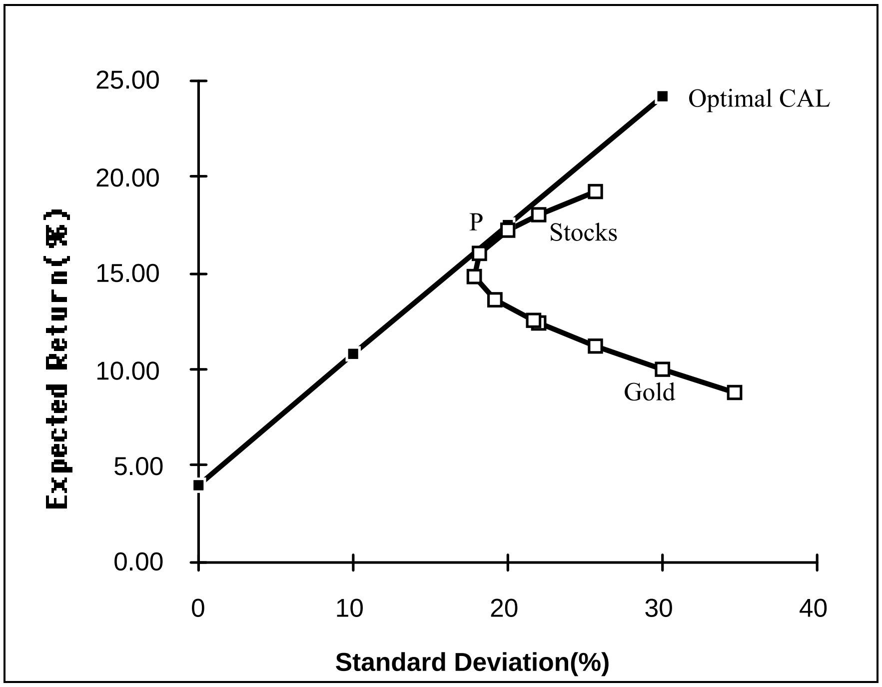
(ii) For A(t) to be a martingale the drift term should be zero or

$a + r - R = 0$

[8]

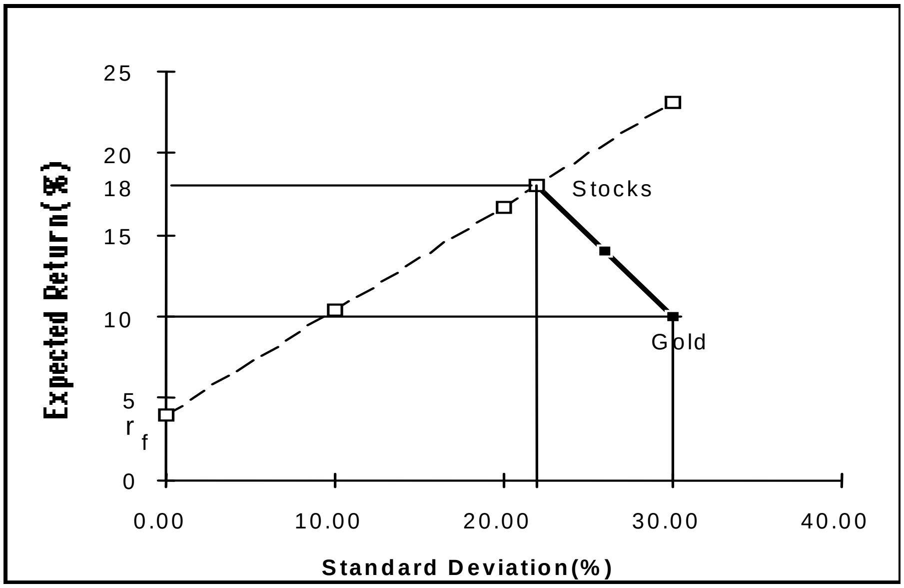
11.

a.



Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.

b. If the correlation between gold and stocks equals +1, then no one would hold gold. The optimal CAL would be comprised of bills and stocks only. Since the set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph), these combinations would be dominated by the stock portfolio. Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.



[7]

[Total Marks 100]

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