Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2010 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Question 1

Variance of temporary annuity due

= Var [$\ddot{a}_{\min(Kx+1,n)}$]

=Var [(1- $v^{\min(Kx+1,n)}$)/d]

= Var ($v^{\min(Kx+1,n)}$)/ d^2

=[$^2Ax:n¬$- $(Ax:n¬)^2$]/d^2

Where $^2Ax:n¬$ is endowment assurance function at rate (1+i)^2 -1

[4]

Question 2

Direct expenses are those that vary with the amount of business written.

Direct expenses are divided into:

- Initial expenses
- Renewal expenses
- Termination expenses

Examples of each:

- Initial expenses – those arising when the policy is acquired e.g. initial commission
- Renewal expenses – those arising regularly during the policy term e.g. renewal commission
- Termination expenses – those arising when the policy terminates; for example – expenses on settling the death claim, maturity and surrender

[4]

Question 3

(i) Policy Reserve at Time 10:

${}_{10}V$ = 100000*$\bar{A}^{1}_{55:\overline{10|}}$ - 1897.24* $\ddot{a}_{55:\overline{10|}}$

Now,

$$\bar{A}^{1}_{55:\overline{10|}} = \sqrt{1.04}\left(A_{55} - \frac{D_{65}}{D_{55}}A_{65}\right)$$

=(1.04)^(0.5) [0.38950- (689.23/1105.41)*0.52786]

=0.06157

So,

${}_{10}V$= 100,000*0.06157 - 1897.24*8.219
= - 9436.42
i.e. a negative reserve of Rs.9436.42.

(ii) Explanation of negative reserve

It may be noted that the policy reserve is negative which resulted from the expected cost of the benefits after ten years being lower than the expected value of the remaining premiums.

When the reserve is negative, it means that the policyholder "owes" that amount of money to the company. Should the policy lapse at such a time, then the debt becomes unrecoverable and the company makes a loss. The company can't impose a surrender penalty with term assurance because the surrender value is usually 0.

Altering the product design

The product terms should be altered so that, at any time during the 20-year policy term, the reserve cannot be negative and the life insurer does not lose on lapse of the policy. In other words, we need to ensure that:

EPV of future benefits is higher than EPV of future premiums payable at any time during the policy term.

The following could be achieved by any one or more of the following combinations:

(1) Reduce the sum assured payable in the first ten years relative to the sum assured payable in the second ten years.

(2) Increase the premiums payable in the first ten years relative to the premiums payable in the second ten years.

(3) Reduce the total premium paying term (which effectively achieves (2) above).

(4) Incorporate an endowment benefit payable on survival to the end of the term. This way, the company could impose a surrender penalty.

[9]

Question 4

$$(aq)_x^{\alpha} = \int_0^1 {}_t(ap)_x\,(a\mu)_{x+t}^{\alpha}\,dt = \int_0^1 ({}_tp_x^{\alpha})({}_tp_x^{\beta})(\mu_{x+t}^{\alpha})\,dt$$

$$= \int_0^1 ({}_tp_x^{\alpha}\,\mu_{x+t}^{\alpha})\,{}_tp_x^{\beta}dt$$

$$\text{But } {}_tp_x^{\alpha}\,\mu_{x+t}^{\alpha} = -\frac{d}{dt}({}_tp_x^{\alpha})$$

$$\Rightarrow {}_tp_x^{\alpha}\ \mu_{x+t}^{\alpha} = -\frac{d}{dt}(1-t^2/x)$$

$$= 2t/x$$

$$(aq)_x^{\alpha} = \int_0^1 \frac{2t}{x}\,(t/x)\,dt$$

$$= \int_0^1 2t^2/x^2\,dt$$

$$= \left[\frac{2t^3}{3x^2}\right]_0^1$$

$$= \frac{2}{3x^2}$$

[6]

Question 5

The benefit amount on death will be Rs. 10,000 for the first year, Rs.10,000 *1.04 for the second year, Rs.10,000 *(1.04)^2 for the third year, etc, and the benefit amount on maturity will be Rs.10,000 *(1.04)^20 .

So the premium equation can be expressed as:

$$P\ddot{a}_{[40]:\overline{20|}} = (10000/1.04)* A^{1\ @0\%}_{[40]:\overline{20|}} + 10000* \frac{D_{60}}{D_{[40]}}^{@0\%} +150 + 300* A^{@4\%}_{[40]:\overline{20|}}$$

The functions evaluated at 0% are:

$$A^{1\ @0\%}_{[40]:\overline{20|}} = \frac{l_{[40]} - l_{60}}{l_{[40]}}$$

=0.05755

And

$$\frac{D_{60}}{D_{[40]}}^{@0\%} = \frac{l_{60}}{l_{[40]}}$$

= 0.94245

Now premium equation becomes:

13.930P=(10000 /1.04)*0.05755 +10000* 0.94245+ 150+ 300 *0.46423
= 10267

And the premium is:

P= 10267/13.930
= Rs. 737

[4]

Question 6

a) Investment return :

Policy Year	Nifty Index (year end)	Annual Return on Nifty
	4000	
1	3800	-5.00%
2	4294	13.00%
3	4766	11.00%
4	4290	-10.00%
5	4977	16.00%

Unit Account Statement:

Policy Year	Annual Premium	Allocation Charge	Administration Charge	Fund Value at start of year	Fund Value at end of year before FMC	FMC	Fund Value at end of year after FMC
1	100,000	5,000	720	94,280	89,566	2,239	87,327
2	100,000	5,000	720	181,607	205,216	5,130	200,085
3	100,000	5,000	720	294,365	326,746	8,169	318,577
4	100,000	5,000	720	412,857	371,571	9,289	362,282
5	100,000	5,000	720	456,562	529,612	13,240	516,372

b) Guaranteed Maturity benefit

Policy Year	Premium	Guarantee amount
1	100,000	102,500
2	100,000	207,563
3	100,000	315,252
4	100,000	425,633
5	100,000	538,774

Policy Year	Allocation Charge	Administration Charge	Total Expense + Commission	FMC	Investment Income	Cost of Guarantee	Profit
1	5,000	720	4,500	2,239	61	0	3,520
2	5,000	720	4,500	5,130	61	0	6,411
3	5,000	720	4,500	8,169	61	0	9,450
4	5,000	720	4,500	9,289	61	0	10,570
5	5,000	720	4,500	13,240	61	22,402	- 7,881

Policy Year	No of Lives at start of year	Mortality rate	No of Deaths	No of Lives at end of year	Profit after allowing for probabilities
1	1	0.002	0.002	0.998	3,520
2	0.998	0.002	0.002	0.996	6,399
3	0.996	0.002	0.002	0.994	9,412
4	0.994	0.002	0.002	0.992	10,507
5	0.992	0.002	0.002	0.990	- 7,818

Net Present Value = Rs 17,184

Value of New Business = 17.18%

c)

Policy Year	Profit before reserves	Reserves	Increase in reserve	Interest earned on reserve	Profit after allowing for reserves	No of Lives at end of year	Profit after allowing for probabilities
1	3,520	2,183	2,183	0	1,337	1.000	1,337
2	6,411	5,002	2,819	109	3,702	0.998	3,694
3	9,450	7,964	2,962	250	6,737	0.996	6,711
4	10,570	9,057	1,093	398	9,876	0.994	9,817
5	- 7,881	-	- 9,057	453	1,629	0.992	1,616

Net Present Value = Rs 16,071

Value of New Business = 16.07%

d) If r is the IRR to the customer then:

100,000 + 100,000/(1+r) + ……….+ 100,000/(1+r)^4 – 516372/(1+r)^5 = 0 (1)

Therefore:
IRR = 1.08%

[20]

Question 7

i. Let P be the single premium

$P = (D_{60}/D_{40}) * [100,000 * \ddot{a}_{60} + P\ A_{60}] + 1000 + 0.01\ P + 200 * (\ddot{a}_{40} – 1)$ @4%

= (882.85/ 2052.96) *[100,000*14.134 + P * 0.45640] + 1000 + 0.01P +200*(20.005-1)

P = 611685 / (1 – 0.218296) = 771,927
[Note: Give credit for round off error]

ii. Reserve per inforce policy at the end of 25 years

${}_{25}V = (100000 + 200) * \ddot{a}_{65} + P\ A_{65}$

= 100200* 12.276 + 771,927 * 0.52786 = 16,37,525

Mortality profit Calculations:

Death strain at risk per policy at beg of year
= Sum payable on death – (reserve at year end)

= P – (16,37,525) = 771,927 – 16,37,525 = - 865,598

Actual death strain = Number of deaths * death strain at risk

= 10 * (- 865,598) = - 86,55,980

Expected death strain = 1000 * q_{64} * death strain at risk
= 1000* 0.012716 * (- 865598) = - 110,06,944

Mortality Profit = EDS – ADS = - 110,06,944 – (-86,55,980) = - 23,50,964

i.e. there was a mortality loss of Rs. 23,50,964 during 25th year.

iii. the insurer has incurred a mortality loss because

- the death strain is negative in 25th year
- actual number of deaths (10) are less than expected number of deaths (1000 * q_{64} = 12.716)
- Since death strain is negative and actual number of deaths are less than expected, the insurer looses money due to additional survivors of 2.716.

[13]

Question 8

(a)

Probability of benefit payment = exp (- $\int_0^{5.5} 0.02t$) = exp (- 0.11) = 0. 8958

Value of the benefit = 500,000 * (1/1.05)^5.5 * 0.8958 = 342,497

(b)

The reserve would be required to be set up in on year before the negative cash flow years i.e. year 3 & 5. Hence the cash flow of years 3 & 5 will be zero.

But the preceding years cash flows i.e. for year 2 and 4 will reduce by Rs.100.
The revised cash flow for year 2 = 200 – 100 = 100
The revised cash flow for year 4 = 100 – 100 = 0

The revised profit vector would be
(-100, 100, 0, 0,0,100)

[5]

Question 9

(a) Single Premium =100,000 *[2V + 2V^2 + 2V^3 ...+ 2V^{20} + 960 * V^{20}] / [1 – 0.05]@5%

= 100,000 * [2 *a_{20} + 960 * V^{20}] / [1 – 0.05] @5%

= =100000*(2*12.4622+960*0.37689)/0.95
= 40,709,347

Single Premium per policy = 40,709,347/ 1000 = 40,709.35

(b) The rationale is that urban lives experience lower mortality than rural lives due to good access of education and medical facilities which affects mortality. If the pattern of mortality is similar for the two group of lives but the average age at death of urban lives is higher than that of rural lives by 5 years, then we can approximate the mortality risk of urban lives by that of rural lives by 5 years younger.

[5]

Question 10

a)

	Outcome	Cash flow*
(1)	HHH	25000,0,0
(2)	HHS	25000,0, -35000
(3)	HHD	25000,0,-50000
(4)	HSH	25000,35000,0
(5)	HSS	25000,-35000,-35000
(6)	HSD	25000,-35000,-50000
(7)	HD	25000,-50000

* Positive sign means cash flow to the company and negative means cash flow to the policyholder

b)

Completing the set of transition probabilities:

p^{HH}_{55+t} + p^{HS}_{55+t} + p^{HD}_{55+t} = 1

Therefore p^{HH}_{55+t} = 0.75

p^{SS}_{55+t} + p^{SH}_{55+t} + p^{SD}_{55+t} = 1

Therefore p^{SS}_{55+t} = 0.3

The probability that each outcome occurs is:

	Outcome	Probability
(1)	HHH	0.5625
(2)	HHS	0.12
(3)	HHD	0.0675
(4)	HSH	0.096
(5)	HSS	0.048
(6)	HSD	0.016
(7)	HD	0.09

c) The net present value of each outcome is

	NPV of profit
(1)	25000
(2)	-6746.03
(3)	-20351.5
(4)	-8333.33
(5)	-40079.4
(6)	-53684.8
(7)	-22619

d)

$$\text{Mean} = \Sigma \text{ NPV} \times \text{liability}$$

= 25000*0.5625 – 6746.03*0.12 + …..
= Rs 6260.77

$$\text{Variance} = \Sigma \text{NPV}^2 \times \text{Probability} - (\text{Mean})^2$$

= 521,714,224.01

Standard Deviation = Rs 22,841.06

[15]

Question 11

Insurance Company received P, so guaranteed maturity benefit = $(1.06)^3$ * P = 1.191016P

The company invests P @ an average interest rate of 6.5% so due to receive 1.20795P

On Death the company does not lose money because it pays out exactly the value of asset available. This occurs with probability ${}_3q_{60}$ = (1 - $\frac{l_{63}}{l_{60}}$) = 0.0463

At maturity (t=3) the insurance company loses money only if yields at the time are j such that:
{1.20795P/ (1+j)} < 1.191016P
i.e. (1+j) > 1.014218

Prob(1+j > 1.014218) for lognormal (1+j)
= Prob (z > [Ln 1.014218 – 0.003] / 0.01) from standard normal
= Prob (z > 1.11) = 1 – 0.86650 = 0.1335
Maturity occurs with probability 1 – 0.0463 = 0.9537
So the overall probability of loss is:
= 0.9537 * 0.1335 = 0.1273

[6]

Question 12

$$\int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \; \bar{a}_{\overline{y+t:n}|} \, dt$$

$$= \int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \int_0^n v^s \; {}_sp_{y+t} \, ds \, dt$$

$$= \int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \left[\int_0^\infty v^s \; {}_sp_{y+t} \, ds - \int_n^\infty v^s \; {}_sp_{y+t} \, ds \right] dt$$

$$= \int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \left(\int_0^\infty v^s \; {}_sp_{y+t} \, ds \right) dt$$

$$- \int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \left(\int_n^\infty v^s \; {}_sp_{y+t} \, ds \right) dt$$

Substituting $r = s + t$ into (1) we get:

(1) $$= \int_0^\infty v^t \; {}_tp_{xy} \; \mu_{x+t} \left(\int_t^\infty v^{r-t} \; {}_{r-t}p_{y+t} \, dr \right) dt$$

$$= \int_0^\infty {}_tp_x \; \mu_{x+t} \; v^t \; {}_tp_y \left(\int_t^\infty v^{r-t} \; {}_{r-t}p_{y+t} \, dr \right) dt$$

$$= \int_0^\infty {}_tp_x \; \mu_{x+t} \left(\int_t^\infty v^r \; {}_rp_y \, dr \right) dt$$

Reversing integrals:

(1) $$= \int_0^\infty v^t \ {}_tp_y \left(\int_0^t {}_rp_x \ \mu_{x+r} \ dr \right) dt$$

$$= \int_0^\infty v^t \ {}_tp_y \ {}_tq_x \ dt$$

$$= \int_0^\infty v^t \ {}_tp_y \left(1 - {}_tp_x\right) dt$$

$$= \int_0^\infty v^t \ {}_tp_y \ dt - \int_0^\infty v^t \ {}_tp_{xy} \ dt$$

$$= \bar{a}_y - \bar{a}_{xy}$$

Substituting $r = s - n + t$ into (2) we get:

(2) $$= \int_0^\infty {}_tp_x \ \mu_{x+t} \ v^t \ {}_tp_y \left(\int_t^\infty v^{r+n-t} \ {}_{r+n-t}p_{y+t} \ dr \right) dt$$

$$= \int_0^\infty {}_tp_x \ \mu_{x+t} \ v^n \ {}_np_y \left(\int_t^\infty v^r \ {}_rp_{y+n} \ dr \right) dt$$

Reversing integrals:

(2) $$= v^n \ {}_np_y \int_0^\infty v^t \ {}_tp_{y+n} \left(\int_0^t {}_rp_x \ \mu_{x+r} \ dr \right) dt$$

$$= v^n \ {}_np_y \int_0^\infty v^t \ {}_tp_{y+n} \ {}_tq_x \ dt$$

$$= v^n \ {}_np_y \left[\bar{a}_{y+n} - \bar{a}_{x:y+n} \right]$$

Finally:

$$(1) - (2) = \bar{a}_y - \bar{a}_{xy} - v^n \ {}_np_y \ \bar{a}_{y+n} + v^n \ {}_np_y \ \bar{a}_{x:y+n}$$

$$= \bar{a}_{y:\overline{n}|} + v^n \ {}_np_y \ \bar{a}_{x:y+n} - \bar{a}_{xy}$$

[9]
Total Marks 100

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