Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2008 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Question 1
(i)

- Premiums (+)
- Investment Income on reserves and cashflows (+)
- Investment Income on solvency margin (+)
- Expenses (-)
- Commission (-)
- Tax (-)
- Claims (-)
- Increase in reserve (-)
- Increase in solvency margin (-)

*(note: no marks for 2 correct cash flows after that half mark for each correct one ; **maximum 3**; no marks for a point if the sign of the cash flow is not given)*

(ii)

Profit Vector

The profit vector is the expected profit at the end of each policy year per policy in force at the beginning of that policy year e.g. ((–800, 186, 178, 206, 215) could be the profit vector for a 5 year product.

Profit Signature

The vector of expected profits per policy issued is called the profit signature. For example if $(PRO)_t$ is the profit vector, ${}_{t-1}P_x * (PRO)_t$ is the profit signature; where ${}_{t-1}P_x$ is the probability that a policy issued to a life aged x is inforce at the beginning of t^{th} year (or at the end t-1 years).

Net present Value of Profit

Net Present Value (NPV) of the profit signature determined using the risk discount rate.

$$\mathbf{NPV} = \sum_{t=1}^{t=\infty} (1+i_d)^{-t} (PS)_t$$

Where i_d is the risk discount rate

Profit Margin
Profit Margin is the expected NPV of the profit signature expressed as a percentage of the expected net present value of the premium income. If the premium paid at the beginning of the *t* th policy year is P_t , this is

$$\frac{\sum_{t=1}^{t=\infty}(1+i_d)^{-t}\ (PS)_t}{\sum_{t=1}^{t=\infty}(1+i_d)^{-(t-1)}\ {}_{t-1}p_x\ P_t}$$

[7]

Question 2

We know that

$l_x = l_0\, e^{-\int \mu_t\, dt}$ where l_0 is the arbitrary radix

therefore $l_x = l_0\, e^{-\int \left(\frac{1}{100-t}\right) dt}$

$l_x = l_0\, e^{[\log_e (100-t)]}$

$= l_0\, e^{\log e \left[\frac{100-x}{100}\right]}$

$= l_0 \left[\frac{100-x}{100}\right]$

$l_x = K\,(100 - x)$

where is K is the arbitrary radix.

[5]

Question 3

i) EPV = 20,000 $a_{\overline{68:65}}$ = 20,000 (a^m_{68} + a^f_{65} - $a_{68:65}$)

= 20,000 (11.412 + 13.871 – 10.112) = 20,000 (15.171)

= 303,420

ii) The office loses money if PV of payments > 320,000

i.e. if 20,000 $a_{\overline{n}|}$ > 320,000 or $a_{\overline{n}|}$ > 16

at 4%pa, a_{26} = 15.9828 and a_{27} = 16.3296 so if the office makes the 27th payment under this annuity, it incurs a loss. It, therefore, makes a profit so long as both lives have died before this time, with probability ${}_{27}q_{\overline{68:65}}$

$${}_{27}q_{\overline{68:65}} = ({}_{27}q_{68}^{m})({}_{27}q_{65}^{f}) = \left(1 - \frac{l_{95}^{m}}{l_{68}^{m}}\right)\left(1 - \frac{l_{92}^{f}}{l_{65}^{f}}\right)$$

$$= \left(1 - \frac{1020.409}{9440.717}\right)\left(1 - \frac{3300.559}{9703.708}\right)$$

$$= (0.891914)(0.65987)$$

$$= 0.5885$$

[6]

Question 4

i. Under a unit linked contract if some of the future net cash flows are negative, a non unit reserve will be required to be set up to meet the cash flow requirement in those years where there is negative net cash flow.

- Stronger reserving basis than used in pricing results into higher reserve requirement.
- Higher reserve requirement defers the profits from earlier years to later years.
- If the risk discount rate is higher than the interest earned rate on reserve, the present value of profits after allowing for reserve will reduce
- Therefore, stronger reserving basis reduces the profitability and vice a versa.

ii.

Reserve at the beginning of year 5, ${}_5V = 0$

${}_4V = (0 - 200) / 1.06 = - 188.68$ (since this is negative, so ${}_4V$ will taken as zero)

${}_3V = (0 + 400) / 1.06 = 377.36$

${}_2V = (377.36 - 1000) / 1.06 = - 587.40$ (since this is negative, so ${}_3V$ will taken as zero)

${}_1V = (0 + 200) / 1.06 = 188.68$

Thus the reserve requirement is (188.68, 0, 377.36, 0, 0)

The net cash flows, after allowing for increase in reserve, are (- 488.68, 0, 622.64, 0, 200)

[8]

Question 5

(Note for Markers: There are two interpretations for timing of expenses (beginning or end of year) in non unit fund cash flows in part b. Both interpretations should be given equal credit. The solution under both interpretations is given below.)

a)

Unit Account Value for Charge Structure A:

Policy Year	Premium	Allocated Premium	Cost of Allocation	Fund b/f	Fund before charge	FMC	Fund c/f
1	100000	80000	76000	0	83600.00	836.00	82,764
2	100000	101000	95950	82764.00	196585.40	1965.85	194,620
3	100000	101000	95950	194619.55	319626.50	3196.27	316,430

Unit Account Value for Charge Structure B:

Policy Year	Premium	Allocated Premium	Cost of Allocation	Fund b/f	Fund before charge	FMC	Fund c/f
1	100000	94000	89300	0	98230.00	982.30	97248
2	100000	94000	89300	97247.70	205202.47	2052.02	203150
3	100000	94000	89300	203150.45	321695.49	3216.95	318479

b) Profit Testing of Charge Structure A

(Interpretation 1: expenses are taken at the beginning of the year)

Policy Year	Prob of inforce at beg	Prob of inforce at end	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	Surrender Charge	FMC	Profit	PV of Profit
1	1.000	0.849	24000	2000	10000	500	840.00	0	836.00	13176.00	11978.18
2	0.849	0.763	4050	500	0	400	243.00	0	1965.85	5358.85	3760.06
3	0.7633	0.762	4050	500	0	300	243.00	0	3196.27	6689.27	3835.90
											19,574

(Interpretation 2: expenses are taken at the end of the year)

Policy Year	Prob of inforce at beg	Prob of inforce at end	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	Surrender Charge	FMC	Profit	PV of Profit
1	1.000	0.849	24000	2000	10000	500	720.00	0	836.00	13056.00	11869.09
2	0.849	0.763	4050	500	0	400	213.00	0	1965.85	5328.85	3739.01
3	0.76325	0.762	4050	500	0	300	213.00	0	3196.27	6659.27	3818.70
											19,427

Profit Testing of Charge Structure B

(Interpretation 1: expenses are taken at the beginning of the year)

Policy Year	Prob of inforce at beg	Prob of inforce at end	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	Surrender Charge	FMC	Profit	PV of Profit
1	1.000	0.849	10700	2000	10000	500	42.00	4500	982.30	3724.30	3385.73
2	0.849	0.763	10700	500	0	400	642.00	1500	2052.02	13994.02	9818.95
3	0.7633	0.762	10700	500	0	300	642.00	0	3216.95	13758.95	7889.96
											21,095

(Interpretation 2: expenses are taken at the end of the year)

Policy Year	Prob of inforce at beg	Prob of inforce at end	Profit on Allocation	Expense	Commission	Death claims	Non Unit interest	Surrender Charge	FMC	Profit	PV of Profit
1	1.000	0.849	10700	2000	10000	500	-78.00	4500	982.30	3604.30	3276.64
2	0.849	0.763	10700	500	0	400	612.00	1500	2052.02	13964.02	9797.90
3	0.76325	0.762	10700	500	0	300	612.00	0	3216.95	13728.95	7872.76
											20,947

(Note: Make some allowance for round off errors in the intermediate and final answers)

The value of new business under charge structure B is higher than that of under charge structure A. Hence charge structure B is more profitable to the Company.

c) New Business Strain

- New business strain under both the charge structures A & B is Nil as the net cashflows are positive in the first policy year.
- Both the charge structures release the profit to the Company in first year itself but the charge structure A releases profit much earlier than that of the charge structure B. Therefore charge structure A is more efficient to the Company in its overall capital management.
- However, the charge structure B is more profitable than A. So the capital efficient charge structure leads to lower profit margin. Company needs to decide whether it wants to promote the capital efficient model or higher profitable sales model.

[12]

Question 6

i) The present value random variable is

$$PVRV = \begin{cases} 100v & \text{if } K_x = 0 \\ 200v^2 & \text{if } K_x = 1 \\ 0 & \text{if } K_x \geq 2 \end{cases}$$

ii) Standard deviation

The EPV of the benefit is

$$EPV = 100vq_x + 200\, v^2 p_x q_{x+1}$$

$$= \frac{100}{1.06} \times 0.025 + \frac{200}{1.06^2} \times 0.975 \times 0.030$$

$$= 7.56497$$

And

$$E(PV^2) = (100\, v)^2 q_x + (200\, v^2)^2 p_x q_{x+1}$$

$$= \left(\frac{100}{1.06}\right)^2 \times 0.025 + \left(\frac{200}{1.06^2}\right)^2 \times 0.975 \times 0.030$$

$$= 1149.24870$$

So variance of the PVRV is:

$Var(rv) = E[PV^2] - \{E[PV]\}^2 = 1092.01993$

And standard deviation = $\sqrt{1092.01993} = 33.0457$

[6]

Question 7

Higher level of education and higher general awareness reduces the probability of illness and death for the following factors:

- Higher income increases the affordability of health care
- Choice of healthy diet reduces the risk of illness and death
- Personal health care and regular exercise to improve fitness level
- Moderation in alcohol consumption and smoking
- Awareness of dangers of drug abuse
- Awareness of safe sexual lifestyle

- Avoiding dangerous activities such as motor racing, hang gliding
- Precautions against infectious deceases

Occupation

- Occupation determines the persons environment where the life passes, say 40 to 50 hours every week
- Occupation may involve exposure to harmful substances such as chemicals, smoke etc.
- Occupation may increases the risk of accident leading to illness or death e.g. workers in mines etc.
- Sedentary occupation such as office work may be less healthy than a fitness instructor
- Occupations normally determines their income which may determine their lifestyle, diet, quality of housing etc. and may ultimately have an inlfluence on the mortality/ morbidity

[5]

Question 8

Advantages of single figure indices are

- Some single figure indices such as crude death rates are easy to calculate
- They provide the overall summary statistic to give you a feel of the overall picture

Disadvantages of single figure indices are

- The summary indices are weighted averages where age is one of the weight dimensions. Summary measures are, therefore, heavily biased towards older ages as higher the age higher the rate
- Summary measures miss any abnormalities in the age-sex specific rates that may exist. Any errors are less likely to be detected in summary indices.

[3]

Question 9

Benefit A:

The formula for the benefit A is as under:

$$\ddot{a}^{(12)}_{\overline{10}|} = \frac{i}{d^{(12)}} a_{\overline{10}|} = 8.2856$$

Thus the value of benefit A is

20,000 * 8.2856 = 165712

Benefit B:
The formula for the benefit B is as under:

$${}_{10}p_{63}^{m}\ {}_{10}p_{60}^{f} v^{10} (\ddot{a}_{73:70} - \frac{11}{24})$$

Where

$${}_{10}p_{63}^{m} = 0.90398$$

$${}_{10}p_{60}^{f} = 0.95372$$

$$\ddot{a}_{73:70} = \frac{2}{5} \times 10.233 + \frac{3}{5} \times 8.110 = 8.9592$$

(Note: above value has been calculated by using interpolation of table values)

$v^{10} = 0.675564$

Thus the value of benefit B will be

20000 * 0.90398 * 0.95372 * 0.675564 * (8.9592 – 11/24) = 99024

Benefit C

There could be one of following three possibilities:

- Male survives and female spouse dies in next 10 years
- Female survives and male dies in next 10 years
- Both survive next 10 years

Thus the value of the benefit would be as under:

$$15,000v^{10}\left[{}_{10}p_{63}^{m}\left(1-{}_{10}p_{60}^{f}\right)\ddot{a}_{73^m}^{(12)}+\left(1-{}_{10}p_{63}^{m}\right){}_{10}p_{60}^{f}\ \ddot{a}_{70^f}^{(12)}\right.$$
$$\left.+{}_{10}p_{63^m:60^f}\left(\ddot{a}_{73}+\ddot{a}_{70}-2\ddot{a}_{73:70}\right)\right]$$
$$=15,000\times 0.67556\times\left[0.90398\times(1-0.95372)\times\left(10.288-\tfrac{11}{24}\right)\right.$$
$$+\left(1-0.90398\right)\times 0.95372\times\left(12.934-\tfrac{11}{24}\right)$$
$$\left.+0.90398\times 0.95372\times\left(10.288+12.934-2\times 8.9592\right)\right]$$

= 62079 [5 – *3 for right formula and 2 for right calculations*]

Thus the total value of benefit is Rs.326815.

Single Premium, P will be calculated as under:

P = Value of total benefits + Value of expenses + Commission
= 326815 + 500 + 0.02 * P

P = Rs.333, 995

[10]

Question 10

Assumptions and definitions

Lx	number of active members attaining age *x*
Ix	number of active members retiring due to ill health between exact ages *x and x* + 1
$\bar{a}_y^i$	value of an annuity of 1 *pa*, payable in accordance with the scheme rules, to a member retiring due to ill health at exact age *y*
$\frac{s_{x+t}}{s_x}$	ratio of earnings between exact ages $x + t$ and $x + t + 1$ to those between exact ages *x* and $x + 1$
Other assumptions	• Retirements occur half way through the year • New member aged x has salary S, which has just been reviewed and which, therefore, represents the salary between age his age x and x+1

Let

$$z_x = \frac{1}{3}(s_{x-3} + s_{x-2} + s_{x-1})$$

The present value at age x of the ill health retirement pension on retirement at age x + t last birthday (where t < 60 - x) will be

$$\frac{(60-x)}{80} \times S \times v^{t+½} \times \frac{i_{x+t}}{l_x} \times \frac{z_{x+t+½}}{s_x} \times \bar{a}^i_{x+t+½} = \frac{(60-x)}{80} S \frac{^zC^{ia}_{x+t}}{^sD_x}$$

where $^zC^{ia}_x = z_{x+½}\ v^{x+½}\ i_x\ \bar{a}^i_{x+½}$.

Summing over all years until age retirement 60, we get

$$\frac{(60-x)}{80} S \frac{^zM^{ia}_x - {}^zM^{ia}_{60}}{^sD_x}$$

where $^zM^{ia}_x = \sum_{t=0}^{64-x} {}^zC^{ia}_{x+t}$.

[7]

Question 11

Selection is the process by which lives are divided into separate groups so that the mortality (or morbidity) within each group is homogeneous.

Selection can arise as a result of decisions by the insurance company, called underwriting.

Selection can also arise as a result of decisions made by the policyholder (self selection).

Lives in different risk classes are normally charged according to different premium scales, which reflect the mortality differences between the classes.

In practice the homogeneous groups are classified in such a way that they avoid possibility of selection by the proposer against the insurer.

Selective Withdrawals in annuity policies :

It is more likely that a life insured may lapse the cover if he or she is in good health condition against those who are not in good health condition. Therefore the average mortality experience of continuing lives (inforce portfolio) is likely to be worse due to selective withdrawals

If the option to surrender the annuity policy is given, it is likely that those who are in bad health condition will more likely to surrender the policy. This will result into lighter mortality experience of the continuing (inforce) polices.

[5]

Question 12

EPV of benefits

$110{,}000\, A_{[50]:10} - 10{,}000(IA)_{[50]:10}$ (function @ 6% pa)

$= 110{,}000\,\{A_{[50]} - v^{10}{}_{10}p_{[50]}\,A_{60}\} - 10{,}000\{\,(IA)_{[50]} - v^{10}{}_{10}p_{[50]}\,(10A_{60} + IA_{60})\}$

$= 110{,}000\, A_{[50]} - 10{,}000(IA)_{[50]} + v^{10}{}_{10}p_{[50]}\,\{10{,}000(IA_{60}) - A_{60}\}$

$= 110{,}000(0.20463) - (10{,}000)(4.84789) + (0.55839)(0.95684)\{10{,}000(5.46572 - 0.32692)\}$

$= 22{,}509.30 - 48{,}478.90 + 27{,}456.09$

$= 1486.49$

EPV of gross premiums

Let P be annual premiums

$P\, \ddot{a}^{6\%}_{50:10} = 7.698\, P$

EPV of expenses

$200 + 0.25P + 0.02P\, a^{6\%}_{[50]:9} + 50\, \ddot{a}^{4\%}_{[50]:9} + 200\, A^{4\%}_{[50]:10}$

$= 150 + 0.23P + 0.02\, P\, \ddot{a}^{6\%}_{[50]:10} + 50\ddot{a}^{4\%}_{[50]:10} + 200\, (A^{4\%}_{[50]} - v^{10}{}_{10}p_{[50]}\, A^{4\%}_{60})$

$= 150 + 0.23P + 0.02P\,(7.698) + 50(8.318) + 200\{0.32868 - (0.67556)(0.95684)(0.4564)\}$

$= 150 + 415.90 + 6.73 + P(0.23 + 0.15396)$

$= 572.63 + 0.38396P$

Equation of value

$7.698P = 1486.49 + 572.63 + 0.38396P$

$7.31404P = 2059.12$

$P = 281.53$pa

(Alternative solution)

EPV of benefits

$110{,}000\ A_{[50]:10} - 10{,}000(IA)_{[50]:10}$ (function @ 6%pa)

$= 110{,}000\ \{A_{[50]} - v^{10}{}_{10}p_{[50]}\ A_{60}\} - 10{,}000\{\ (IA)_{[50]} - v^{10}{}_{10}p_{[50]}\ (10A_{60} + IA_{60})\}$

$= 110{,}000\ A_{[50]} - 10{,}000(IA)_{[50]} + v^{10}{}_{10}p_{[50]}\ \{10{,}000(IA_{60}) - A_{60}\}$

$= 110{,}000(0.20463) - (10{,}000)(4.84789) + (0.55839)(0.95684)\{10{,}000(5.46572 - 0.32692)\}$

$= 22{,}509.30 - 48{,}478.90 + 27{,}456.09$

$= 1486.49$

EPV of gross premiums

Let P be annual premiums

$P\ \ddot{a}^{(12)\ 6\%}_{50:10} = (7.698 - \frac{11}{24}) = 7.24\ P$

EPV of expenses

$200 + 0.25P + 0.02P\ a^{6\%}_{[50]:9} + 50\ \ddot{a}^{4\%}_{[50]:9} + 200\ A^{4\%}_{[50]:10}$

$= 150 + 0.23P + 0.02\ P\ \ddot{a}^{(12)\ 6\%}_{50:10} + 50\ddot{a}^{4\%}_{[50]:10} + 200\ (A^{4\%}_{[50]} - v^{10}{}_{10}p_{[50]}\ A^{4\%}_{60})$

$= 150 + 0.23P + 0.02P\ (7.24) + 50(8.318) + 200\{0.32868 - (0.67556)(0.95684)(0.4564)\}$

$= 150 + 415.90 + 6.73 + P(0.23 + 0.1448)$

$= 572.63 + 0.3748P$

Equation of value

$7.24P = 1486.49 + 572.63 + 0.3748P$

$6.8662P = 2059.12$

$P = 299.94$pa

[8]

Question 13

i) Let P_x be the annual premium under the whole life assurance issued to a life aged x. Adding it to the prospective reserves, at the end of t years for a whole life assurance, we get,

$${}_tV_x + P_x = A_{x+t} - P_x \ddot{a}_{x+t} + P_x$$

$$= A_{x+t} - P_x (\ddot{a}_{x+t} - 1)$$

$$= A_{x+t} - P_x a_{x+t}$$

writing

$$A_{x+t} = vq_{x+t} + vp_{x+t} A_{x+t+1}$$

and $a_{x+t} = vp_{x+t} \ddot{a}_{x+t+1}$

$${}_tV_x + P_x = vq_{x+t} + vp_{x+t} A_{x+t+1} - P_x vp_{x+t} \ddot{a}_{x+t+1}$$

$$= vq_{x+t} + vp_{x+t} (A_{x+t+1} - P_x \ddot{a}_{x+t+1})$$

$$= vq_{x+t} + v(1 - q_{x+t})\, {}_{t+1}V_x$$

$$= v\, {}_{t+1}V_x + vq_{x+t}(1 - {}_{t+1}V_x)$$

Therefore

$$({}_tV_x + P_x)(1+ i) = {}_{t+1}V_x + q_{x+t} (1 - {}_{t+1}V_x)$$

ii) The reserves at the end of t years increased by the premium then due and accumulated with interest for a year should provide a) the reserves at the end of the (t + 1) years and b) the excess of the sum assured of 1 over the reserves at the end of (t + 1) years in case of death.

iii) Actual death strain: The actual death strain is the actual difference between the sums assured and policy reserves at the end of year of claim, for death claims and is given by

$$\sum_{claims} (S - {}_{t+1}V)$$

Expected death strain: The expected death strain is the expected difference between the sums paid by way of death claims and the reserves held for these claims. It is calculated by multiplying the probability of a claim by the difference between the sum assured and the policy reserve, at the end of the year of claim, and summing for all policies, and is given by

$$\sum q(S - {}_{t+1}V)$$

iv) The reserves as at 31.12.07 in respect of each type of policy are as under:

Whole life assurance

$$1 - \frac{\ddot{a}_{40}}{\ddot{a}_{30}} = 1 - \frac{20.005}{21.834} = 1 - 0.9162315 = 0.0837684$$

Deferred annuity payable yearly from 60th birthday

$\ddot{a}_{60} = 14.134$

Whole Life Assurance

Death claims during 2007 = 5000

Reserves released for their deaths = 5000(0.0837684)

= 418.84

Therefore Actual death strain = 5000 – 418.84

= 4581.16

The sum assured in force as at 1.1.07 = Rs.505,000

The reserves as at 31.12.07 for sum assured in force at 1.1.07

= 505,000 (0.0837684) = 42303.06

Therefore expected death strain

= q_{39} (505,000 – 42303.06)

= (0.000870) (462696.94)

= 402.55

Therefore, expected mortality profit from mortality in 2007

= EDS – ADS

= 402.55 – 4581.16

= - 4178.61 i.e. a mortality loss ……….A

Deferred Annuity

Actual reserves released on death = 1200 $\ddot{a}_{60}$

= 1200 (14.134) = 16960.80

Reserves as at 31.12.07 for annuities in force as at 1.1.07 = 61200 $\ddot{a}_{60}$

Expected release of release on death

= q_{59} (61200 $\ddot{a}_{60}$) = (0.007140) (61200 x 14.134)

= 6176.1057

Profit on annuities = 16960.80 – 6176.1057 = 10784.69 ………..B

Total profit = A + B

= -4178.61 + 10784.69

= 6606.08

[18]

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