# INSTITUTE OF ACTUARIES OF INDIA <br> EXAMINATIONS 

$20^{\text {th }}$ May 2008
Subject CT8 - Financial Economics
Time allowed: Three Hours (14.30 - 17.30 Hrs)
Total Marks: 100

## INSTRUCTIONS TO THE CANDIDATES

1) Do not write your name anywhere on the answer sheet/s. You have only to write your Candidate Number on each answer sheet/s.
2) Mark allocations are shown in brackets.
3) Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.
4) Fasten your answer sheet/s together in numerical order of questions. This, you may complete immediately after expiry of the examination time.
5) In addition to this paper you should have available Graph paper, Actuarial Tables and an electronic calculator.

## Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of IAI"

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

## AT THE END OF THE EXAMINATION

Please return your answer sheet/s and this question paper to the supervisor separately.

## Q1)

(a) Current price of zero-coupon bond of maturity value 100 maturing in time 5 is 0.72 . Current price of zero-coupon bond of maturity value 100 maturing in time 6 is 0.68 . Calculate the forward rate for the $5^{\text {th }}$ year.
(b) Define market price of risk.
(c) What is the one-factor term structure and what are its limitations?

Q2) An investor buys, for a premium of 187.06, a call option on a non-dividend paying stock whose current price is 5,000 . The strike price of the call is 5,250 and the time to expiry is 6 months. The risk free rate of return over this period is $5 \%$ p.a.

The Black-Scholes price for a call option on a non-dividend paying share is

$$
\begin{aligned}
& \qquad \mathrm{c}_{\mathrm{t}}=\mathrm{St} * \Phi\left(\mathrm{~d}_{1}\right)-\mathrm{K} *(\exp (-\mathrm{r} *(\mathrm{~T}-\mathrm{t}))) * \Phi\left(\mathrm{~d}_{2}\right) \\
& \text { where } \mathrm{d}_{1}=\left\{\log (\mathrm{St} / \mathrm{K})+\left(\mathrm{r}+\left(1 / 2 \sigma^{2}\right)\right) *(\mathrm{~T}-\mathrm{t})\right\} /(\sigma * \sqrt{(\mathrm{~T}-\mathrm{t})}) \\
& \mathrm{d}_{2}=\mathrm{d}_{1}-(\sigma * \sqrt{(\mathrm{~T}-\mathrm{t})}) \\
& \text { and } \mathrm{St}=\text { current price } \\
& \sigma=\text { volatility } \\
& \mathrm{T}-\mathrm{t}=\text { time to expiry } \\
& \mathrm{K}=\text { strike price } \\
& \mathrm{r}=\text { risk free rate }
\end{aligned}
$$

(a) Calculate the price of a put option with the same time to maturity and strike price as the call.
(b) The investor buys a put option with strike price 4,750 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (a).
[You need to estimate the implied volatility to within $1 \%$ p.a. of the correct Value.]
Q3) A financial institution uses a four-state discrete time Markov Chain model to value its portfolio.
At the beginning of each year, the institution assigns each of the bond issuer one of the following categories:

State 1: The institution expects to receive full payments due that year
State 2: The institution expects to receive $75 \%$ of any payments due that year
State 3: The institution expects to receive $50 \%$ of any payments due that year
State 4: The institution expects to receive no payment from the issuer that year.
The risk-neutral probabilities of moving from different states are:
Probability of moving to default state in one year time is $5 \%$.

Probability of moving from state 1 to state 2 in one year is $20 \%$.
Probability of moving from state 1 to state 3 in one year is $10 \%$.
Probability of moving from state 2 to state 3 in one year is $20 \%$.
Probability of rating improvement in one year is only limited to moving one state above and that probability for each state is $15 \%$.
(1) The institution is considering purchasing a 3 -year bond issued by a company currently rated in state 1 . The bond pays annual interest of $6.5 \%$ at the end of each year and will be redeemed at $110 \%$ of the face value at the end of 3 years. The face value of the bond is Rs. 100 . Assume the spot yield on 1, 2 and 3 years govt. bonds as $6.25 \%$.
a) Calculate the risk free value of the bond that the institution should pay?
b) Comment on the answer in (1)(a).
(2) After negotiations, the institution agrees to purchase the bonds at a price of Rs. 95.7.

Calculate the credit spread for this bond.
Q4)
(a) Explain what is meant by a recombining binomial tree. State the advantage and disadvantage of using a recombining tree [vis-à-vis a non-recombining tree] to model share price movements?
(b) A trader in derivatives is using a two-step binomial tree to determine the value of a 6-month European put option on a non dividend-paying share. The put option has a strike price of Rs. 950 . The trader assumes that during the first 3 months, the current share price of Rs. 1000 will either increase by $10 \%$ or decrease by $5 \%$. The continuously compounded risk free rate during this period [per 3 months] is $1.75 \%$. During the following 3 months, the trader assumes that the share price will either increase by $20 \%$ or decrease by $10 \%$. The risk free rate during this period is expected to be $2.5 \%$ [per 3 months].
i) Calculate the value of the put option.
ii) The trader believes that a more accurate value of the put option can be determined by dividing the term of the option into "months". State the disadvantages of applying this modification to the model; and suggest an alternative model based on months that might be more efficient numerically.

## Q5)

(a) What is an arbitrage opportunity \& why it is difficult to find an arbitrage opportunity in an efficient market?
(b) On May 1, 2008,

Share price of Infosys is:
Bid Price
Offer Price
Rs. 1400.00
Rs. 1400.70

The Future contract price is:
Bid Price
Offer Price
Rs. 1410.60
Rs. 1411.00

The contract expires on May 31, 2008. At the expiry of the contract, both future price $\&$ share price are equal.

The risk free rate is $6 \%$ pa continuously compounded.
Is there any arbitrage opportunity available?
We can assume that:

- No margin is paid while entering into a Future contract.
- No commission is paid while entering or exiting any contract.
(c) Define the lower \& upper bounds on European put option for a non-dividend paying share.
(d) Describe Put-call parity in a no-arbitrage situation on European options on a non-dividend paying share.


## Q6)

(a) Explain the different forms of efficient market hypothesis (EMH).
(b) Does weak form of EMH imply that the strong form is applicable? Does the strong form of EMH imply that the weak form is applicable? Explain.
(c) How do the following relate to the EMH?

- Technical Analysis
- Fundamental Analysis
- Insider Trading


## Q7)

(a) An investor is contemplating investment in a portfolio with a return of $100 \times \mathrm{x}^{2}-4,000$; where x is uniformly distributed between 1 and 10. Calculate the following:

- Variance of return
- Downside semivariance of returns.
(b) Calculate the VaR at $10 \%$ confidence level.

Q8) Two securities $A$ and $B$ have the following characteristics.
$\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=13 \% ; \mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)=7 \%$
$\sigma_{\mathrm{A}}=30 \% ; \sigma_{\mathrm{B}}=10 \% ; \rho_{\mathrm{AB}}=\mathrm{x}$
(a) The correlation of the securities is -1 . Is it possible to construct a risk less portfolio comprising these two securities? If yes, what is the proportion of asset A in such a portfolio?
(b) Derive and expression for the proportion of security A in the portfolio that minimizes the risk. What proportion of a portfolio should consist of security A for minimization of risk if the correlation of the securities is 0.1 ?
(c) The return of securities in a market is given by a single-index model as follows:
$\mathrm{R}_{\mathrm{i}}=\alpha_{\mathrm{i}}+\beta_{\mathrm{i}} \mathrm{R}_{\mathrm{M}}+\varepsilon_{\mathrm{i}}$
Derive an expression for the covariance of 2 securities $i$ and $j$ in terms of $\beta_{i}, \beta_{j}$ and $V_{M}$, which is the variance of market returns.

## Q9)

(a) Explain the terms diversifiable risk and non-diversifiable risk of a security under CAPM.
(b) What is the market price of risk under CAPM? A market comprises 2 risky assets A and B; with the following returns in different states of the world.

| State | Probability | Asset A | Asset B |
| :--- | :--- | :--- | :--- |
| 1 | 0.2 | $-1 \%$ | $-2 \%$ |
| 2 | 0.4 | $3 \%$ | $5 \%$ |
| 3 | 0.4 | $6 \%$ | $8 \%$ |
| Market <br> Capitalisation |  | 25,000 | 75,000 |

The risk free rate is $3 \%$. Calculate the market price of risk.

