Institute of Actuaries of India

Subject ST6 – Finance & Investment B

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INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Arpan Thanawala
Chairperson, Examination Committee

Q.1)

i) The value of this arrangement at the outset can be found by discounting using market rates,

$$V_0 = -\frac{L}{1.07^3} + \frac{L(1+i)^2}{1.08^5}$$

Let –L be the cash flow at time 3 and $+L(1+i)^2$ at time 5, L is the principal and *i* is the agreed rate of interest.

The interest rate specified in FRAs is usually chosen so that the value of agreement is zero at the outset. Hence $V_0 = 0$

$$-\frac{L}{1.07^3} + \frac{L(1+i)^2}{1.08^5} = 0$$

$$-\frac{L}{1.225} + \frac{L(1+i)^2}{1.4693} = 0$$

$$+\frac{L}{1.225} + \frac{L(1+i)^2}{1.4693}(i.e)\frac{(1+i)^2.1.225}{1.4693} = 1 \Rightarrow i = 9.5184\%$$

ii)

$$V = -\frac{L}{1.065} + \frac{L(1+i)^2}{1.0675^3} =$$

$$= L\left[-\frac{1}{1.065} + \frac{(1+i)^2}{1.0675^3}\right]$$

$$= L\left[-0.9389 + \frac{1.1992}{1.2165}\right]$$

$$= L[-0.9389 + 0.9858]$$

$$= L(0.0469)$$

$$L = 10cr$$

$$\therefore V = 0.469crore$$

$$Rs.46.9lakhs$$

So it is worth Rs. 46.9 lakhs to the party making the earlier payment and Rs. 46.9 to the counter party

[5]

Q.2) The following clues should enable you to match the prices to the contracts:

- The strike rates under (*i)* and (*ii)* are very similar. However, the rate for (*i)* is slightly lowers as 4% *pa* convertible quarterly corresponds to 3.969% continuously-compounded, whereas 3% *pa* convertible monthly corresponds to 3.990% pa continuously-compounded. So the price of *ii* will be slightly lower than the price of (*i)*.
- *(iv)* is deeply in-the-money and so will have the highest price
- *(iii)* is also in-the-money, unlike (*i)* and (*ii)* and so is likely to have a higher price than (*i)* and (*ii)*.
- The two strike prices for a collar are usually chosen to make the price zero. (The two rates will usually straddle the current interest rate)
- A fixed-for-floating swap – *ie* pay fixed and receive floating - can be replicated by +1 cap and -1 floor (with the same interest rate0. so its price will be the price of (*i)* minus the price of (*iii)*.

  Strictly speaking, the value of an interest rate swap to a party that is paying fixed and receiving floating is equal to the value of **plus** one cap and **minus** one floor, ie (i-iii) = -Rs.5000. Thus, in this instance it is the value to the counterparty that pays floating and receives fixed that must be +Rs 5000 ie *(vi)*.

- i = Rs. 13000
- ii = Rs. 12000
- iii = Rs. 18000
- iv = Rs. 30000
- v = Rs. 0
- vi = Rs. 5000

[9]

Q.3)

i) a) **Exchange of a fixed-rate and floating-rate bond**

Under the interest rate swap, Agarwal agrees to pay Ram cash-flows equal to interest at a predetermined fixed rate on Rs 10crore for 3 years. At the same time, Ram agrees to pay Agarwal cash-flows equal to interest at a floating rate on the Rs. 10 crore.

If we assume no possibility of default, this interest rate swap can be valued as a long position in one bond compared to a short position in another bond, since the notional principal is Rs 10crore in both cases.

Agarwal is long in the floating rate bond and short in the fixed rate bond.

i) b) **Fixed interest rate**

The value of the fixed-rate bond, assuming coupon payment R is:

$$\left[\frac{R}{1.0401}+\frac{R}{1.0410^2}+\frac{10+R}{1.0416^3}\right]$$

Thus, the coupon payments R can be found from:

$$10=\left[\frac{R}{1.0401}+\frac{R}{1.0410^2}+\frac{10+R}{1.0416^3}\right]$$

Solving this gives R = Rs. 415,632.

i) c) **Value of swap to Investor A**

The floating payment received at 1 January 2007 is determined by the 1-year LIBOR rate at 1 January 2006, of 4.01%. So the value fo the floating rate bond at 1 July 2006 is:

$$B_{fl}=\frac{10(1+0.0401)}{1.0410^{1/2}}=Rs10.1941crore$$

The value of the fixed interest bond at 1 July 2006 is:

$$B_{flx}=\frac{0.415632}{1.0410^{1/2}}+\frac{0.415632}{1.0418^{1\frac{1}{2}}}+\frac{10.415632}{1.0424^{2\frac{1}{2}}}=10.1868$$

Thus, the value of the swap Agarwal on 1 July 2006 is :

$$V_{swap}=B_{fl}-B_{flx}=10.1941-10.1868=Rs.0.0073crore$$

ii) **Value the swap as a series of forward rate agreements**

An FRA is an agreement that a specified fixed interest rate will apply to a specified principal over an agreed future time period.

An FRA to borrow at a fixed rate can be financed by lending at the same floating rate over the same period.

There will be no net cashflow at the start of the forward period, whilst the net cash-flow at the end of the period will be the principal times the difference between the fixed and floating rates – ie the same as under a swap.

So a swap can be valued as a combination of a series of forward rate agreements.

FRA for 2006/07

The floating payment received at 1 January 2007 is determined by the 1-year LIBOR rate at 1 January 2006 of 4.01 %.

Thus, the value of the net cash-flow received by Agarwal at 1 January 2007 is:

$$\frac{10crore\times(0.0401-0.0415632)}{1.0410^{1/2}}=-Rs14,341$$

FRA for 2007/08

The floating payment expected at 1 January 2008 (based on current spot rates) is determined by the forward rate for the year 2007 at 1 July 2006. The forward rate is given by:

$$\frac{1.0418^{1\frac{1}{2}}}{1.0410^{1/2}} - 1 = 0.0422$$

Thus, the value of the net cash-flow received by A at 1 January 2007 is:

$$\frac{10crore \times (0.0422 - 0.0415632)}{1.0418^{1\frac{1}{2}}} = Rs.5{,}991$$

FRA for 2008-09

The floating payment expected at 1 January 2009 is determined by the 1-year LIBOR rate for the year 2007 at 1 July 2006. The forward rate for this period is give by:

$$\frac{1.0424^{2\frac{1}{2}}}{1.0418^{1\frac{1}{2}}} - 1 = 0.04330$$

Thus, the value of the net cash-flow expected by A at 1 January 2008 is;

$$\frac{10crore \times (0.04330 - 0.0415632)}{1.0424^{2\frac{1}{2}}} = Rs.15661$$

The total value of the swap to Investor A at 1 July 2006 is thus:

$$-14341 + 5991 + 15661 = Rs.7311$$

which agrees with the answer in (i)(c)

[10]

Q.4) The distribution of V_t in <u>two months</u> time has an exponential distribution with mean 'm'.

$$m = V_0 e^{\mu t} = 10e^{0.15 \times 2/12} = 10 \times 1.0253 = 10.253crore$$

The distribution function of the exponential distribution with mean 'm' is

$$F(x) = 1 - e^{-\frac{x}{m}}$$

So the 10th percentile (ie the level it will remain above 90% of the time) is found from

$$F(x_{0.1}) = 1 - e^{-x_{0.1}/m} = 0.1$$

$$\therefore -m\log(0.9) = 1.0803core$$

the var@90% confidence level is 10-1.0803=8.9197 crore

The calculated VAR is very large relative to the value of the asset. This is due to high showness of exponential distribution and implies a high probability of making a large loss.

[5]

Q.5) Using the expected return and volatility for the first security, the market price of risk for BSE Sensex is:

$$\frac{\mu_1 - r}{\sigma_1} = \frac{0.20 - 0.06}{0.25} = \frac{0.14}{0.25} = 0.56$$

When there is just one source of randomness, derivatives on the same underlying asset have the same market price of risk:

$$\frac{\mu_1 - r}{\sigma_1} = \frac{\mu_2 - r}{\sigma_2} = 0.56$$

$$\mu_2 = 0.56x0.35 + 0.06 = 0.256$$

Thus, the expected return from the second security = 25.6%

[6]

Q.6)

i) The probability that the call option will be exercised is the probability that $S_T > K$, where S_T is the stock price at time T. In a risk neutral world:

$$\ln S_T \sim N[\ln S_0 + (r - \sigma^2/2)T, \sigma^2 T]$$

The probability that $S_T > K$ is the same as probability that $\ln S_T > \ln K$. Probability of $\ln S_T > \ln K$ is:

$$1 - \Phi[\frac{\ln K - \ln S_0 - (r - \sigma^2/2)T}{\sigma\sqrt{T}}]$$

$$= \Phi[-\frac{\ln K - \ln S_0 - (r - \sigma^2/2)T}{\sigma\sqrt{T}}]$$

$$= \Phi[\frac{\ln S_0 - \ln K + (r - \sigma^2/2)T}{\sigma\sqrt{T}}]$$

$$= \Phi[\frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$= \Phi(d_2)$$

The expected value at Time T in a risk neutral world of a derivative security which pays off Rs. 300 when $S_T > K$ is therefore

$$300\,\Phi(d_2)$$

From the risk neutral valuation, the present value of the derivative is:

$$300\, e^{-rT}\Phi(d_2)$$

ii) a) The required probability is the probability of the stock price being above 2000 in three months time. Suppose that the stock price in three months is S_T .

$$\ln S_T \sim N[\ln 1900 + (0.20 - \frac{0.30^2}{2})0.25, 0.30^2 x 0.25]$$

$$\ln S_T \sim N(7.511, 0.0225)$$

Since ln2000 = 7.601, the required probability is:

$$1 - \Phi(\frac{7.601 - 7.511}{\sqrt{0.0225}}) = 1 - \Phi(0.6003) = 1 - 0.7258 == 0.2742$$

ii) b) In this case the required probability is the probability of the stock price being less than 2000 in three months. It is

1-0.2742 = 0.7258.

[10]

Q.7) The delta of the portfolio is:

-4000 x (-0.30) – 2000x0.80 – 1000x0.60 – 1000x0.50 = -1500[1]

The Gamma of the portfolio is:

-4000x1.20 – 2000x0.50 – 1000x2.00 -1000x1.60 = -9400

The kappa of the portfolio is:

-4000x0.50-2000x0.60-1000x1.50-1000x1.20 = -5900

Let w_1 be the position in the first traded option and w_2 be the position in the second traded option.

To make the portfolio Gamma neutral, we require that:

$$1.00w_1 + 0.60w_2 - 9400 = 0$$
$$w_1 + 0.6w_2 = 9400 \qquad (1)$$

To make the portfolio Kappa neutral, we require that:

$$0.50w_1 + 0.50w_2 - 5900 = 0$$
$$w_1 + w_2 = 11800 \qquad (2)$$

Solving (1) and (2), we get

w_1 = 5800 and w_2 = 6000

The whole portfolio then has a delta of:
-1500 + 0.50x5800 + 0.20x6000 = 2600
Therefore, the portfolio can be made Delta, Gamma and Kappa neutral by taking a long position in 5800 of the first traded option, a long position in 6000 of the second trade option and a short position in $2600.

[10]

Q. 8)

a) By writing covered call options, Rajesh receives premium income of Rs.80,000. If, in August, the price of the stock is less than or equal to Rs. 2100, then Rajesh will have his stock plus the premium income. But the *most* he can have at that time is (Rs. 2,100,000 + Rs. 80,000) because the stock will be called away from him if the stock price exceeds $45. The payoff structure is:

Stock price	Portfolio value
less than Rs. 2100	1000 times stock price + Rs. 80,000
greater than Rs. 2100	Rs. 2,100,000 + Rs. 80,000 = Rs. 2,180,000

This strategy offers some extra premium income but leaves Rajesh subject to substantial downside risk. At an extreme, if the stock price fell to zero, Rajesh would be left with only Rs. 80,000. This strategy also puts a cap on the final value at Rs. 2,180,000, but this is more than sufficient to purchase the house.

b) By buying put options with a Rs. 1900 strike price, Rajesh will be paying Rs. 80,000 in premiums in order to insure a minimum level for the final value of his position. That minimum value is: (Rs. 1900 □ 1000) – Rs. 80,000 = Rs. 1,820,000.
This strategy allows for upside gain, but exposes Rajesh to the possibility of a moderate loss equal to the cost of the puts. The payoff structure is:

Stock price	Portfolio value
less than Rs. 1900	Rs. 1,900,000 – Rs. 80,000 = Rs. ,820,000
greater than Rs. 1900	1000 times stock price – Rs. 80,000

c) The net cost of writing call and buying put is zero. The value of the portfolio will be as follows:

Stock price	Portfolio value
less than Rs. 1900	Rs. 1,900,000
between Rs. 1900 and Rs. 2100	1000 times stock price
greater than Rs. 2100	Rs. 2,100,000

If the stock price is less than or equal to Rs. 1900, then the Strategy C preserves the Rs. 1,900,000 principal. If the price exceeds Rs. 2100, then Rajesh gains up to a cap of Rs. 2,100,000. In between, Rs. 1900 and Rs. 2100, his proceeds equal 1000 times the stock price.
The best strategy in this case would be C since it satisfies the two requirements of preserving the Rs. 1,900,000 in principal while offering a chance of getting Rs. 2,100,000. Strategy A should be ruled out since it leaves Rajesh exposed to the risk of substantial loss of principal.
Our ranking would be: (1) strategy C; (2) strategy B; (3) strategy A.

[10]

Q.9)

a) Asian options are options where the payoff depends on the average price of the nderlying asset during at least some part of the life of the option. The payoff from an average price call is max($S_{average}$ – K, 0) and that from an average price put is max(K-$S_{average}$,0), where $S_{average}$ is the average value of the underlying asset calculated over the predetermined averaging period.
Example: A call option on Reliance stock has a payoff on 1 July equal to max($S_{average}$ – 500, 0) where $S_{average}$ is the arithmetic mean of the closing price on 30 April, 31 May and 30 June. The closing prices on the three dates are 510, 520 and 530. So the payoff is max(520-500, 0) = 20.

b) A chooser option has the feature that, after a specified period of time, the holder can choose whether the option is a call or a put.
Example: On 1 January a trader pays Rs. 50 for a chooser option on March put/call option on a particular share with strike Rs. 500. These options are already trading in the market. He must choose by 1 February whether he wants a call or a put option. On 31 January the share price is Rs. 510, the put price is Rs. 15 and the call price is Rs. 40. So he elects to have a call option. No payment is required at this time. On the expiry date in March the share price is Rs. 520 and he receives a payoff of Rs. 20.

c) A numeraire is a quantity whose values are used as the units for expressing the price of a security.
Example: Suppose f and g are the prices of two traded securities. We define $\phi = \frac{f}{g}$.
The variable ϕ is the relative price of f with respect to g. It can be thought of as measuring the price of f in units of g rather than rupees. The security price g is referred to as the numeraire.

d) A trinomial tree is an alternative to a binomial tree, where there are three choices at each branch.
Example: A stock price is currently Rs. 100. Over each of the next two 3-month periods it is expected to go up by 10% or remain the same or down by 10% with probabilities of 30%, 40% and 30% respectively.

[8]

Q.10) Let's evaluate both the transactions.
Selling the stock
1) Receives 650 * (1-1%) = 643.50
2) Loss of dividend of 45 as on 15th Jan 2007 whose value as on 1st Jan = 44.90 (@5%, 44.89 @6%). Since the friend will be net lender rather than borrower the applicable rate of interest is 5%. (1 for identifying loss of dividend, 1 mark for identifying the applicable rate of interest, 1 for identifying that the dividend adjustment occurs during the mid of the month and the amount has to be discounted or compounded appropriately)

Buying the future
3) Pay the brokerage - .5% of 594 = 2.97
4) Pay the margin – 32.55
5) Pay (594-32.55) on maturity and get the shares. The value of this amount as on 1st Jan 2007 is 559.08 (@5%)

In the nutshell, his expected gain from the strategy is 643.5 – 44.9 – 2.97 - 32.55 - 559.08 = 4 today
Alternative solution:
Let's evaluate both the transactions.
Selling the stock
1) Receives 650 * (1-1%) = 643.50
2) Loss of dividend of 45 as on 15th Jan 2007 whose value as on 31st Jan = 45.10 (@5%). Since the friend will be net lender rather than borrower the applicable rate of interest is 5%. (1 for identifying loss of dividend, 1 mark for identifying the applicable rate of interest, 1 for identifying that the dividend adjustment occurs during the mid of the month and the amount has to be discounted or compounded appropriately)

Buying the future
3) Pay the brokerage - .5% of 594 = 2.97
4) Pay the margin – 32.55
5) Pay (594-32.55)=561.45 on maturity and get the share.
6) Net cash as on 1st Jan is (643.50 – 2.97 – 32.55) = 607.98
7) The amount above is invested @5% and the total amount with interest is 610.56
8) After making the total payment of 561.45 to get the share your friend is left with 49.11 and a share.

Alternatively, without implementing this strategy he would have hold a share and dividend worth 45.10. Therefore, he should go for this strategy if he wants to increase his expected income and his gain would be 4.01 (end of the month) per share.

The risks in implementing this strategy are

i) The discount is not as substantial as you may infer by looking at the prices.

ii) Even if the prices are known for calculating the gain/loss before putting the order on the exchange, the actual transacted price may be different compared to the price used in calculating the gains.

iii) The company may either reduce or increase the amount of dividend after the transactions. The decrease is beneficial under the strategy but not the increase.

iv) Taxes have been ignored in the above calculations and they may decrease/increase the benefits.

The daily settlements have been ignored in the above calculations. This may give rise to further cash-flow management problems and interest gain/losses on the same.

[12]

Q.11)

a) The stock price tree:

Stock Price movement from one node to the other two nodes

		200
	150	160
100	110	120

Conditional Risk neutral probabilities

Conditional Risk neutral probabilities of moving from one node to the other two nodes

		0.5
	0.25	0.5
1	0.75	0.3
		0.7

(Note that though the realistic probability of moving from one node to the other two nodes does not change but risk neutral probabilities in this case are different for different nodes. Also, henceforth the arrows are not shown but should be understood from the context.)

Value of the derivative (calculated using discounted expected value under the risk neutral probability)

Derivative Value

		1000
	470	128
117.92	32	0

For example 32 = (128 * 0.3 + 0 * 0.7) / (1 + 0.20)

Since the fair (arbitrage) value of the derivative comes to be 117.92 which is higher than the market traded value of 100, I can make an arbitrage profit and should buy the derivative. Though the value here is not derived using the replicating portfolio method, but there exists a replicating portfolio to achieve the final output.

b) Realistic probabilities

Realistic Probabilities

		0.09
	0.3	0.21
1	0.7	0.21
		0.49

Risk Neutral probabilities

Risk neutral Probabilities

		0.125
	0.25	0.125

1	0.75	0.225
		0.525

$E_Q(X_T)$ = 0.125* 1000 + 0.35 * 128 = 169.8

$E_P\left(\frac{dQ}{dP}X_T\right)$ = (.125/.09 * 1000) * 0.09 + (0.35/0.42*128)*0.42 + (.525/0.49*0)*0.49 = 169.8

The radon-nikodym process (ς_t)

Rado Nikodym process

		1.389
	0.833	0.595
1	1.071	1.071
		1.071

Left hand side = $E_Q(X_T \mid F_s)$ = 1.2 * derivative value = 564 (on node 170) and 38.4 (on node 32)

Right hand side = $\varsigma_s^{-1}E_P(\varsigma_T X_T \mid F_s)$ (on node 170) = 1/ 0.833 * (0.3 * 1.389 * 1000 + 0.7 * 0.595 * 128) = 564

(On node 32) = 1/1.071 * (0.3 * 1.071 * 128 + 0.7 * 1.071 * 0) = 38.4

[15]

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