The Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

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INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

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Q. 1) Since the select period is one year, we can write:

$$\bar{A}^{\;1}_{[45]:\overline{1}|} + {}_{1|}\bar{A}_{45:\overline{19}|} = \bar{A}^{\;1}_{[45]:\overline{1}|} + \frac{D_{46}}{D_{[45]}} * \bar{A}_{46:\overline{19}|}$$

Where

$$\bar{A}^{\;1}_{[45]:\overline{1}|} = v^{1/2} * q_{[45]} = v^{1/2} * 0.9 * q_{45}$$

= 0.980581 * 0.9 * 0.00266

= 0.0023475

$$\frac{D_{46}}{D_{[45]}} = \frac{l_{46}}{l_{[45]}} * v = p_{[45]} * v = (1 - q_{[45]}) * v$$

= (1- 0.9*0.00266)*0.961538
= 0.959236

$$\bar{A}_{46:\overline{19}|} = 1 - \delta * \bar{a}_{46:\overline{19}|}$$
=1 – 0.0392207 * 12.74
=0.500328

Giving:

$$\bar{A}_{45:\overline{20}|} = 0.0023475 + 0.959236 * 0.500328 = 0.48228$$

[8]

Q.2) Equivalence principle $\Rightarrow$ EPV Premiums = EPV Benefits + EPV expenses

Let P be the quarterly premium

EPV Premiums:

$$4P\ddot{a}^{(4)}_{[45]:\overline{20}|} = 4P[\ddot{a}_{[45]:\overline{20}|} - \tfrac{3}{8}(1 - \frac{D_{65}}{D_{[45]}})]$$

$$= 4P[13.785 - \tfrac{3}{8}(1 - \tfrac{689.23}{1,677.42})] = 54.2563P$$

EPV Death Benefit:

210,000 $\bar{A}^{\;1}_{[45]:\overline{20}|}$ - 10,000 * $(I\bar{A})^{1}_{45:\overline{20}|}$

$$\bar{A}^{1}_{[45]:\overline{20}|} = (1.04)^{0.5}\left[A_{[45]} - v^{20}\,{}_{20}p_{[45]}\,A_{65}\right]$$

$= (1.04)^{0.5}$ [0.27583 – 0.4563869*(8821.2612/9798.0837)*0.52786]
= 0.0601062

$$=(1.04)^{0.5}\left[(IA)_{[45]} - v^{20}\,{}_{20}p_{[45]}\left\{(IA)_{65} + 20^{*}A_{65}\right\}\right]$$

$=(1.04)^{0.5}$[8.33865 -0.4563869*(8821.2612/9798.0837)*{7.89442+20*0.52786}]
= 0.7721077

210,000 * 0.0601062 – 10,000 * 0.7721077 = 4901.23

EPV annuity: $\frac{D_{65}}{D_{[45]}} * \left(23000 * \ddot{a}_{65} + 2000 * \left(I\ddot{a}\right)_{65}\right)$

= (0.410887) * (23000 * 12.276 + 2000 * 113.91)

= 209622.20

EPV expenses :
Death Claim = 250 * ${}_{20}q_{[45]}$ = 250* (1- 0.90030) = 24.92
Annuity = 0.02 * EPV annuity = 0.02 * 209622.2 = 4192.44

Premium related :

$$(0.05)(4P\ddot{a}^{(4)}_{[45]:\overline{20}|}) + (0.30)(4P\ddot{a}^{(4)}_{[45]:\overline{1}|})$$

$$= (0.05)(54.2563P) + (0.3)(4P)(1 - \tfrac{3}{8}[1 - \frac{D_{[45]+1}}{D_{[45]}}])$$

= 2.712815 P + 1.182171 P = 3.8950 P

Other :

160+ 40 * $\ddot{a}_{[45]:\overline{20}|}$ @ 0%

$=160 + 40 \{ (1 + e_{[45]}) - \frac{l_{65}}{l_{[45]}}(1 + e_{65}) \}$

=160 + 40 { 35.282 – 0.90030 * 17.645} = 935.85

Equivalence principle

54.2563 P = 4901.23 + 209622.20 + 24.92 + 4192.44 + 3.8950 P + 935.85

50.3613 P = 219676.64

P = 4362.01

Quarterly premium is Rs. 4,362.

[14]

Q.3)

(i) ***Derivation of annuity value***

One approach is the following (where all functions are at 1% interest and allow for adjusted mortality):

$\ddot{a}_{40:\overline{20}|} = 1 + v\, p_{40} + v^2\ {}_2p_{40}\, \ddot{a}_{42:\overline{18}|} = 17.598$

$\ddot{a}_{42:\overline{18}|} = \dfrac{17.598 - 1 - (1.01)^{-1} * 0.999063}{(1.01)^{-2} * 0.998050} = 15.954$

Also,

$\ddot{a}_{[40]:\overline{20}|} = 1 + v\, p_{[40]} + v^2\ {}_2p_{[40]}\, \ddot{a}_{42:\overline{18}|}$

$= 1+(1.01)^{-1}*0.999212 + (1.01)^{-2} * 0.998251*15.954$

$= 17.601$

(ii) ***Initial expense supportable***

We need to set the present value of premiums equal to the present value of benefits, renewal expenses and initial expenses, and solve for the initial expense amount.

We first need to establish the basic annuity and assurance functions, allowing for the extra force of mortality. The extra force of 0.00956945 corresponds to an extra 1% interest on survival functions evaluated at 4%, since

$1.04 * \exp\{0.00956945\} = 1.05$

Thus, evaluating at 4% up to age 55 and at 5% thereafter:

$\ddot{a}_{[40]:\overline{20}|} = \ddot{a}_{[40]:\overline{15}|} + \dfrac{D_{55}}{D_{[40]}} * \ \ddot{a}_{55:\overline{5}|}5\%$

$= 11.461 + (1105.41/2052.54)*4.503 = 13.885$

Using the premium conversion relationship, we have

$A_{[40]:\overline{20}|} = 1 - d * \ddot{a}_{[40]:\overline{20}|} = 0.46596$

The EPV of the renewal expenses is

$75 * [\ {}_1p_{[40]} * 1/(1.04) + {}_2p_{[40]} * 1.03/(1.04)^2 + \ldots\ldots + {}_{19}p'_{[40]} * 1.03^{19} / (1.04)^{20}]$

Where p’ allows for the extra mortality after age 55

$= 75/ 1.03\ [{}_1p_{[40]} * 1.03/(1.04) + {}_2p_{[40]} * 1.03^2/(1.04)^2 + \ldots\ldots + {}_{19}p'_{[40]} * 1.03^{20} / (1.04)^{20}]$

$= (75 / 1.03) * a_{[40]:\overline{19|}}\, i^*$

Where $i^* = (1.03/1.04) = 0.0097 \approx 1\%$

We now need to solve the following equation of value, where the unknown is the initial expense amount *IE*:

$P\ \ddot{a}_{[40]:\overline{20|}} = 100000\ A_{[40]:\overline{20|}} + 0.025\ P\ a_{[40]:\overline{19|}} + IE + (75/1.03)\ a_{[40]:\overline{19|}\,1\%}$

So :

$3950\ (\ddot{a}_{[40]:\overline{20|}} - 0.025 * a_{[40]:\overline{19|}}) = 100{,}000\ A_{[40]:\overline{20|}} + IE + (75/1.03)\ a_{[40]:\overline{19|}\,1\%}$

$= 3950\ (13.885 - 0.025 * 12.885) = 100{,}000 * 0.46596 + IE + (75/1.03) * 16.601$

Which we solve to get IE = 5,769

Hence the policy can support initial expenses of Rs.5,769 on these premium rates.

[10]

Q.4)

$$\bar{a}_x = \int_0^\infty v^t *{}_t p_x\, dt = \int_0^\infty e^{-\delta t} * e^{-\mu t}\, dt$$

(since the force of mortality is constant)

$$= \int_0^\infty e^{-(\mu+\delta)t}\, dt = \frac{1}{(\mu+\delta)} = 12.5 \Rightarrow \mu+\delta = 0.08 \Rightarrow \mu = \delta = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = 0.04 * 12.5 = 0.5$$

$${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{0.04}{0.04+2*0.04} = \frac{1}{3}$$

$$Var(\bar{a}_{\overline{T|}}) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{1/3-(0.5)^2}{(0.04)^2} = 52.0833333$$

The standard deviation of $\bar{a}_{\overline{T(x)|}}$ = 7.2168784

[4]

Q. 5)

$$a_{x:\overline{20|}} = \ddot{a}_{x:\overline{20|}} - 1 + \frac{D_{x+20}}{D_x} = \ddot{a}_{x:\overline{20|}} - 1 + A_{x:\overline{20|}}^{\ \ 1} \qquad \text{(I)}$$

$$A_{x:\overline{20|}} = 1 - d\,\ddot{a}_{x:\overline{20|}} \Rightarrow \ddot{a}_{x:\overline{20|}} = \frac{1-A_{x:\overline{20|}}}{d} \qquad \text{(II)}$$

$$A_{x:\overline{20}|} = A^{1}_{x:\overline{20}|} + A_{x:\overline{20}|}^{\;\;1} \qquad \text{(III)}$$

$$A_{x} = A^{1}_{x:\overline{20}|} + A_{x:\overline{20}|}^{\;\;1} * A_{x+20} \qquad \text{(IV)}$$

Putting the values given in the question,

$$0.28 = A^{1}_{x:\overline{20}|} + 0.25 * 0.40 \Rightarrow A^{1}_{x:\overline{20}|} = 0.18$$

Substituting this value in equation III we have

$$A_{x:\overline{20}|} = 0.18 + 0.25 = 0.43$$

Putting this value in equation II gives $\ddot{a}_{x:\overline{20}|} = \dfrac{1-0.43}{(0.05/1.05)} = 11.97$

Putting this value into equation I we have the required answer as
11.97-1+0.25 =11.22

[4]

Q. 6)

(i) Define a service table:
l_{26+t} = no. of members aged 26 + t last birthday
r_{26+t} = no. of members who retire age 26 + t last birthday
s_{x+t} / s_x = ratio of earnings in the year of age $x + t$ to $x + t + 1$ to the earnings in the year of age x to $x + 1$

Define

$$z_{26+t} = \tfrac{1}{3}\left(s_{26+t-3} + s_{26+t-2} + s_{26+t-1}\right)$$

$\bar{a}^{r}_{26+t}$ = value of annuity of 1 p.a. to a retiree aged exactly 26 + t.

Past service:
Assume that retirements take place uniformly over the year of age between 60 and 65. Retirement for those who attain age 65 takes place at exact age 65.

Consider retirement between ages 26 + t and 26 + t + 1, $34 \le t \le 38$.

The present value of the retirement benefits related to past service:

$$\frac{50000*5}{60} * \frac{z_{26+t+\frac{1}{2}}}{s_{25.25}} * \frac{v^{26+t+\frac{1}{2}}}{v^{26}} * \bar{a}^{r}_{26+t+\frac{1}{2}}$$

$$= \frac{50000*5}{60} * \frac{{}^{z}C^{ra}_{26+t}}{{}^{s}D_{26}}$$

Where

$${}^{z}C^{ra}_{26+t} = z_{26+t+\frac{1}{2}}\, v^{26+t+\frac{1}{2}}\, r_{26+t}\, \bar{a}^{r}_{26+t+\frac{1}{2}}$$

And $\quad {}^{s}D_{26} = s_{25.25}\, v^{26}\, l_{26}$

For retirement at age 65, the present value of the benefits is:

$$\frac{50000*5}{60} * \frac{z_{65}}{s_{25.25}} * \frac{v^{65}}{v^{26}} * \frac{r_{65}}{l_{26}} \bar{a}^{r}_{65}$$

$$= \frac{50000*5}{60} * \frac{{}^{z}C^{ra}_{65}}{{}^{s}D_{26}}$$

where

$${}^{z}C^{ra}_{65} = z_{65}\, v^{65}\, r_{65}\, \bar{a}^{r}_{65}$$

Summing over all ages, the value is:

$$\frac{50000*5}{60} * \frac{{}^{z}M^{ra}_{60}}{{}^{s}D_{26}}$$

where

$${}^{z}M^{ra}_{60} = \sum_{t=34}^{39} {}^{z}C^{ra}_{26+t}$$

Future service:

Assume that retirements take place uniformly over the year of age, between ages 60 and 65. Retirement at 65 takes place at exactly age 65.

If retirement takes place between ages 60 and 61, the number of future years service to count is 34. If retirement takes place at age 61 or after, the number of future years service to count is 35.

For retirement between ages 60 and 61, the present value of the retirement benefits is:

$$\frac{50000*34}{60} * \frac{z_{60+\frac{1}{2}}}{s_{25.25}} * \frac{v^{60+\frac{1}{2}}}{v^{26}} * \frac{r_{60}}{l_{26}} * \bar{a}^{i}_{60+\frac{1}{2}}$$

$$= \frac{50000*34}{60} * \frac{{}^{z}C^{ra}_{60}}{{}^{s}D_{26}}$$

For retirement at later years, the formula is similar to the above, with 35 in place of 34.

Adding all these together gives:

$$\frac{50000}{60} * \frac{1}{{}^{s}D_{26}} * \{ 34\ {}^{z}C^{ra}_{60} + 35\ ({}^{z}C^{ra}_{61} + \ldots\ldots\ldots + {}^{z}C^{ra}_{65})$$

$$= \frac{50000}{60} * \frac{1}{{}^{s}D_{26}}\ {}^{z}\overline{M}^{ra}_{60}$$

Where

$${}^{z}\overline{M}^{ra}_{60} = \sum_{t=0}^{5} (35*\ {}^{z}C^{ra}_{60+t} - {}^{z}C^{ra}_{60})$$

(ii) Define a service table, with l_{26+t} and s_{x+t} / s_x defined as in part (i). In addition, define d_{26+t} as the number of members dying aged 26 + t last birthday.

Assume that deaths take place on average in the middle of the year of age. The present value of the death benefit, for death between ages 26 + t and 26 + t + 1, is

$$50000 * 4 * \frac{s_{26.25+t}}{s_{25.25}} * \frac{v^{26+t+\frac{1}{2}}}{v^{26}} * \frac{d_{26+t}}{l_{26}}$$

$$= 50000 * 4 * \frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}}$$

where

$${}^{s}C^{d}_{26+t} = s_{26.25}\, v^{26+t+\frac{1}{2}}\, d_{26+t}$$

Adding the present value of benefits for all possible years of death gives

$$50000 * 4 * \sum_{t=0}^{38} \frac{{}^{s}C^{d}_{26+t}}{{}^{s}D_{26}}$$

$$= 200000 * \frac{{}^{s}M^{d}_{26}}{{}^{s}D_{26}}$$

where

$${}^{s}M^{d}_{26} = \sum_{t=0}^{38} {}^{s}C^{d}_{26+t}$$

[18]

Q. 7) If the annuity were payable until Y's 65th birthday (or her earlier death), then the expected present value of the benefit would be:

$$10000 *(a_{60(f):\overline{5|}} - a_{60(m):60(f):\overline{5|}})$$

However, if X dies before age 65 and Y is still alive at age 65, the annuity continues until Y reaches age 75 or until her earlier death. The expected present value of this part of the benefit is:

$$10000 * v^5 \, {}_5q_{60(m)} \, {}_5p_{60(m)} \, a_{65(f):\overline{10|}}$$

So the single premium is given by

$$P = 10000 \,(a_{60(f):\overline{5|}} - a_{60(m):60(f):\overline{5|}} + v^5 \, {}_5q_{60(m)} \, {}_5p_{60(m)} \, a_{65(f):\overline{10|}})$$

Now

$$a_{60(f):\overline{5|}} = a_{60(f)} - v^5 \, {}_5p_{60(f)} \, a_{65(f)}$$

$$= 15.652 - 1.04^{-5} * \frac{9703.708}{9848.431} * 13.871$$

$$= 4.419$$

$$a_{60(m):60(f):\overline{5|}} = a_{60(m):60(f)} - v^5 \, {}_5p_{60(m)} \, p_{60(f)} \, a_{65(m):65(f)}$$

$$= 13.090 - 1.04^{-5} * \frac{9647.797}{9826.131} * \frac{9703.708}{9848.431} * 10.958$$

$$= 4.377$$

$$a_{65(f):\overline{10}|} = a_{65(f)} - v^{10}\ {}_{10}p_{65(f)}\ a_{75(f)}$$

$$= 13.871 - 1.04^{-10} * \frac{8784.955}{9703.708} * 9.933$$

$$= 7.796$$

So,

$$P = 10000\ [4.419 - 4.377 + 1.04^{-5} * (1 - \frac{9647.797}{9826.131}) * \frac{9703.708}{9848.431} * 7.796]$$

= Rs.1,566

[7]

Q. 8) Only decrement 1 operates before $t = 0.7$

${}_{0.7}(aq)^{1}_{40} = 0.7 * (aq)^{1}_{40} = 0.7 * 0.10 = 0.07$ since decrements are Uniformly Distributed through the year

Probability of reaching $t = 0.7$ is 1-0.07 = 0.93

Decrement 2 operates only at $t = 0.7$, eliminating 0.125 of those who reached 0.7, we have

$$q^{2}_{40} = (0.93) * (0.125) = 0.11625$$

[4]

Q. 9)

(i) ***Unit provisions***

The following table shows the figures required:

Year	Unit Prov at start	Unit Fund before AMC	AMC	Unit Fund after AMC
1	15000.00	1350.00	490.50	15859.50
2	15859.50	1427.36	518.61	16768.25
3	16768.25	1509.14	548.32	17729.07
4	17729.07	1595.62	579.74	18744.95
5	18744.95	1687.05	612.96	19819.03

(ii) ***Net present value***

The following table shows the calculation of the profit signature.

Year	Commission	Initial Expenses	Renewal Expenses	Interest	AMC	In force Profit	Prob in force	vt
1	750	150	50.00	-85.50	490.50	-545.00	1.00000	0.892857
2	0	0	52.50	-4.73	518.61	461.38	0.94525	0.797194
3	0	0	55.13	-4.96	548.32	488.24	0.89350	0.711780
4	0	0	57.88	-5.21	579.74	516.65	0.84458	0.635518
5	0	0	60.78	-5.47	612.96	546.71	0.79834	0.567427

where probability in force at start of year $t = (0.95 \times 0.995)^{t-1}$

The net present value of the contract at a risk discount rate of 12% is:

$(-545.00 / 1.12) + (436.12 / 1.12^2) + (436.24 / 1.12^3) + (436.35 / 1.12^4) + (436.46 / 1.12^5)$

$= 696.54$

[7]

Q. 10)

(i) ***Required provisions***

The provisions required at the end of year 2 and year 1 are:

$${}_2V = \frac{6}{1.05} = 5.71$$

$${}_1V = \frac{1}{1.05}\left(12 + 0.98 * \frac{6}{1.05}\right) = 16.76$$

(ii) ***NPV of profits before and after zeroisation***

Before zeroisation, the net present value (based on a risk discount rate of 10%) is:

$$\text{NPV} = \frac{-25}{1.08} + \frac{-12*0.98}{1.08^2} + \frac{-6*0.98^2}{1.08^3} + \frac{25*0.98^3}{1.08^4} + \frac{35*0.98^4}{1.08^5}$$

$= 1.46$

After zeroisation the profit in year 1 will become:

$$\textit{Profit in Year } 1 = -25 - 12*\frac{0.98}{1.05} - 6*\frac{0.98^2}{1.05^2} = -41.43$$

So the profit vector will become:

Year	*In force profit*
1	–41.43
2	0
3	0
4	25
5	35

So the NPV after zeroisation will be:

$$\frac{-41.43}{1.08} + 0 + 0 + \frac{25 * 0.98^3}{1.08^4} + \frac{35 * 0.98^4}{1.08^5}$$

= 0.91

As expected, the NPV after zeroisation is smaller because the emergence of the profits has been deferred and the risk discount rate is greater than the accumulation rate.

[7]

Q. 11) If S_t is surplus in year t per policy in force at begin year t then:

$({}_{t-1}V + P - E_t)*1.07 = q_t(10000 + {}_tV) + (1 - q_t)*{}_tV + S_t$

Where ${}_tV$ etc is relevant reserve, P the required premium, E_t is expenses for year t and q_t the relevant mortality for year t

So $S_t = (P - E_t)*1.07 + {}_{t\,1}V*1.07 - {}_tV - 10000*q_t$

We need to sum ${}_{t-1}p_{[x]}*S_t*v_t$ at 10% for t = 1, 2, 3 and set to zero.

$${}_1V = 10000*\left(1 - \frac{\ddot{a}_{[57]+1:\overline{2|}}}{\ddot{a}_{[57]:\overline{3|}}}\right) \quad = 10000*\left(1 - \left(1 + v * {}^{l_{59}}\!/\!{}_{l_{[57]+1}}\right)/\, 2.873\right)$$

$$= 10000 * (1 - {}^{1.956}\!/\!{}_{2.873}) = 3191.79$$

${}_2V = 10000 * (1 - {}^{1}\!/\!{}_{2.873}) = 6519.32$ and ${}_3V = 10000$ using 4% interest.

The following table can now be completed:

Year end t	*1*	*2*	*3*
Prem-Expense	0.8*P	0.95*P	0.95*P
${}_{t-1}V_{[57]}$	0	3191.79	6519.32
$10000*q_{[57]+t\,1}$	41.71	61.80	71.40
Interest	0.056*P	0.0665*P+223.43	0.0665*P+456.35
${}_tV_{[57]}$	3191.79	6519.32	10000.00
S_t	0.856P- 3233.50	1.0165*P - 3165.90	1.0165*P - 3095.73
${}_{t-1}p_{[57]}$	1.00000	0.99583	0.98967
${}_{t-1}p_{[57]}*S_t$	0.856*p - 3233.50	1.0123*P- 3152.70	1.006*P- 3063.75

Therefore:
$(0.856*P - 3233.5)*v + (1.0123*P - 3152.70)*v^2 + (1.006*P - 3063.75)*v^3 = 0$ at 10%

i.e. $2.3706*P = 7846.92$

P = Rs.3,310.10

[12]

Q. 12)

- Insurance works on the basis of pooling independent homogeneous risks
- The central limit theorem then implies that profit can be defined as a random variable having a normal distribution
- Life insurance risks are usually independent
- Risk classification ensures that the risks are homogeneous
- Lives are divided by risk factors
- More factors implies better homogeneity
- But the collection of more factors is implied by
  - Cost of obtaining data
  - Problems with accuracy of information
  - The significance of the factors
  - The desires of the marketing department

[5]

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