# Actuarial Society of India

## EXAMINATION

## 23$^{rd}$ May 2007

## Subject ST6 — Finance and Investment B

Time allowed: Three hours (14.15* pm – 17.30 pm)

### INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer sheet.*

2. *\* You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *The answers are not expected to be any country or jurisdiction specific However, if Examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.*

4. *You must not start writing your answers in the answer sheet until instructed to do so by the supervisor.*

5. *Mark allocations are shown in brackets.*

6. *Attempt all questions, beginning your answer to each question on a separate sheet.*

7. *Candidates should show calculations where this is appropriate.*

8. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

### Professional Conduct:

*It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI.*

**Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.**

### AT THE END OF THE EXAMINATION
Please return your answersheets and this question paper to the supervisor seperatly.

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**In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.**

**Q.1**   Ram and Gopal have entered into a forward rate agreement at time 0, for the period starting from the year 3 till the end of year 5,. The spot yields for the next five years (expressed as annual effective rates up to the end of the year) are summarized by the vector (6%, 6.5%, 7%, 7.5%, 8%).

   i.   What should be the effective annual interest rate chosen for this contract?    (2)

   ii.   If the principal is Rs. 10 Crore, calculate the value of this agreement to the parties involved at the end of year 2, given that the 5-year term structure applicable at that time is (6%, 6.25%, 6.5%, 6.75%, 7%). Comment briefly on your answer.    (3)

**[5]**

**Q.2**   The following interest rate derivatives, all with a 5-year term are sought from a banker:
   i.   A 4% cap on quarterly interest payments
   ii.   A 4% cap on monthly interest payments
   iii.   A 4% floor on quarterly interest payments
   iv.   A 5% floor on quarterly interest payments
   v.   A 2.5% / 4% collar on quarterly interest payments
   vi.   A fixed-for-floating 4% swap on quarterly interest payments.

The principal and all other details were the same for each contract. The quoted interest rates are compounded with the same frequency as the payments The current continuously-compounded short rate in the market is 3%.

The prices for these six contracts are as given below, but please note that they are not in the same order as the derivatives given above:

        Rs. 13,000        Rs.  5,000
        Rs.    0        Rs. 30,000
        Rs. 12,000        Rs. 18,000

For each contract, find the price from the set of prices, given above    **[9]**

**Q.3**   The following table shows reference interest rates at 1 January 2006 and 1 July 2006 for a variety of terms – all rates are expressed as effective annual rates.

| Maturity date | Rate at 1 January 2006 | Rate at 1 July 2006 |
|---|---|---|
| 1 July 2006 | 3.98% | - |
| 1 January 2007 | 4.01 % | 4.10 % |
| 1 July 2007 | 4.08 % | 4.14 % |
| 1 January 2008 | 4.10 % | 4.18 % |
| 1 July 2008 | 4.12 % | 4.22 % |
| 1 January 2009 | 4.16 % | 4.24 % |

A 3-year interest rate swap was arranged on 1 January 2006 based on a principal of Rs. 10 crore under which Agarwal agreed to pay a series of annual fixed interest payments to Ram in return for a series of floating payments based on annual reference rates. The first payment is to be on 1 January 2007.

   i.   (a) Explain how the swap can be viewed as the exchange of a
            fixed-rate and a floating –rate bond                                          (2)
        (b) Hence, calculate the annual coupon payment (to the nearest
            rupee) on the assumption that it is chosen so that initial
            Value of the swap is zero to both parties.                            (2)
        (c) Calculate the value of the swap to Agarwal on 1 July 2006     (2)

   ii.   Explain how the swap is similar to a series of FRAs and, by valuing the swap in this way, verify your answer from (i)                 (4)
                                                            **[10]**

**Q.4**   $V_t$, the market value of a particular security $t$ years in the future, is modeled by assuming that $V_t$ has an exponential distribution with mean $V_0 e^{\mu t}$, where $\mu$ is a positive constant.

   i.   Calculate the value at risk at the 90% confidence level for this security using a time horizon of two months, given that the current market value is $V_o$ = Rs. 10 Crore and $\mu = 0.15$.                           (3)
   ii.   Comment on your answer in (i)                      (2)
                                                            **[5]**

**Q.5**   Consider two securities, both of which are positively dependent on the performance of BSE Sensex and depend on no other stochastic variables. Suppose that first one has an expected return of 20% per annum and a volatility of 25% per annum, and the second one has a volatility of 35% per annum. Assume that the instantaneous risk free rate of interest is 6% per annum. What is the market price of risk for BSE Sensex? What is the expected return from the second security?                               **[6]**

**Q.6**   i.   Show that the probability that a European call option on a stock will be exercised in a risk neutral world is $\Phi(d_2)$. What is an expression for the value of a derivative on the stock that pays off Rs. 300 if the price of the stock is greater than exercise price (K) and zero otherwise? Assume that the stock price follows geometric Brownian motion.   (5)

     ii.   A stock price follows geometric Brownian motion with an expected return of 20% per annum and a volatility of 30% per annum. The current price of the stock is Rs. 1900.

        (a)     What is the probability that a European call option on the stock with an exercise price of Rs. 2000 and a maturity date in three months will be exercised?

        (b)     What is the probability that a European put option on the stock with the same exercise price and maturity date will be exercised?            (5)
                                                               **[10]**

**Q.7** A financial institution has the following portfolio of over the counter options on US dollar:

| Type of Option | Option Delta | Option Gamma | Option Kappa | Size of Position |
|---|---|---|---|---|
| Put | -0.30 | 1.20 | 0.50 | -4000 |
| Call | 0.80 | 0.50 | 0.60 | -2000 |
| Call | 0.60 | 2.00 | 1.50 | -1000 |
| Call | 0.50 | 1.60 | 1.20 | -1000 |

Two traded options are available on US dollar. The first traded option is available with a delta of 0.50, a gamma of 1.00 and a kappa of 0.50. The second traded option is available with a delta of 0.20, a gamma of 0.60 and a kappa of 0.50. What is the position in the two traded options and in US dollar would make the portfolio delta, gamma, and kappa neutral? **[10]**

**Q.8** Rajesh, a manager at Infosys Technologies Limited, received 1000 shares of company stock as part of his compensation package. The stock currently sells for   Rs. 2000 a share. Rajesh would like to defer selling the stock until the next July. In August, however, he will need to sell all his holdings to provide for a down payment on his new house. Rajesh is worried about the price risk involved in keeping his shares. If the value of his stock holdings falls below Rs. 1,900,000, his ability to come up with the necessary down payment would be jeopardized. On the other hand, if the stock value rises to Rs. 2,100,000, he would be able to make a small cash reserves even after making the down payment. Rajesh considers three investment strategies:

a) Strategy A is to write August call options on the Infosys shares with strike price Rs. 2100. These calls are currently selling for Rs. 80 each.

b) Strategy B is to buy August put on Infosys with strike price Rs. 1900. These options also sell for Rs. 80 each.

c) Strategy C is to write the August calls with strike price Rs. 2100 and to buy the January puts with strike price Rs. 1900.

Evaluate each of these strategies with respect to Rajesh's investment goals. What are the advantages and disadvantages of each? Which would you recommend?

**[10]**

**Q.9** Briefly explain the following with the help of an example:
a) Asian Options                                                                                                    (2)
b) Chooser Options                                                                                                (2)
c) Numeraires                                                                                                         (2)
d) Trinomial Trees                                                                                                (2)

**[8]**

**Q.10** You are an investment actuary specializing in derivatives in Timbaktoo. A friend of yours comes to you for advice. He is holding 1000 shares of Great Actuarial Consultacy (GAC) and he found that futures are trading at a substantial discount to the market. On hearing this, you asked your secretary to collect information on the same and she provided you the following information as on 1st Jan 2007.

GAC – Spot Price 650.00 / 652.00
GAC - Futures (1 month i.e. 31$^{st}$ Jan 2007) 590 / 594
Interest Rates (for all terms) – 5% / 6% (p.a. simple)
Ex-Dividend Date – 15$^{th}$ Jan 2007
Dividend per share - 45
Brokerage on Spot Transaction – 1% of the spot value
Brokerage on Future Transaction – 0.5% of the future price
Margin required at any point in time – 5% of the underlying value as atthe close of the day (assume 651)
For purpose of this question assume that futures will NOT be settled daily and margin will be released at the maturity along with the settlement.

Please advice your friend on the strategy of selling the stock and buying the future? Outline the risks your friend takes while implementing this strategy?
[Hint: 20/22 means you can sell at 20 but buy at 22 ]

**[12]**

**Q.11**  Your friend has taken you to Mars. You were surprised to find that stock markets on Mars are open only on the 1$^{st}$ of every month. Fortunately, you landed on one such day. You headed straight to the market and found that stock price of Universal Consulting (UC) moves only by Martian dollar (M$) 10 or by M$ 50. The probability of movement by M$10 is 70%. The interest rates were 20% per month compounded monthly for all terms. The current stock price for UC was M$100. You found an unusual European option on UC which was being traded at M$100 with a remaining term of 2 months. The payoff from this option was given by the formula given below:

$X_T = S_T^2$ x Max( $S_T$ – 150, 0) / 2000 where T represents the expiry date (maturity date) and $X_T$ represents the payoff on maturity.

a)  Will you buy the option? Why or why not?                                              (7)

b)  Show that for this tree
   a.  $E_Q(X_T) = E_P\left(\dfrac{dQ}{dP} X_T\right)$                                    (3)

   b.  $E_Q(X_T \mid F_s) = \varsigma_s^{-1} E_P(\varsigma_T X_T \mid F_s)$ where $\varsigma_t = E_P\left(\dfrac{dQ}{dP} \mid F_t\right)$ for T = 2    (5)

[Hint: Q stands for risk neutral probability measure whereas P stands for realistic probability measure]

**[15]**

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