

Actuarial Society of India

EXAMINATIONS

21st May 2007

Subject CT8 – Financial Economics

Time allowed: Three Hours (14.30 – 17.30 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

- 1) *Do not write your name anywhere on the answer sheets. You have only to write your Candidate's Number on each answer sheet.*
- 2) *Mark allocations are shown in brackets.*
- 3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
- 4) *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*
- 5) *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

“It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI.”

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheets and this question paper to the supervisor separately.

- Q.1)** Asset liability modeling entails quantifying the expected cost of guarantees as well as the capital requirements necessary to ensure solvency under extreme scenarios (e.g. 1 in 1000 year event).
- (i) (a) Assume that there is a guaranteed equity fund as part of an unit-linked insurance plan. Describe the limitations of the lognormal random walk process for modeling asset prices in the above instance, with respect to the homoscedasticity of this process. (2)
- (ii) Assume that the capital guarantee is now on a balanced fund in an unit-linked insurance plan, where the underlying portfolio comprises a mix of equities, corporate bonds, gilts and money instruments. As a trial set of data for the newly built ALM model, the Appointed Actuary has asked you to generate 2500 sets of 30 years worth of yearly future returns for each asset class, using a Lognormal Random Walk Model.
- (a) List the key model inputs and assumptions you would be required to make in order to complete the task (the variables, not the actual estimates). (3)
- (b) Describe how the recent dramatic acceleration in Indian economic growth might influence your decisions around the equity and gilt assumptions? (2)
- (c) The Appointed Actuary has also requested that you point out key shortcomings, aside from any data collection/availability/relevance issues, of adopting the lognormal random walk model in generating the required returns for the various asset classes suggested. These shortcomings should be over and above those alluded to in (i) (a) (3)
- (d) The Chief Investment Officer is skeptical of how, in an efficient market, a statistical asset model can be utilized to determine the optimal investment strategy of the underlying portfolio. Does he have grounds for concern? Give an example. (1)
- (e) Would this be any different if the Wilkie Model were used? Explain your answer in the context of the equity risk premium and the model's consistency with the weak form of the efficient markets hypothesis. (1)
- [12]**

Q. 2)

- (i) Draw a diagram to show the upper and lower bounds (ignoring transaction costs) on the possible value of an American Call option, as a function of the price of the underlying security. The bounds illustrated should be independent of any model for the price process followed by the underlying security. Define all notation used. (2)
- (ii) Explain the rationale behind the bounds shown (in terms of arbitrage profits). (2)
- (iii) Under what conditions might it be optimal to exercise an American Call Option ahead of expiry? (1)
- [5]**

Q. 3) The following are prices, very shortly before their October expiry date, for options in the Company X.

Company X (Current Share Price Rs 310)				
Exercise Price	Call Options Expiry Date		Put Options Expiry Date	
	October	January	October	January
300	A	B	C	D
330	E	F	G	H

(i) Give a rough estimate of A. Say which entries of B, C, D, E, F, G and H would you expect to be greater than A, less than A or which could be either. Give reasons. **[4]**

Q. 4)

(i) Define and describe the following partial derivative terms with respect to an option price $f(t, S_t)$ and the underlying share (S_t at time t) : Delta, Gamma, Vega and Theta **(4)**

(ii) (a) Describe the relevance of Gamma in ensuring a cost efficient dynamic delta hedging strategy **(1)**

(b) Explain why it is impossible to complete a Gamma-hedge using the underlying asset? **(1)**

(c) Explain the impact on a hedging program involving European call options where S_t is close to the strike price near maturity. **(1)**

(iii) Under what conditions is Theta equal to the risk free rate of growth on a portfolio? **(1)**

(iv) Use put-call parity to derive, for a dividend paying stock, the relationship between:

(a) the Delta of a European call and the Delta of a European Put

(b) the Gamma of a European call and the Gamma of a European Put

(c) the Vega of a European call and the Vega of a European Put

(d) the Theta of a European call and the Theta of a European Put **(2)**

[10]

Q. 5) Consider the following conditions:

Current asset price = ₹ 150

European call strike price = ₹ 160

European call maturity = 2 years

European put strike price = ₹ 145

European put maturity = 3 years

Asset price volatility = 15%

Risk-free force of interest = 1%

Dividend rate (cc) = 0

(i) Show that the values of the European call and put options are approximately 9.82 and 10.83 respectively. (8)

(ii) Assume α of the portfolio is invested in the above European calls
 β of the portfolio is invested in the above European puts
 γ of the portfolio is invested in the underlying asset
 The total portfolio value is Π

Assume also that the Delta and Gamma values for the call and put options are as follows:

$$\Delta \text{ call} = 0.4586$$

$$\Delta \text{ put} = -0.3535$$

$$\Gamma \text{ call} = 0.012470$$

$$\Gamma \text{ put} = 0.009539$$

Solve for α , β and γ such that the overall portfolio is delta and gamma hedged under the conditions in (i). Describe the portfolio composition. (3)

[11]

Q. 6 An asset has a current price of Rs 40. Over the next year, the price will either increase to Rs 48 with probability p , or decrease to Rs 38 with probability $(1-p)$. The effective risk-free rate of interest for the next year is r .

(i) Value a European Put on this asset with one year to maturity and a strike price of Rs 42, under the following scenarios:

(a) $r = 5\%$, $p=90\%$

(b) $r = 10\%$, $p = 90\%$

Use a different approach for each answer. (2)

(ii) In what direction is the value of the put likely to change under the following scenarios (justify your answers):

(a) r remains constant, but p increases

(b) r increases and p increases (explain in context of a replicating portfolio) (2)

(iii) Assume 4 scenarios where $r = 5\%$ and p is either 90%, 66%, 50% or 25% and another 4 scenarios where $r = 10\%$ and p is either 90%, 66%, 50% or 25%. Which of these 8 scenarios are plausible models of an investment universe populated by non-satiated risk-averse investors? Explain. (2)

[6]

- Q. 7)** A large listed Indian car manufacturer has significant sales in Europe, and hence its share price is positively correlated to the Rupee:Euro exchange rate.

The company's equity currently trades at Rs 100. If the Rupee/Euro trades at Rs 100 in one year, the share will trade at Rs 150. If the Rupee/Euro is at Rs 80 in one year, the share will trade at Rs 130m and if it is at Rs 60, the share price will remain unchanged.

The current Rupee:Euro exchange rate is Rs 70/€. In one year's time, the exchange rate will either be Rs 100/€, Rs 80/€ or Rs 60/€. Interest rates in India are 12% and EU 3%.

A French Mutual Fund wants to profit from the upside potential on the equity stock price movements of this car manufacturer. Hence it has decided to purchase a European Call on the equity stock of this company with one year to expiry and a strike price of €1.4.

- (i) The Fund Manager wants you to value the option by constructing a portfolio of physical assets (i.e. Rupees cash, shares in the Indian Company, and Euros cash) that would replicate the pay-off profile of the option.

Write down and solve the system of linear equations under the three possible outcomes such that the portfolio value will always have the same profile as the call option. (4)

- (ii) Find the arbitrage free price of the portfolio and comment on the nature of your results (2)

[6]

- Q. 8)** On 1 January 2007, you have two sets of interest rate data relating to government-issued instruments available to you:

Maturity	Zero Coupon Bond Price	Date	Short rate
1	0.934649	1 Jan 2006	6.32%
2	0.873633	1 Feb 2006	8.038%
3	0.817517	1 Mar 2006	7.799%
4	0.766170	1 April 2006	9.877%
5	0.719183	1 May 2006	9.103%
6	0.676067	1 June 2006	7.557%
7	0.636355	1 July 2006	4.519%
8	0.599629	1 Aug 2006	5.605%
9	0.565531	1 Sept 2006	6.945%
10	0.533765	1 Oct 2006	5.605%
		1 Nov 2006	6.177%
		1 Dec 2006	6.016%
		1 Jan 2007	6.701%

Assume $\alpha = 0.25$.

- (i) Fit a Vasicek model to the yield curve data, stating any assumptions made. (4)
- (ii) The SDE for the Vasicek model is the same as for the Ornstein-Uhlenbeck process. Use this to determine the distribution for $r(t+\Delta t)$ at time t for the Vasicek model. (2)
- (iii) Using the derived distribution in (ii), describe how you would fit the Vasicek model on 1 Jan 2007 to the short rate history given. Assume investors are risk neutral. Describe the methodology and assumptions made, do not perform the calculations. (4)
- (iv) Write down the stochastic differential equation for the Cox-Ingersoll-Ross model and use it to explain, in the context of the Vasicek SDE, why the C-I-R model eliminates a key problem with the Vasicek model. State any assumptions necessary. (2)
- (v) Explain how the C-I-R model, and its assumption of no negative short rates, could lead to under-estimating interest rate exposure. (2)
- [14]**

Q. 9)

- (i) List the possible outcomes of a default on a corporate bond. (2)
- (ii) Describe how the Jarrow-Lando-Turnbull (J-L-T) credit risk model is an example of a “reduced form model”. (2)
- (iii) How might the J-L-T model be generalized to allow for changes in credit spreads under different economic conditions? Give a practical example of such a change (2)
- [6]**

Q. 10)

- (a) Define single-index model and express it in the form of an equation defining all the terms? (2)
- (b) Calculate the expected return, variance of return on security i and the covariance of the returns on the securities i and j ? (8)
- [10]**

Q. 11) Consider a zero-coupon corporate bond that promises to pay a return of 12% next period. Suppose that there is a 20% chance that the issuing company will default on the bond payment, in which case there is an equal chance of receiving a return of either 8% or 0%.

- (i) Define “shortfall probability”? (1)

- (ii) Calculate values for the following measures of investment risk:
- (a) downside semi-variance
 - (b) shortfall probability based on the risk-free rate of return of 8.5%
 - (c) the expected shortfall below the risk-free return conditional on a shortfall occurring. (5)
- [6]

Q. 12)

- (i) Describe briefly the three forms of the efficient market hypothesis and the relationship between them? (4)
 - (ii) What are the major difficulties involved with testing for informational efficiency? (4)
 - (iii) Describe three examples of events to which investment markets sometimes appear to under-react. (2)
- [10]
