Actuarial Society of India

EXAMINATIONS

18th May 2007

Subject CT6 – Statistical Methods

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer sheets. You have only to write your Candidate's Number on each answer sheets.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheets and this question paper to the supervisor separately.

Q. 1)		
(i)	Explain the term "Insurable Interest".	(2)
(ii)	State any three criteria that a risk event should ideally meet in order to be insurable.	(3)
		[5]
Q. 2)		
(i)	The prior distribution of the probability of Head in a random toss of a coin is has density $2p, \quad 0 < p < 1$. It took five tosses of the coin to get the first Head. What is the posterior mean of the probability of Head?	(4)
(ii)	The aggregated claims from a portfolio of insurance policies are $X_1,....,X_n$ for the years 1,…,n, respectively. The aggregate claim X_{n+1} for the year n+1 has to be estimated. It is known that , given a fixed value of the random parameter θ, the claims $X_1,.....,X_{n+1}$ are conditionally independent and normally distributed with mean θ and variance θ^2. The prior distribution of the parameter θ is exponential with mean μ . (a) Obtain the unconditional mean of X_{n+1}. (b) Obtain the unconditional variance of X_{n+1}. (c) Find an estimate of θ which is a linear function of $X_{1,}....,X_n$ and μ, and minimizes the mean squared difference between θ and the linear estimate.	 (1) (2) (5)
		[12]
Q. 3)		
(i)	Describe the insurance cover provided under each of the following two category of products: a. Liability b. Property Give two example of insurance cover in each of the above two categories.	(4)
(ii)	The table below shows the cumulative incurred claims on a portfolio of general insurance polices:	

Accident Year	Delay Year		
2004	2,748	3,819	3,991
2005	2,581	4,014	
2006	3,217		

	The company decides to apply the Bornhuetter-Ferguson method to calculate the reserves, with the assumption that the Ultimate Loss Ratio is 85%. Calculate the reserve at the end of the year 2006, if the earned premium is 5,012 and the paid claims are 1,472.	(6)
		[10]
Q. 4)	What do you mean by "Statistical games"? Explain how statistical games differ from the game theory.	**[4]**
Q. 5)	The no claims discount (NCD) system operated by an insurance company has three levels of discount: 0%, 25% and 50%. If a policyholder makes a claim they remain at or move down to the 0% discount level for two years. Otherwise they move up a discount level in the following year or remain at the maximum 50% level. The probability of an accident depends on the discount level: *Discount Level* — *Probability of accident* 0% — 0.25 25% — 0.2 50% — 0.1 The full premium payable at the 0% discount level is 750. Losses are assumed to follow a lognormal distribution with mean 1,451 and standard deviation 604.4. Policyholders will only claim if the loss is greater than the total additional premiums that would have to be paid over the next three years, assuming that no further accidents occur.	
(i)	Calculate the smallest loss for which a claim will be made for each of the four states in the NCD system.	(2)
(ii)	Determine the transition matrix for this NCD system.	(6)
(iii)	Calculate the proportion of policyholders at each discount level when the system reaches a stable state.	(3)
(iv)	Determine the average premium paid once the system reaches a stable state.	(1)
(v)	Describe the limitations of simple NCD systems such as this one.	(2)
		[14]
Q. 6)		
(i)	In a group of policies, the monthly number of claims for a single policy has a Poisson distribution with parameter λ, where λ is a random variable with the following density function: $f(\lambda) = 2e^{-2\lambda}, \lambda > 0.$	

	Calculate the probability of n claims on a policy picked at random from the group of **claims**. (Policyes)	(3)
(ii)	Find the moment generating function for the aggregate claims distribution if the claims have a gamma distribution with mean 2 and variance 2.	(5)
		[8]
Q. 7)		
(i)	Claims occur as a Generalized Pareto distribution with parameters $\alpha = 6$, $\lambda = 200$ and $k = 4$, A proportional reinsurance arrangement is in force with a retained proportion of 80%. Find the mean and variance of the amount paid by the insurer and the reinsurer on an individual claim.	(5)
(ii)	The loss from a risk has the exponential distribution with mean μ. An insurer covering the risk has offers from two competing reinsurers. The first reinsurer would provide excess of loss reinsurance with retention limit μ. The second reinsurer would provide proportional reinsurance. The expected value of the direct insurer's share of loss is the same under both the reinsurances. Which reinsurance will result in smaller variance of the direct insurer's share of loss?	(6)
		[11]
Q. 8)	You are using simulation to find the mean and variance of a random variable that has the density function, $$f(x) = \frac{k}{1+e^{-x}} \quad (-1 < x < 1).$$	
(i)	Determine the value of the constant k	(3)
(ii)	Generate five pseudo-random values from this distribution based on the U(0,1) values 0.017, 0.757, 0.848, 0.531 and 0.321.	(4)
(iii)	You have generated 1,000 such numbers and found that $\sum x = 165.681$ and $\sum x2 = 339.275$. Use these statistics to find a 95% confidence interval for the mean of the distribution and a point estimate for the standard deviation.	(4)
		[11]
Q. 9)	A moving average time series is defined by the relationship, $$X_t = \varepsilon_t + 0.25\varepsilon_{t-1} + 0.5\varepsilon_{t-2} + 0.25\varepsilon_{t-3}$$ Where $\varepsilon_t \sim N(0,\sigma^2)$ denotes white noise.	
(i)	Find the mean and variance of X_t	(2)
(ii)	Calculate the autocorrelation function ρ_k, at all lags k.	(4)

(iii)	Identify *p*,*d* and *q* for the ARIMA(*p*,*d*,*q*) process $\{Y_t\}$ where Y_t is defined by $Y_t = 0.5\, Y_{t-1} + 0.5\, Y_{t-2} + X_t$.	(4)
		[10]
Q. 10)	An insurer covers a portfolio of risks where claims arrive as a Poisson process at the rate of one per year, and the claim size is Rs. 1000 (fixed). The insurer has an initial asset of Rs. 400 and receives premium at the rate of Rs. 1200 per year. Determine the probabilities of	
(i)	no claim in the first year,	(1)
(ii)	exactly one claim in the first year,	(1)
(iii)	ruin within the first year.	(3)
		[5]
Q. 11)	A random variable X has density of exponential family form: $$f(x) = \frac{1}{\mu} e^{-x/\mu} \qquad (x > 0).$$	
(i)	Show that $f(x)$ can be written in the form of an exponential family of distributions, and identify the canonical parameter.	(3)
(ii)	Determine the variance function and hence determine the variance of *X*.	(3)
(iii)	Let *Y* be another random variable defined as $Y = X + \alpha$, where *X* has the density *f* with $\mu = 1$, and α is an unspecified parameter. Can the density of *Y* be written in the form of an exponential family? Explain.	(4)
		[10]

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