Actuarial Society of India

EXAMINATIONS

16th May 2007

Subject CT4– Models

Time allowed: Three Hours (10.00 am – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Do not write your name anywhere on the answer sheet/s. You have only to write your Candidate's Number on each answer sheet/s.***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***
5. ***In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer sheets and this question paper to the supervisor separately

Q. 1) The experience is to be graduated by reference to the standard table, using a range of possible functions, including the following:

$$(1) \quad \tilde{q}_x = q_x^s(\alpha + \beta x)$$

$$(2) \quad \tilde{q}_x = \alpha\, q_x^s + \beta x$$

where (q_x) is the graduated rate of mortality at age x.

(a) What is graduation and Explain why it might be necessary to graduate an experience several times, using different functions, in the way described here. (3)

(b) Obtain and simplify the simultaneous equations required in order to find the values of the parameters "α" and "β" for function (1), using a weighted least-squares method. (3)

(c) It has been suggested that a more successful graduation would be achieved by using mathematical formulae which do not include any reference to a standard table. Discuss the merits of this suggestion. (2)

[8]

Q. 2) A small life insurance company established its operation 5 years back and the number of pension policies on its books at the end of march each year is given below

Date	***Number of pension policies***
31 March 2003	**14,937**
31 March 2004	**25,173**
31 March 2005	**35,298**
31 March 2006	**45,473**
31 March 2007	**55,601**

i). Describe how you would use a simple mathematical model to project the number of pension policies as on 31 march 2008. You are not required to fit any model. (3)

ii). Outline the limitations of using the model in (i) to project the number of pension policies beyond 31 March 2008. (2)

iii). It is further assumed that the number of pension policies will be approaching a maximum level by the year 2020. Show how you would adapt the model to take into account the assumption that the maximum number of pension policies will be 1,00,000. (3)

[8]

Q. 3) i). Define the Standardised Mortality Ratio (SMR) and show that it can be expressed as a weighted average of the ratio of the age-group specific mortality rates in the study population to those in the standard population. Describe the weights used in the calculation of this average. (3)

ii). You are a member of a committee responsible for monitoring the trend in assured lives mortality rates. You have been presented with the following ratios of actual to expected mortality rates on the basis of a standard table constructed twenty years ago, and the total expected deaths over the period 2003–2006 based on the same table. (2)

Age	Ratio of Actual to Expected Mortality Rates		Total expected deaths for the period 2003-2006 (thousands)
	2003 and 2004	2005 and 2006	
15–44	1.70	2.00	10
45+	0.85	0.75	30
			Total 40

Calculate the SMR for each period.

iii). The committee is in the habit of measuring the change in mortality by calculating a "Comparative Mortality Factor" (CMF). This is calculated "by applying the observed mortality rates to a standard population and then comparing the result to that obtained when the expected mortality rate is applied to the same population". In this case the standard population was that upon which the standard table was based on twenty years ago. The CMF for the two periods were 0.94 and 0.88 respectively, which lead the committee to conclude that mortality is improving. Explain the difference between the results of your SMR calculation and these figures. Indicate, with a reason, which you think provides the better results. (3)

[8]

Q. 4) i). Suppose that N lives are observed between exact ages x and $x + 1$.Some of these lives possess more than one life insurance policy; let the proportion with i policies be π_i ($i = 1, 2, 3, ...$). Let C_i be the number of policies under which claims are made. Show that

$$E[C_i] = \sum_{i=1}^{\infty} i\pi_i N q_x$$

and

$$\mathrm{Var}(C) = \sum_{i=1}^{\infty} i^2 \pi_i N q_x (1 - q_x)$$

(4)

ii). Suppose instead that the total number of lives observed is

$$\sum_{i=1}^{\infty} i\pi_i N$$

and that each life possesses one life insurance policy. Let C_2 be the number of policies under which a claim is made. Show that

(a) $E[C_2] = E[C_1]$

(b) $$\frac{\text{Var}[C_1]}{\text{Var}[C_2]} = \frac{\sum_{i=1}^{\infty} i^2 \pi_i}{\sum_{i=1}^{\infty} i \pi_i}$$ (3)

iii). State the consequence of the result in (ii)(b) above for the statistical testing of a graduation of crude rates of mortality based on observations of policies rather than lives. (1)

[8]

Q. 5) An investigation of the mortality of assured lives has provided the following data:

θ_x number of deaths during the investigation period aged x last birthday,

E_x initial exposed to risk of death contributed during the investigation by lives while aged x last birthday,

q_x^s mortality rate at age x according to the A1967/70 Ultimate mortality table (the standard table).

The following is an extract from the data:

Age x	θ_x	$E_x \; q_x^s$
45	24	20.35
46	17	18.61
47	15	13.09
48	44	31.32
49	12	10.77
50	8	11.24
51	40	48.38
52	41	36.25
53	27	42.98
54	50	66.31
55	42	59.27
56	13	25.30
57	88	74.41
58	76	91.65
Total	**497**	**549.93**

i) Test the hypothesis that the true underlying mortality rates of the experience are equal to those defined by the A1967/70 Ultimate mortality table, using the following statistical tests:

(a) the chi-squared test,
(b) the cumulative deviations test, (6)
(c) the grouping of signs test

(2)

ii) State clearly the conclusions that you draw from your investigation.

[8]

Q. 6) You are required to investigate the mortality experience of a large portfolio of whole life and temporary assurance policies. The following information is available:

(A) the number of policies in force at each 1 January from 1 January 1991 to 1 January 2006 inclusive, subdivided by age nearest birthday and curtate duration on the relevant 1 January (2)

(B) for each calendar year from 1991 to 2006 inclusive, the number of deaths, subdivided by age nearest birthday at death and curtate duration at death

i. State, with reasons, the parts of the policy data, or the special groups of lives, which you consider should be excluded or investigated separately.

ii. Derive formulae for crude estimates of *m*-type select mortality rates using the given data. Obtain the ages and durations to which your rates apply. You should define all symbols used and state all assumptions made. (8)

[10]

Q. 7) Answer the following:

i). Write down the conditional distribution of [N (t + s) - N (t)] given Ft, where $s > 0$ and $\{N(t): t \geq 0\}$ is a Poisson process with rate λ and $\{Ft: t \geq 0\}$ is the filtration associated with N. (2)

ii). Use your answer in i), to find $E(\theta^{N(t+s)}|Ft)$. (1)

iii). Define S = {000, 001, 010, 011, 100, 101, 110, 111} where each state represents one corner of a cube in terms of the {x, y, z} co-ordinates. Obtain the transition matrix of the Markov chain in which from any corner, independent of the past, the next transition is equally likely to any of the three adjacent corners. (3)

iv). Recalculate the transition probability matrix as obtained in part iii) for the case when the probability is inverse to the distance traveled. (Note you can assume that the distance is measured in a straight line from the start of the journey). (4)

[10]

Q. 8) For a particular data entry operator, any period j is either error free (Yj = 0) or gives rise to one error (Yj = 1). The probability of having no error in the next period is estimated using the operator's past record as follows (all values of Yj are either 0 or 1):

$$P\,[Yn+1 = 0|Y1 = y1,\ Y2 = y2,\ ....,\ Yn = y_n] = pe^{-\lambda(\,y1+y2+...+yn)}$$

where $0 < p < 1$ and $\lambda \geq 0$. The cumulative number of errors committed by the operator over the time period 1 to n is given by

$$Xn = \sum_{j=1}^{n} Yj\ .$$

i. Verify that the Markov property holds for the sequence $\{Xn : n \geq 1\}$. (2)

ii. Explain why the sequence $\{Yn : n \geq 1\}$ does not form a Markov chain. (1)

iii. Write down the transition matrix for the Markov chain X. (2)

iv. State with reasons the following:

(a) Whether the Markov chain X is time homogeneous.

(b) Whether the Markov chain X is irreducible.

(c) Whether the Markov chain X admits a stationary probability distribution. (3)

v. Starting from state $X_t = j$, calculate the probability of making no further errors for the next n periods by the operator. (2)

[10]

Q. 9) The levels of discount in car insurance scheme for no claims discount are given as 0%, 15%, 30% and 40%. If a driver does not make a claim in any one year, he moves up a level (for example from 15% to 30%); if he is already at 40% he stays there. If he makes a claim, he moves down one level; if he is already at 0% he stays there.

A somewhat reckless driver has a probability of 0.5 of making a claim in any one year. Further, the full premium is Rs. 20,000/- and the driver is currently at the 0% level.

Calculate

a. The average premium he pays per year in the long run. (4)

b. The expected time it will take him to reach the 40% level for the first time. (4)

c. The expected value of the total amount of premiums he will pay till he reaches the 40% level for the first time. (4)

[12]

Q. 10) The price of OPEC crude is referenced relative to their initial value such that at the start time (t= 0) the value of the OPEC crude is also 0. Further, note that the prices are measured at discrete points in time and at each discrete point in time, the OPEC crude price increases in value by one unit of currency with probability p or decreases in value by one unit of currency with probability q = 1-p.

Let $p_0(n)$ denote the probability that OPEC crude is at its original value after n units of time and $f_0(n)$ be the probability that the value of OPEC crude has returned to their original value for the first time after n units of time. Also, define

$$P_0(s) = \sum_{n=0}^{\infty} p_0(n)s^n \qquad \text{and } F_0(s) = \sum_{n=0}^{\infty} f_0(n)s^n.$$

Now,

a) Show that $P_0(s) = 1 + P_0(s)F_0(s)$ (4)

b) Show that $P_0(s) = (1 - 4pqs^2)^{-1/2}$ (4)

c) Show that $F_0(s) = 1 - (1 - 4pqs^2)^{½}$ (4)

d) Having proven the three expressions given above, deduce the probability that the price of OPEC crude will ever return to its original value. (3)

e) Also, what is special about the case p = 1/2 (3)

[18]

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