Actuarial Society of India

EXAMINATIONS

22nd June 2005

Subject ST6 – Finance and Investment B

Time allowed: Three Hours (2.15* - 5.30 pm)

INSTRUCTIONS TO THE CANDIDATE

1. ***You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.***

2. ***You must not start writing your answers until instructed to do so by the supervisor.***

3. ***The answers are not expected to be any country or jurisdiction specific. However, if examples/illustrations are required for any answer, the country or jurisdiction from which they are drawn should be mentioned.***

4. ***Mark allocations are shown in brackets.***

5. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

6. ***Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.***

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

AT THE END OF THE EXAMINATION

Hand in BOTH your answer script and this question paper to the supervisor.

Q.1 A one step binomial "tree" model with a time step δt is being used to value a European derivative on a non-dividend-paying share. The initial share price is S_0. The share price at expiry is assumed to be either S_0u or S_0d, where $d < e^{räô} < u$ and r is the risk-free force of interest. The corresponding payoffs from the derivative are h_u and h_d.

(a) Determine the constituents of the replicating portfolio for this derivative, based on this model. **[2]**

(b) Stating any assumption you make, deduce that the initial value of the derivative is:

$$V_0 = \frac{qh_u + (1-q)h_d}{e^{rdt}} \text{ where } q = \frac{e^{rdt} - d}{u - d}$$

[3]

(c) Explain the relationship between the result in (b) and the derivative pricing formula:

$$V_t = e^{-r(T-t)} E_Q\left[H \mid F_t\right]$$

[3]

(d) Explain clearly why the condition that $d < e^{räô} <$ u is imposed. **[2]**

Total [10]

Q.2 The increase in the price of a share over the next year is believed to have a mean of 20% and a standard deviation of 15%.

a) Determine the value of u and d for a one-step binomial tree model that are consistent with the mean and standard deviation of the return on the underlying share, assuming that the share price is thrice as likely to go up than to go down. **[3]**

b) Hence calculate the value of each of the following options, given that the current share price is 300, the risk-free force of interest is 6% per annum and dividends can be ignored:

i. a one-year European call option with a strike price of 325

ii. a one-year European put option with a strike price of 325.

[5]

c) Verify numerically that the put-call parity relationship holds in this case. **[2]**

Total [10]

Q.3

a)

i) State (without derivation) the partial differential equation derived from Ito's lemma that is satisfied by the prices of derivatives in a Black-Scholes world, defining the variable concerned. **[3]**

ii) State the main assumptions underlying this equation. **[3]**

iii) State the issues relating to σ when this model is used in practice and how these are addressed. **[3]**

b) State the main constraints on the behaviour of a stock price for Ito's lemma to be applicable. **[3]**

Total [12]

Q.4

a) Explain why the value of a floating-rate par bond immediately after an interest payment date will equal the nominal amount.

[2]

b) In the context of interest rate caps and floors, explain what is meant by:

i. the "tenor" of a contract

ii. "caplets" and "floorlets". **[4]**

Total [6]

Q.5 The following are the prices of the four most actively traded bonds in a small island economy. All bonds are redeemable at par (1000) and pay annual coupons of 8%, except for Bond B, which is a zero-coupon bond.

	Term	Price
Bond A	1	1017.11
Bond B	5	697.68
Bond C	6	1115.07
Bond D	15	1376.07

a) Estimate the 1-year and 6-year spot rates, expressing your answers as continuously-compounded rates. **[2]**

It is known that the instantaneous forward rates for terms up to 6 years is constant with a value f and that the instantaneous forward rate for terms of more than 6 years is constant with a f^*.

b) Estimate f and f*. **[3]**

c) Hence sketch graphs, on the same diagram, of the instantaneous forward rate and the spot rate. **[2]**

d) Comment on the pattern of the rates and suggest a possible reason for the shape of your graph. **[2]**

Total [9]

Q.6

a) Distinguish between the credit risks faced by:

i. the writer and the purchaser of an over-the-counter option

ii. the two counterparties to an over-the-counter forward contract.

[4]

b) Discuss how the credit risks in a(ii) could be reduced by:

i. reducing the probability of default

ii. reducing the underlying exposure

[2]

c) "Sunshine Pension Fund" has entered into an agreement with XYZ bank, which is an investment bank whereby Sunshine Pension Fund will transfer ownership of a zero-coupon bond to XYZ bank in return for cash. In three days time the XYZ bank will return the zero-coupon bond to Sunshine Pension Fund in return for the initial cash plus interest at the repo rate.

Discuss the credit risks faced by XYZ Bank and suggest ways in which they could be reduced. **[5]**

Total [11]

Q.7 Consider a one-year European call and put options on Company X stock when the stock price is \$30, the strike price is \$30, the risk-free rate is 5% per annum and the volatility is 25% per annum

a) Calculate the delta of call and put options. Assume that prices of options on Company X stock follow the Black-Scholes model **[1]**

b) Suppose that an investor holds 200 shares in Company X, and that he decides to delta hedge these shares using the put option. How many put options does he require? Does he need to buy or sell them? **[2]**

c) Suppose that another investor holds 300 shares in Company X, and that she decides to delta hedge these shares using the call option. How many call options does she require? Does she need to buy or sell them? **[2]**

Total [5]

Q.8 An oil consumer has a requirement for 80,000 barrels of oil and has purchased 20 futures contracts.

An oil speculator has purchased 40,000 barrels of oil and has a short holding of 30 futures contracts.

Each futures contract is for 2,000 barrels.

a) Calculate the hedge ratio for each of these portfolios. **[2]**

b) The volatilities of the spot price and the futures price over the relevant period are $\sigma_s = 2$ and $\sigma_f = 3$. The correlation between these prices is 0.78. **[2]**

Calculate the minimum variance hedge ratio.

c) Comment on your answers to (a) and (b) and also on how each party would be affected by an adverse movement in the spot price. **[3]**

Total [7]

Q.9 You are an investment advisor to a merchant bank. Your team has observed that the instantaneous forward rate curve implied by the current prices of bonds has the following properties:

- the short rate is 5%
- the long rate is 6%
- the highest rate occurs for bonds with a term of 15 years.

You wish to fit a Vasicek model to $f(\tau)$, the τ - year forward rates, using the following formula:

$$f(t) = \mathrm{R}e^{-at} + L(1 - e^{-at}) + \frac{b}{a} e^{-at}(1 - e^{-at})$$

a) State the values of the parameters R and L that are consistent with your observations.
[2]

b) Show that, in order for the curve to have a stationary point at $\tau = 15$, the following must be satisfied:

$$b = \frac{(R - L)a}{2e^{-15a} - 1}$$

[3]

c) For each of the cases $\alpha = 0.06$ and $\alpha = 0.04$, evaluate the corresponding values of β from the equation in (b) and the values of f (15). Use the values of the parameters R and L states in (a). **[2]**

State the properties and limitations of the Vasicek model that are illustrated by your calculations in (c). **[5]**

Total [12]

Q.10 You are investment advisor to Mukund Housing Society (MHS), which has issued recently a two year bond that entitles the holder to the return on the Nifty index up to a maximum level of 40% growth over the two year period. The bond has a guaranteed minimum level of return so that investors will receive at least x% of their initial investment back. Investors cannot redeem their bonds prior to two years. Assume that the continuously-compounded risk-free interest rate over this period is 6% per annum and that the initial value of NSE Nifty is 2000.

a) Explain how MHS can hedge its position using a combination of call and put options on the index, stating the quantities of the securities involved.
[3]

b) Determine (to the nearest whole number percentage) the value of x at which MHS makes neither a profit nor a loss.

Assume that prices of options on the index follow the Black-Scholes model based on a volatility of 20% per annum. Ignore dividends and expenses. **[5]**

c) MHS decides to issue bonds each requiring an initial investment of Rs.1000 and to set the guarantee level at 90%. Calculate, based on the same assumptions, the initial profit MHS expects to make per bond sold. **[2]**

d)

i) State why MHS has imposed the restriction on redemption before the end of the two years.

ii) Discuss the alternatives for MHS if it did permit early redemption.

[4]

e) Outline the remaining sources of risk MHS is exposed to. **[2]**

f) Explain why MHS does not need to make an assumption about the growth of the Nifty index in determining x. **[2]**

Total [18]

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