Actuarial Society of India

EXAMINATIONS

20th June 2005

Subject CT8 – Financial Economics

Time allowed: Three Hours (2.30 – 5.30 pm)

INSTRUCTIONS TO THE CANDIDATES

1. *Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.*

2. *Mark allocations are shown in brackets.*

3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4. *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5. *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1   Consider a Zero-coupon corporate bond that promises to pay a return of 8 % next period. Suppose that there is a 10% chance that the company will default on the bond payment, in which case there is an equal chance of receiving a return of 4% or 0%.

a)   Calculate values for the following measures of investment risk:

(1) Downside semi variance

(2) Shortfall probability based on a risk free rate of return of 5% pa

(3) The expected shortfall below the risk free return conditional on a short fall occurring   **[6]**

b)   Discuss the usefulness of "down side semi variance" as a measure of investment risk.   **[2]**

Total [8]

Q.2   **a)**   Explain "Market Price of Risk" in the context of the Capital Asset Pricing Model.   **[3]**

b)   When is it optimal to consider only the mean –variance of return from a portfolio when choosing investments?   **[3]**

c)   Show that the CAPM result can be written as a single index model; and hence it is consistent with the Arbitrage Pricing Theory.   **[4]**

Total [10]

Q.3   **a)**   Distinguish between the lognormal model of security prices and Wilkie model with respect to consistency with market efficiency.   **[5]**

b)   Find the mean and variance of the stochastic integral

$$I = \int_0^1 t \, dB_t$$

[3]

Total [8]

Q.4   **a)**   If U [w] denotes an investors' utility function, where w denotes the level of wealth, state the conditions [in terms of derivatives of the utility function] for

(i)   Absolute risk aversion

(ii) Relative risk aversion   **[1]**

b)   State the assumptions underlying the Mean Variance Portfolio theory.   **[2]**

c)   The shares of the two corporations namely J and S have the following risk and return statistics:

$$E(r_j) = 14\% \qquad E(r_s) = 16\%$$
$$\sigma_j = 22\% \qquad \sigma_s = 25\%$$
$$\rho_{JS} = 0.5$$

Determine the minimum risk portfolio. What are the expected return and standard deviation of that portfolio?   **[8]**

Total [11]

Q.5   **a)**   Let B_t [t $\geq$0] be a Standard Brownian Motion process starting with B_0 =0

i)   What is the probability that B_2 takes a positive value?   **[1]**

ii) What is the probability that B_2 takes a value in the interval [-1, 1]?   **[2]**

iii) Prove that the probability that B_1 and B_2 both take positive values is 3/8.   **[4]**

iv) Show that if B_{50} is 3.04, the probability that B_{100} is negative is approximately 1/3.   **[1]**

b) Use *Ito's lemma* to show that the continuous‑time lognormal model of security prices can be derived from the assumption that the security prices follow a geometric Brownian motion process. **[5]**

Total [13]

Q.6 **a)** What is meant by saying that the market is "arbitrage‑free"? **[1]**

b) The increase in the price of a share over the next year is believed to have a mean of 10% and a S.D of 10%

  i) Determine the values of u and d for a one‑step binomial tree model that are consistent with the mean and standard deviation of the return on the underlying share, assuming that the share price is twice as likely to go up than to go down. **[5]**

  ii) Hence calculate the value of each of the following options:
  - a one year European Call Option with a strike price of 275.
  - a one year European Put Option with a strike price of 300

  For this purpose, assume a current share price of 250 and the continuously compounded risk free rate of interest to be 7.5% pa. Also assume that dividends can be ignored. **[6]**

Total [12]

Q.7 **a)** What is meant by "volatility" of share prices? **[1]**

b) One of the assumptions underlying the Black and Scholes option pricing model is that the price of the asset underlying follows a geometric Brownian motion Discuss briefly what this assumption means; and why this assumption may not be valid in practice. **[4]**

c) An European put option on a share has one month to expiry and an exercise price of 2.00. It is assumed that the share price at expiry will be either 1.50 or 2.50. The continuously compounded risk‑free interest rate is 1 percent per month.

  i) Use a hedging argument to value the put option assuming that the underlying share is currently priced at 2.19. **[4]**

  ii) Use risk neutral valuation to value the put option assuming that the underlying share is priced currently at 2.21 **[2]**

  iii) Hence estimate the options delta when the share is priced at 2.20. **[1]**

Total [12]

Q.8 **a)** Explain what is meant by a one‑factor model of interest rates and briefly discuss the limitations of such models **[6]**

b) The Redington model of interest rates assumes that the force of interest takes a value that is independent of the term of investment. Explain the advantages and disadvantages of this model. **[4]**

c) S_t denotes the price of a security at time t. The discounted security process $e^{-rt} S_t$ [where r denotes the continuously – compounded risk-free interest rate] is a martingale under the risk‑neutral measure Q.

B_t denotes the accumulated value at time t of an initial investment of 1 unit of cash.

    i) Write down an expression for B_t **[1]**

    ii) Show that the discounted cash process is also a Q–martingale.   **[2]**

    iii) Deduce that the discounted value of any self-financing portfolio

    [where transactions are made only by switching funds between the security and cash, with no addition or withdrawals of funds from the portfolio] will also be a Q-martingale?   **[3]**

Total [16]

Q.9 **a)** Explain the difference between a recombining and a non-recombining binomial tree   **[2]**

    b) Explain briefly Wilkie model of investment returns   **[4]**

    c) A European call option and a European put option each have a strike price of 105. Find the expected pay-off for these options if the share price at expiry can be assumed to be uniformly distributed between 90 and 110.   **[4]**

Total [10]

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